Perennial groundwater yield planning for complex nonlinear aquifers: Methods and examples

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(Received; accepted 28 September 1994)

Optimal perennial groundwater yield pumping strategies were computed for a complex multilayer aquifer with: (i) confined and unconfined flow, and (ii) many flows typically described by piecewise-linear (nonsmooth) equations. The latter flows account for over 50% of the aquifer discharge from the test area, the eastern shore of the Great Salt Lake in Utah. Normally utilized response matrix (RM) and embedding (EM) simulation/optimization modelling procedures did not converge to optimal solutions for this area; they diverged or oscillated. However, the newly presented linear RM and EM approaches satisfactorily addressed the nonlinearities posed by over 2000 piecewise-linear constraints for evapo-transpiration, discharge from flowing wells, drain discharge, and vertical interlayer flow reduction due to desaturation of a confined aquifer. Both presented modelling approaches converged to the same optimal solution. Superposition was applied to the nonlinear problem by: making a cycle within the RM analogous to an iteration in a simulation model (such as MODFLOW); and using a modified MODFLOW to develop influence coefficients. The EM model contained about 40000 nonzero elements and 12000 single equations and variables, demonstrating its suitability for large scale planning.

INTRODUCTION

A common management goal in arid and semi-arid regions is to fully analyze water resources for economic and social benefit. A groundwater management plan should also consider aquifer physical limitations and legal and economic constraints. Groundwater management plans are increasingly developed using simulation/optimization (S/O) models, which couple groundwater simulation ability with mathematical optimization capabilities. They simultaneously compute the best management strategy for the specified objectives and constraints, and predict aquifer response to the strategy.

S/O models are frequently classified as using either the embedding (EM) approach or the response matrix (RM) approach, based on how groundwater head response to hydraulic stress is simulated in the model. The EM approach incorporates finite-difference or finite-element approximations of the groundwater flow equation as constraints for each cell and stress period. The RM approach uses superposition and influence coefficients generated by pre-optimization simulations.

Most S/O models assume system flows are linear, and employ linear constraints, generally within linear programming (LP) models, to compute optimal pumping strategies. S/O models addressing contaminant transport frequently use nonlinear constraints. However, real aquifer systems frequently are complex and have nonlinear flow processes. Flow in unconfined aquifers is nonlinear if transmissivity changes significantly in response to pumping.

Other nonlinear flows are defined in normal simulation models using piecewise-linear equations. Derivatives of these nonsmooth functions are not continuously differentiable. For example, the equation describing flow to drains in many codes is piecewise-linear (it has two joined linear segments). If aquifer head is below the drain elevation, there is no groundwater
flow to the drain. If head is above the drain, flow is linearly proportional to the head difference.

Traditionally, linear S/O models have used cycling to address unconfined (nonlinear) aquifers. Cycling involves (i) assuming aquifer parameter values and equations; (ii) computing an optimal pumping strategy and system response to the strategy; (iii) comparing the strategy with a previously computed strategy and either stopping or returning to step (i).

Piecewise-linear equations cannot be solved by LP models directly. In S/O models, such flows have frequently been assumed to be insignificant or to be insignificantly affected by the optimal pumping strategy. Mixed integer programming (MIP) models have been used to address small numbers of piecewise-linear constraints. Common experience is that MIP models can have difficulty converging if there are many such constraints.\textsuperscript{11} Cycling has been used to address relatively minor piecewise-linear flow rates, or situations where the model does not have difficulty using one particular linear segment. For example, if water levels drop far below the ground surface, evapotranspiration (Et), flow from drains and artesian flow are all zero. This facilitates convergence to an optimal solution.

The more nonlinear the aquifer system, the more difficult it is to apply linear optimization models to compute optimal groundwater pumping strategies. There are difficulties in using large numbers of nonlinear equations. It is necessary to develop improved ways of addressing common piece-wise flows within linear S/O models.

The major goal of this paper is to explain and illustrate how to optimize groundwater planning for complex aquifer systems containing many flows which normally are described via piecewise expressions.

To achieve this objective it was necessary to:

(i) show how to adapt both response matrix (RM) and embedding (EM) approaches for that task;
(ii) discuss why one might pick one approach over the other for a particular situation; and
(iii) illustrate application of the selected approach to the East Shore area of Utah's Great Salt Lake, for a range of management scenarios.

Currently, flows described by piece-wise expressions (Et, drainage, and free artesian flow) account for more than half of the discharge from the aquifer system underlying the eastern shore of Utah's Great Salt Lake. That area is not amenable to normal cyclical embedding (EM) or response matrix (RM) approaches. When we applied the cyclical EM approach used by Gharbi and Peralta\textsuperscript{11} or a normal cyclical RM approach\textsuperscript{12} to this study area, they diverged or oscillated rather than converging to an optimal solution.

Both the enhanced EM and RM cycling procedures presented here achieve convergence in a procedure somewhat analogous to an iteration in a normal simulation model. During each cycle, the enhanced EM procedure can apply a particular linear segment beyond its normally reasonable range. The enhanced RM procedure requires that, during a particular cycle, precisely the same linear segments are employed during computation of all influence coefficients. These segments are selected before influence coefficient generation, based on initially assumed or previously computed pumping rates and resulting heads. Obviously, in both approaches, inappropriate segments (and resulting segments) may be used at some time during the cycling, although self-correction occurs through cycling. Ultimately all segments are appropriately applied and convergence occurs.

RELEVANT RESEARCH

The EM approach was first applied to groundwater management by Agnoad and Remson.\textsuperscript{1} Because of numerical difficulties with optimization algorithms resulting from the large dimensionality,\textsuperscript{12,34,35} the EM approach has historically been used primarily for small scale and steady-state models. However, it has been more recently applied to larger scale problems. Cantiller et al.\textsuperscript{6} used a one-layer, 1,595 cell EM model for planning conjunctive use of surface and groundwater for 13,006 square miles of the Mississippi alluvial plain. Steady-state EM models have been most useful in