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Rapid surface-water volume estimations in beaver ponds

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Abstract. Beaver ponds are surface-water features that are transient through space and time. Such qualities complicate the inclusion of beaver ponds in local and regional water balances, and in hydrological models, as reliable estimates of surface-water storage are difficult to acquire without time- and labour-intensive topographic surveys. A simpler approach to overcome this challenge is needed, given the abundance of the beaver ponds in North America, Eurasia, and southern South America. We investigated whether simple morphometric characteristics derived from readily available aerial imagery or quickly measured field attributes of beaver ponds can be used to approximate surface-water storage among the range of environmental settings in which beaver ponds are found. Studied were a total of 40 beaver ponds from four different sites in North and South America. The simplified volume–area–depth (V–A–h) approach, originally developed for prairie potholes, was tested. With only two measurements of pond depth and corresponding surface area, this method estimated surface-water storage in beaver ponds within 5% on average. Beaver pond morphometry was characterized by a median basin coefficient of 0.91, and dam length and pond surface area were strongly correlated with beaver pond storage capacity, regardless of geographic setting. These attributes provide a means for coarsely estimating surface-water storage capacity in beaver ponds. Overall, this research demonstrates that reliable estimates of surface-water storage in beaver ponds only requires simple measurements derived from aerial imagery and/or brief visits to the field. Future research efforts should be directed at incorporating these simple methods into both broader beaver-related tools and catchment-scale hydrological models.

1 Introduction

The volume of water stored at the surface of wetlands, ponds, and lakes (as a function of stage) is of great concern to those responsible for assessing risks and balancing water supplies to societal demands. Arriving at reliable estimates of such storage is difficult without some knowledge of the feature’s morphometry, i.e. information that is often time consuming and impractical to acquire, especially when the features are numerous and transient through space and time (Milly et al., 2008). This is particularly true for beaver ponds owing to their cyclic creation and abandonment.

Beaver dams and their associated ponds are ubiquitous in streams and wetlands in the Northern Hemisphere and southern South America (Whitfield et al., 2015). Beaver dam densities have been reported to exceed 40 dams per kilometre (Macfarlane et al., 2017), making them one of the most frequent obstructions to flowing water (Naiman et al., 1986; Pollock et al., 2003). Beaver dams increase the open-water area within watersheds (Hood and Bayley, 2008) and ponds bring numerous ecosystem benefits (Johnston, 2012), but beaver ponds can also be viewed as burdensome or even dangerous from an anthropomorphic perspective (Butler and Malanson, 2005; Green and Westbrook, 2009). Such concerns, whether positive or negative, generally centre around the pond’s capacity to store water and sediment, highlighting the need for quick and accurate surface-water storage estimation methods.

Numerous hydrological investigations have sought to estimate surface-water storage in other types of wetlands (Trigg et al., 2014; Xu and Singh, 2004). For hydrological mod-
ellers, an ideal approach is one that overcomes the need for often time-intensive topographic surveys and that is more practical for use in models at varying scales and locations. Previous studies have set about this by defining statistical relationships between surface area and volume for wetlands of specific physiographic regions (Gleason et al., 2007; Hubbard, 1982; Lane and D’Amico, 2010; Wiens, 2001). Such approaches have been found useful for modelling entire watersheds (Gleason et al., 2007), but limited for estimating storage in individual wetlands because depth and basin morphometry (i.e. surface area, volume, depth) are not considered (Huang et al., 2011; Lane and D’Amico, 2010; Wiens, 2001). Brooks and Hayashi (2002) presented an equation that includes depth and basin morphometry, but to use it, basin morphometry must be predefined and no such information yet exists for beaver ponds.

Another approach, the simplified volume–area–depth (V–A–h) method (Hayashi and van der Kamp, 2000), accounts for depth and calculates basin morphometry for each individual wetland. Requiring only two measurements of depth and surface area, it has been shown to provide reliable estimates of surface-water storage in the pothole wetlands of the North American prairies for which it was designed (Minke et al., 2010). Prairie potholes are depressional wetlands that have fairly regular shapes, i.e. concave profiles with smooth slopes. Beaver ponds, by contrast, typically encompass a bathymetry that is far more complex because their size and shape is controlled by the dimensions of the dam and the land surface that becomes flooded upon dam establishment (Johnston and Naiman, 1987). Whether statistical or analytical approaches can reliably estimate water storage in beaver ponds has yet to be determined. Our goal was thus to explore tools useful for estimating surface-water storage in beaver ponds. We studied beaver ponds across much of their habitat range and (i) evaluated the utility of the simplified V–A–h method in estimating surface-water storage, (ii) evaluated correlations between surface-water storage and beaver pond morphometry, and (iii) described beaver pond morphometry in relation to surface-water storage capacity.

2 Methods

2.1 The simplified V–A–h method

The simplified V–A–h method is based on a simple power equation (Hayashi and van der Kamp, 2000), where the area of a pond (A), at a given height above the pond bottom (h), is described as

\[ A = s \left( \frac{h}{h_0} \right)^{2/p}, \]

where \( h_0 \) is the unit height of the water surface (e.g. 1 m for SI units), \( s \) is a scaling coefficient that represents the area of a circle \( (m^2) \) with a radius that corresponds to \( h_0 \), and \( p \) is a dimensionless morphometry coefficient that represents the shape of the bathymetric curve (i.e. the area–depth relationship of the pond). The volume of the pond is then determined by integrating all the area profiles below \( h \) to give

\[ V(h) = \int_0^h s \left( \frac{h}{h_0} \right)^{2/p} dh = \frac{s}{1+2/p} \left( \frac{h^{1+2/p}}{h_0^{2p}} \right), \]  

Using Eqs. (1) and (2) requires parameterizing the \( s \) and \( p \) coefficients. The simplified V–A–h method arrives at these values by rearranging Eq. (1) to give (Minke et al., 2010)

\[ s = A_1 \left( \frac{h_1}{h_2} \right)^{-2/p}, \]

and

\[ p = 2 \left( \frac{\log (h_1/h_2)}{\log (A_1/A_2)} \right), \]

where \( A_1 \) and \( A_2 \) are surface areas of the pond at corresponding depths of \( h_1 \) and \( h_2 \), respectively, and \( h_1 < h_2 \). With only two measurements of area and depth in time, Eqs. (3) and (4) can be used to calculate \( s \) and \( p \) coefficients that are then reinserted into Eqs. (1) and (2) to define the entire area–depth and volume–depth relationship of the pond.

2.2 Beaver pond morphometry

2.2.1 Metrics for surface-water volume estimations

A beaver pond’s capacity to store surface water is defined simply by its bathymetry, and can be directly calculated if an accurate topographic survey is available. The problem here relates to how well we can approximate that volume given some simple measures of the dam and pond dimensions. To discover if metrics exist, a series of morphometric variables were generated in addition to the \( p \) coefficient described in Eq. (1). They include the maximum dam height \( (h_{\text{max}}) \) defined as the difference in elevation \( (m) \) between the dam crest and the lowest point in the pond, the maximum surface area \( (m^2) \) of the pond \( (A_{\text{max}}) \) at \( h_{\text{max}} \), and the length \( (m) \) of the dam \( (D_{\text{len}}) \) measured along its crest. Regression analysis was then used to determine if any of the variables are correlated to the maximum volume of the pond \( (V_{\text{max}}) \).

2.2.2 Morphometric analysis

Understanding the underlying mechanics of the simplified V–A–h method and how morphometry relates to a pond’s capacity to store water requires a deeper analysis of the bathymetric curve. The bathymetric curve is equivalent to the hypsometric curve defined by Strahler (1952) as the ground surface area of a land mass with respect to elevation. To compare curves for ponds of different size and relief, it is necessary
to express the variables as relative depth ($R_D$) and relative area ($R_A$) as

$$R_D = \frac{h}{h_{\text{max}}},$$

(5)

and

$$R_A = \frac{a}{A_{\text{max}}},$$

(6)

where $h$ is the stage (m) elevation of the pond and $a$ is the corresponding surface area ($m^2$) at any given $h$. For ease of visual interpretation, we express the bathymetric curve as $R_D$ vs. $1 - R_A$ (Fig. 1). Power functions described by Eq. (1) can then be fit to a bathymetric curve with the following equation:

$$R_D = (1 - R_A)^{p/2},$$

(7)

where the $p$ coefficient here is equal to the $p$ coefficient in Eq. (1). This allows for a visual aid in the analysis of error by superimposing estimated curves produced via either Eq. (1) or Eq. (4) to the pond’s actual bathymetric curve. It also eliminates issues of scale between different ponds so that bathymetric curves can be visually compared to one another.

From the relative bathymetric curve, it is possible to compute the bathymetric integral ($B_I$), a modified form of the hypsometric integral defined as the measure of land mass volume with respect the entire reference solid created by the maximum dimensions of the pond (Fig. 1; Strahler, 1952):

$$B_I = \frac{V_{\text{land}}}{h_{\text{max}}A_{\text{max}}} = \int_0^1 R_A dR_D.$$  

(8)

Equation (10) produces values between 0 and 1, with 1 representing a reference solid entirely composed of land mass. Using the $B_I$, we introduce a new metric that represents the pond’s bathymetric capacity to store water ($B_{WC}$). Since the total volume of the reference solid is comprised of either land or water, the $B_{WC}$, relative to the reference solid, is expressed as

$$B_{WC} = 1 - B_I = \frac{V_{\text{max}}}{h_{\text{max}}A_{\text{max}}}.$$  

(9)

The $B_I$ and $B_{WC}$ are quantitative measurements of the pond’s morphometry and capacity to store water, respectively. The value in using these metrics is that they facilitate the comparison of surface-water storage capacity among beaver ponds and other wetland types.

Figure 1. Perceptual diagram of the relationship between morphometric variables. The area ($a$) at a given stage of the pond ($h$) is a point on the bathymetric curve (thick black line), where $R_A$ is the relative area and $R_D$ is the relative depth. The bathymetric integral ($B_I$) is the integration of everything below the bathymetric curve and the pond’s capacity to store water ($B_{WC}$) is the integration of everything above the bathymetric curve. The morphometry ($p$) coefficient represents the shape of the bathymetric curve in the power function equation (red-dashed line; Eq. 7). The reference solid is the box created by multiplying the maximum height of the dam ($h_{\text{max}}$) by the maximum surface area created by the pond ($A_{\text{max}}$), and is entirely comprised of land ($V_{\text{land}}$) and/or water ($V_{\text{max}}$) proportional to $B_I$ and $B_{WC}$.
Finally, we described the shape of the beaver pond surface using a dimensionless shape index \( S_t \), which is essentially the ratio of the pond perimeter to the circumference of a circle with the same area (Hutchinson, 1957):

\[
S_t = \frac{P}{2\sqrt{\pi A_{\text{max}}}},
\]

(10)

where \( P \) is the perimeter of the pond (m). Ponds with \( S_t = 1 \) have shapes that are perfectly circular, whereas ponds with \( S_t > 1 \) are increasingly complex. Pond shape is an important metric as much of the interaction between surface water and groundwater happens at the shoreline (Shaw and Prepas, 1990). We chose \( S_t \) as it is easy to interpret and enables a relative comparison between the shapes of beaver ponds and other types of wetlands (Minke et al., 2010).

2.3 V–A–h models for surface-water storage estimation in beaver ponds

Three versions of the power function model described by Eq. (1) were tested in this study. They are referred to as the full, simplified, and optimized models. The simplified model is the actual test of the simplified V–A–h method and the other two models were included to aid in the analysis of this approach.

The full model is a power function fitted to the complete data set of each pond’s bathymetry (i.e. empirical fit). We arrive at values for \( s \) and \( p \) by fitting a simple power function, \( y = ax^b \), to the pond’s bathymetric curve, and assume \( a = s \) and \( b = 2/p \) in accordance with Eq. (1). Non-linear least-squares regression was used to determine the best fit; the ability of this model to make accurate area and volume estimates depends on its “goodness of fit” to the data set. Analysis of the full model was included to (i) identify the \( p \) coefficient that best describes each beaver pond’s morphometry and (ii) assess the overall suitability of power functions to describe beaver pond bathymetry.

The simplified model is a power function using \( s \) and \( p \) coefficients created from the same two relative measurements of depth (i.e. \( h_1 \) and \( h_2 \) as a percentage of \( h_{\text{max}} \)) in each pond. Minke et al. (2010) evaluated the simplified V–A–h method by applying it to two scenarios: a dry one where \( h_1 \) and \( h_2 \) are taken at 0.1 m and 25% of \( h_{\text{max}} \), and a wet one where \( h_1 \) and \( h_2 \) are taken at 50% and 75% of \( h_{\text{max}} \). They found that estimation errors were lowest using the wet scenario; therefore, we chose this scenario to simulate the application of the simplified V–A–h method as it may be practically used in the field.

The optimized model differs from the simplified model through parameterizing coefficients via the optimum combination of \( h_1 \) and \( h_2 \) for each pond. This required calculating \( s \) and \( p \) coefficients at every possible combination of \( h_1 \) and \( h_2 \) along the bathymetric curve (note that \( h_1 \) and \( h_2 \) are expressed as a percentage of \( h_{\text{max}} \) from 1 to 100; therefore, the total number of combinations where \( h_1 < h_2 \) is 5000 for each pond). Each set of \( s \) and \( p \) coefficients was then inserted into Eqs. (1) and (2) to estimate area and volume, respectively, and the set that produced the least combined area and volume error was selected as the optimum. The optimum model was included in this study to discover how best to use the simplified V–A–h method with regards to differences in pond morphometry.

Error for all three models was evaluated using root mean square error (\( E_{\text{RMS}} \)), defined as

\[
E_{\text{RMS}} = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (D_{\text{ACT}} - D_{\text{EST}})^2},
\]

(11)

where \( m \) is the number of data points, \( D_{\text{ACT}} \) is the point on the actual bathymetric curve calculated from the pond itself, and \( D_{\text{EST}} \) is the point on the estimated bathymetric curve derived from the \( s \) and \( p \) coefficients at a given combination of \( h_1 \) and \( h_2 \). Finally, to allow for coherent comparisons of error among the different beaver ponds, the magnitude of error, referred to as \( A_{\text{ERR}} \) (%) for area and \( V_{\text{ERR}} \) (%) for volume, was calculated by dividing the \( E_{\text{RMS}} \) by the actual area and volume of the pond at 80% of \( h_{\text{max}} \). This particular depth was chosen to avoid inconsistencies in error magnitudes that arise when the evaluation depth is set too close to the minimum and maximum (Minke et al., 2010).

2.4 Test sites

Forty beaver ponds were selected for this study and simulated in digital elevation models (DEMs). Our sample design captured the range of structures built by beaver along streams with mineral and organic substrates in both mountainous and lowland terrain. Beaver ponds were thus analyzed from multiple locations where bathymetric data existed, which included Kananaskis Provincial Park, Alberta, Canada; Escondido, Tierra del Fuego, Argentina; the Logan River watershed, Utah, USA; and Voyageurs National Park, Minnesota, USA. Details of the location, terrain, number of ponds, survey methods, and survey resolution for each site are provided in Table 1.

2.5 DEM creation and manipulation for variable calculations

Sites selected for this study were former beaver ponds that had drained sufficiently to either reveal pond bottom bathymetry or allow it to be surveyed. Beaver ponds extracted from lidar, when available, were fully drained with visible relic dams, whereas some ponds surveyed by total station and real-time kinetic geographical positioning system (rtkGPS) often were still full with water up to their crest elevations, but not enough to impede point collection by wading. DEMs that relied on total station and rtkGPS surveys were created with Surfer\textsuperscript{®} v10 (Golden Software, Colorado) using ordinary kriging. The beaver ponds were then isolated...
Table 1. Site locations, characteristics, and details of topographic pond surveys. “n” is the number of ponds studied at each site.

<table>
<thead>
<tr>
<th>Site</th>
<th>Latitude and longitude (degree, ′)</th>
<th>n</th>
<th>Soil substrate type</th>
<th>Terrain</th>
<th>Survey method</th>
<th>DEM resolution (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kananaskis Provincial Park, AB, Canada</td>
<td>51°3.553′N, 114°52.009′W</td>
<td>10</td>
<td>Organic</td>
<td>Mountainous</td>
<td>rtkGPS</td>
<td>1</td>
</tr>
<tr>
<td>Escondido, Tierra del Fuego, Argentina</td>
<td>54°36.908′S, 67°44.540′W</td>
<td>3</td>
<td>Organic</td>
<td>Mountainous</td>
<td>rtkGPS</td>
<td>1</td>
</tr>
<tr>
<td>Logan River Watershed, UT, USA</td>
<td>41°50.327′N, 111°33.668′W</td>
<td>14</td>
<td>Mineral</td>
<td>Mountainous</td>
<td>Total station</td>
<td>0.1</td>
</tr>
<tr>
<td>Voyagerus National Park, MN, USA</td>
<td>48°32.773′N, 93°4.328′W</td>
<td>1</td>
<td>Organic</td>
<td>Lowland</td>
<td>Lidar</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>48°27.975′N, 92°53.864′W</td>
<td>3</td>
<td>Mineral</td>
<td>Lowland</td>
<td>Lidar</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>48°30.405′N, 92°40.331′W</td>
<td>1</td>
<td>Mineral</td>
<td>Lowland</td>
<td>Lidar</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>48°31.262′N, 92°52.794′W</td>
<td>1</td>
<td>Mineral</td>
<td>Lowland</td>
<td>Lidar</td>
<td>1</td>
</tr>
</tbody>
</table>

from the unneeded areas of the DEM by extracting all the points in the raster below and upstream of the dam crest (i.e. \(h_{\text{max}}\)). This was done in ArcGIS v10.2 (ESRI, 2015) as was the calculation of the morphometric variables. The V–h relationship, as well as bathymetric curve of each pond, was calculated at 5 cm increments using a script written in Python™ that utilizes the “volume” feature of ArcGIS Toolbox. The V–h relationship and bathymetric curve of each pond were the primary inputs for the three models, which were built and run in RStudio (RStudio Team, 2015). Finally, the bathymetric curve for each pond was established using linear interpolation to create 100 points, i.e. 1–100% of \(h_{\text{max}}\).

3 Results

3.1 Beaver pond morphometry

Pond morphometric characteristics are provided in Table 2 and examples of the DEMs from each location are provided in Fig. 2. The 40 ponds well represented the various types of beaver ponds that are created in riverine and wetland habitats (Baker and Hill, 2003), with maximum dam heights (\(h_{\text{max}}\)) ranging from 0.25 to 2 m and dam lengths (\(D_{\text{len}}\)) spanning 3–308 m, with medians of 0.83 and 40 m, respectively. Pond volumes (\(V_{\text{max}}\)) ranged between 1 and 9001 m\(^3\) and showed strong power correlations to \(D_{\text{len}}\), \(h_{\text{max}}\), and \(A_{\text{max}}\) (Fig. 3). Among the ponds, there was considerable variability in shape as \(S_I\) values ranged from 1.5 to 5.3 (mean = 2.6). No strong correlations (i.e. \(-0.10 > R^2 < 0.10\)) were found between \(S_I\) and the other morphometric variables used in this study (i.e. \(p\), \(B_I\), \(B_{WC}\), \(D_{\text{len}}\), \(h_{\text{max}}\)).

The \(p\) coefficients for the beaver ponds followed a log-normal distribution, and ranged from 0.45 to 2.58 (median of 0.91) (Fig. 4). Of the 40 beaver ponds analyzed, 70% (28) had \(p\) coefficients that were < 1, indicating that beaver ponds tend to have convex bathymetries. Most beaver ponds tended to be more convex than they are concave, given the shape of the bathymetric curves (Fig. 5) and the range of \(B_I\) (0.45–0.85; median of 0.69). In all but one case, \(V_{\text{land}}\) was greater than 50% of the total volume of space, indicating that most beaver ponds are shallow, which limits the volume of surface water they can store. This phenomenon is well described by the strong exponential relationship between the \(p\) coefficient \((R^2 = 0.96)\) and \(B_I\) and \(B_{WC}\) (Fig. 6). Soil substrate type (Table 1; organic vs. mineral) did not affect the value of the \(p\) coefficient, as evidenced by a t test \((P = 0.97)\).
3.2 Surface-Water Storage Estimations

The full model had the least AERR, and the optimized model had the least VERR (Fig. 7; Table 3). The highest AERR and VERR was associated with simplified model estimates, which also produced the greatest variability of error among the different ponds. With regards to study locations, full VERR ranked as Escondido < Voyageurs < Logan < Kananaskis, whereas full AERR ranked Logan < Escondido < Kananaskis < Voyageurs. Overall, the beaver ponds in Kananaskis proved most difficult to model (i.e. highest VERR and AERR overall); however, mean error for the full model remained below 5% for both area and volume estimates.

Compared to the full model (Fig. 7), the simplified model had higher VERR in 65% of cases (26 ponds) and higher AERR in 98% of cases (39 ponds), whereas the optimized model had lower VERR in 100% of cases but slightly (<1%) higher AERR in 100% of cases. The optimum p coefficients for volume tended to be slightly different than the optimum p coefficients for area, which are the coefficients derived from the empirical fit of the Full model. The optimum model proved useful for revealing the two points on the bathymetric curve that can be used to obtain the optimum p coefficient for volume estimates. Pond 7 had the largest AERR and VERR (Fig. 7), and therefore was selected for more detailed study (Fig. 8). The optimum points were found at the approximate location of where the empirical fit intersects with the bathymetric curve. Thus, using the optimum points in Eq. (4) computes a p coefficient that is closest to the same coefficient generated by the curve fitted by non-linear least-squares regression. The points used by the simplified model for Pond 7 fall on segments of the bathymetric curve that are farther away in distance from the empirical fit; hence, the p coefficient generated by these points creates a curve that is farther away from the bathymetric curve, which ultimately leads to a less accurate estimate of volume.

In a number of ponds, the empirical fit nearly overlapped the entire bathymetric curve, and in such cases, there were many combinations of h₁ and h₂ that produced reasonable estimates of volume. For example, Pond 10 had the lowest full AERR and VERR of all the beaver ponds. In this case, there were 1899 combinations of h₁ and h₂ that produced reasonable estimates of volume. For example, Pond 10 had the lowest full AERR and VERR of all the beaver ponds. In this case, there were 1899 combinations of h₁ and h₂ that produced reasonable estimates of volume. For example, Pond 10 had the lowest full AERR and VERR of all the beaver ponds. In this case, there were 1899 combinations of h₁ and h₂ that produced reasonable estimates of volume.
Figure 3. Power regression relationships between the maximum volume of the beaver ponds ($V_{\text{max}}$) and (a) the length of the beaver dams ($D_{\text{len}}$), (b) the product of the maximum depth of the ponds ($h_{\text{max}}$) and the length of the beaver dams, (c) the maximum surface area ($A_{\text{max}}$) of the ponds, and (d) the product of the maximum surface area and maximum depth of the pond.

Figure 4. Distribution of morphometry ($p$) coefficients (full model) for all beaver ponds sampled ($n = 40$).

4 Discussion

The simplified $V$–$A$–$h$ method estimated surface-water storage in the beaver ponds with high accuracy. Also, strong statistical relationships were found between surface-water storage capacity in beaver ponds and the dimensions of the dam and pond. The beaver ponds studied have a convex shape that permits less water storage than do other open-water wetland types. Surface-water storage estimates can be made in beaver ponds without the need for topographic surveys if pond morphology is used instead.

4.1 $V$–$A$–$h$ Model Performance in Beaver Ponds

The low full $A_{\text{ERR}}$ and $V_{\text{ERR}}$ overall indicates that beaver pond morphometry is adequately described by power functions. This is because the bathymetric curve proved resilient to fluctuations in “elevation” inherent to the impounded land surface. Also, the dams, intricate canals and holes that beavers create in the areas they inhabit (Hood and Larson, 2015) do not warp the shape of the bathymetric curve enough that a power function becomes inappropriate to sufficiently describe it. However, it appears that volume estimations are more resilient to aberrations in the bathymetric curve than are area estimates. The power functions in the full model are fitted to pond bathymetry. When the power curve moves up and down, $A_{\text{ERR}}$ will increase, but sometimes the $V_{\text{ERR}}$ can decrease because volume is the integration of everything above the bathymetric curve. When the curve moves slightly up or down from the empirical fit, irregularities on the bathymetric curve are captured, which improves volume estimations at the sacrifice of area estimates. This explains why the optimum $p$ coefficients for volume are different than they are for area. It also explains why, in many cases, the...
Figure 7. Volume ($V_{ERR}$) and area error ($A_{ERR}$) from each beaver pond using the three different approaches (a–f). Plots on the bottom show the difference in volume (g) and area (h) error of the simplified (solid circles) and optimized (open circles) models relative to the full model (the full model is represented by the solid black line at zero on the y axis). Bars and solid circles are colour coded by location as per the legend at the top of the figure.

Figure 8. Comparison of the bathymetric curve for Pond 7 with the full and simplified curve. The top shows the area ($A_{ERR}$) and volume ($V_{ERR}$) error associated with the simplified curve that was calculated using simplified depths $h_1$ and $h_2$ and the bottom shows the error associated with the full curve and the optimum location for depths $h_1$ and $h_2$. $R_D$ is relative depth, $R_A$ is relative area, $B_I$ is the bathymetric integral, and $p$ is the morphometry coefficient.

simplified model had $V_{ERR}$ that was less than 10 %, while $A_{ERR}$ was greater than 25 %. Without a complete set of pond bathymetry, it is unlikely that users of the simplified V–A–h method would be able to discern the optimum points for $h_1$ and $h_2$; however, as long the chosen values for $h_1$ and $h_2$ are selected within the range identified here (i.e. 18–74 % of $h_{max}$ for $h_1$ and 42–98 % of $h_{max}$ for $h_2$), fairly accurate estimates of surface-water storage should be expected. Overall, the simplified model performed reasonably, exceeding 10 % $V_{ERR}$ in only three cases. Given that the simplified V–A–h method appears to work well across the broad range of beaver pond bathymetry reported here, and across a wide range of prairie potholes (e.g. Minke et al., 2010), it should be a robust enough approach to be used other open-water wetlands.

4.2 Beaver pond morphometry and surface-water storage capacity

Our results show that $p$ coefficients in beaver ponds are lower overall than those reported in prairie wetlands (Hayashi and van der Kamp, 2000) and those reported in forest pools in New England (Brooks and Hayashi, 2002). Because of the strong exponential relationship between $p$ coefficients and $B_{WC}$, we can conclude that beaver ponds typically store less water. For example, the prairie potholes studied by Hayashi and van der Kamp (2000) had a median $p$ coefficient of 3.22. Using Fig. 6, this $p$ coefficient is equivalent to a $B_{WC}$ of 0.61, which is almost double the median beaver pond $B_{WC}$ equivalent of 0.32. The most likely explanation for this is the ontogeny of beaver ponds compared to other open water wetland types. Beaver ponds occur via inunda-
Table 2. Pond morphometric characteristics, including the full model morphometry ($p$) and scaling ($s$) coefficients, shape index ($S_i$), bathymetric integral ($B_I$), bathymetric water capacity ($B_{WC}$), length of the dam ($D_{len}$), and maximum depth ($h_{max}$), area ($A_{max}$), and volume ($V_{max}$) of the ponds.

<table>
<thead>
<tr>
<th>Location</th>
<th>Pond no.</th>
<th>$S_i$</th>
<th>$B_I$</th>
<th>$B_{WC}$</th>
<th>$p$</th>
<th>$s$ ($m^2$)</th>
<th>$D_{len}$ (m)</th>
<th>$h_{max}$ (m)</th>
<th>$A_{max}$ ($m^2$)</th>
<th>$V_{max}$ ($m^3$)</th>
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</table>

The different origins of prairies and beaver ponds are reflected in the shape of the bathymetric curves and the strong statistical relationships between surface-water storage capacity and the dimensions of the dam and pond. Beaver ponds built on deeper and narrower stream channels tend to have lower $p$ coefficients than ponds built on wider, fewer constrained channels. This happens because there is a rapid expansion of surface area inundated as the dam exceeds the height and width of the stream channel; a phenomenon that is well described by the “power” relationships between $D_{len}$, $h_{max}$, $A_{max}$ and $V_{max}$. Pond 12 is a good example of this; the $p$ coefficient was high-
est (2.58) and a distant outlier compared to the other ponds. The uniqueness of this site is that the beaver built a small dam (0.3 m) with excavated peat and impounded groundwater seepage rather than damming channel flows. Even though the dam is relatively small, it has a large $B_{WC}$ (0.55) relative to the other ponds because the dam is entirely dedicated to impounding a mostly flat land surface. In contrast, Pond 6, which was also built in a peatland, has a much lower $B_{WC}$ (0.26) because most of the dam height (2 m) is dedicated to impounding water in an incised stream channel. An advantage of using the $B_{WC}$ metric over pond volumes is that it allows for a comparison of surface-water storage capability in a way that is independent of pond size and shape.

4.3 Tools for surface-water storage estimation in beaver ponds

There are a variety of ways our results can be used to estimate surface-water storage in beaver ponds under different data availability scenarios. In situations where only aerial or remotely sensed imagery is available (i.e. world wide), dam length and pond area can be approximated and used in the generalized power regression relationships presented in Fig. 3. This is a quick and easy way to incorporate beaver pond surface-water storage capacity into land use planning decisions and watershed-scale hydrological models. However, this approach is not suitable for detailed study in individual beaver ponds as it does not account for pond morphometry (Huang et al., 2011; Wiens, 2001). Including dam height should improve estimates. Measuring dam height in the field is quick and straightforward, but it can also be reasonably approximated with remotely sensed imagery alone using spectral-depth correlation methods (e.g. Passalacqua et al., 2015). If dam heights are available, we recommend using our median $p$ coefficient (0.91) for beaver ponds in the equation presented by Brooks and Hayashi (2002):

$$V_{max} = \frac{A_{max} \times h_{max}}{1 + 2/p}.$$  \hspace{1cm} (12)

This equation is a modified form of Eq. (2) used to estimate surface-water storage capacity. It is easily incorporated into spatially distributed hydrological models. Fang et al. (2010) had success in using this approach, albeit for prairie potholes, in their Cold Regions Hydrological Model.

With a moderate amount of data, the simplified V–A–h method offers an alternative that produces surface-water storage estimates with minimal error. The advantage of this method over the others is that it is robust, it is customized to each pond’s basin morphometry, and it calculates a coefficient of scale (i.e. $s$ coefficient) for use in estimating surface-water storage across the range of pond stages, unlike the generalized power regression models and Eq. (12), which are limited to estimates of $V_{max}$. Combined with a few field visits and something as simple as automated water level observations, the simplified V–A–h method can be a powerful tool. But, it also has practical application in relatively data rich environments. For example, many lidar data sets are collected when beaver ponds are not fully drained. If the beaver pond is not entirely full, the measurements for $A_2$ and $h_2$ can be measured within the vertical distance between the crest of the dam and the surface of the water, thus allowing for an appropriate $p$ coefficient to be derived. Furthermore, the simplified V–A–h method is increasingly practical with the advent of new technologies. For example, structure from motion software facilitates the creation of high resolution DEMs from ordinary photographs (Javernick et al., 2014). Theoretically, with both tools, one field visit to collect a few pictures and depths measurements should be all that is needed to make reliable estimates of wetland surface-water storage.

4.4 Implications of study results

The results of our study provide some simple tools that enable surface-water storage in beaver ponds to be estimated without the need for topographic surveys. This allows environmental managers to better assess the risks and benefits associated with beaver ponds that appear on landscapes, and allows for the easy inclusion of the surface-water storage component of beaver ponds into hydrological models at various scales. This study also demonstrates that beaver pond morphometry is different than other types of wetlands, which requires consideration. For example, based on this analysis we might expect beaver ponds to reach their capacity faster during rainfall events, while impounding larger surface ar-

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Table 3. V–A–h model performance comparisons based on the mean ($\pm$ standard deviation) volume (VERR) and area (AERR) error magnitude. 

<table>
<thead>
<tr>
<th>Site</th>
<th>n</th>
<th>Full VERR (%)</th>
<th>Full AERR (%)</th>
<th>Simplified VERR (%)</th>
<th>Simplified AERR (%)</th>
<th>Optimized VERR (%)</th>
<th>Optimized AERR (%)</th>
</tr>
</thead>
<tbody>
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<td>Kananaskis</td>
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<td>3.8 ± 2.1</td>
<td>7.2 ± 6.0</td>
<td>14.6 ± 12.3</td>
<td>2.3 ± 1.6</td>
<td>4.2 ± 2.5</td>
</tr>
<tr>
<td>Escondido</td>
<td>3</td>
<td>3.1 ± 1.4</td>
<td>3.8 ± 0.7</td>
<td>4.3 ± 2.5</td>
<td>6.7 ± 3.8</td>
<td>1.6 ± 0.7</td>
<td>4.0 ± 0.9</td>
</tr>
<tr>
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<td>3.8 ± 1.8</td>
<td>4.1 ± 1.6</td>
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<td>2.1 ± 1.2</td>
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n is the number of ponds studied at each site.
beaver ponds store less surface-water than other wetland types, their relevance to local and regional water balances should not be underestimated. Beaver population recovery, post fur trade, has led to the creation of between 9494 and 42 236 km$^2$ of new beaver ponds globally (Whitfield et al., 2015). Using the estimates of Whitfield et al. (2015) and our median $p$ coefficient (0.91) and median dam height (0.83 m) in Eq. (12), we crudely estimate that between 2.5 and 11 km$^3$ of water are stored in beaver ponds.

5 Conclusions

The primary goal of this study was to test the utility of readily applicable tools for estimating surface-water storage in beaver ponds. We examined whether the simplified V–A–h method was appropriate for this purpose and described beaver pond morphology to explore its relationship to surface-water storage capacity. A number of valuable insights were revealed. The simplified V–A–h method proved to be a simple and effective tool as it was able to estimate beaver pond surface-water storage with an average volume error of 5%. The median basin coefficient for beaver ponds was 0.91, suggesting that they tend to have a convex basin morphometry, and that they typically store less water than other wetlands studied in the same way. Pond capacity was strongly correlated to the dimensions of the dam and surface area of the pond, further cementing the idea that beaver ponds exhibit characteristic traits in pond morphometry that make reliable estimates of surface-water storage possible without the need for topographic surveys. Future research efforts should be directed at applying these simple methods more remotely, and incorporating them into both broader beaver-related planning tools and catchment-scale hydrological models.

6 Data availability

DEMs for beaver ponds in Voyageurs National Park are made publicly available by the Minnesota Department of Natural Resources at http://www.mngeo.state.mn.us/chouse/metadata/lidar_arrowhead2011.html, 2011. DEMs for ponds in the Logan River watershed are available by contacting Joseph M. Wheaton (joe.wheaton@esu.edu), and DEMs for ponds in Kananaskis and Escondido are available by contacting Cherie J. Westbrook (cherie.westbrook@usask.ca).

Competing interests. The authors declare that they have no conflict of interest.

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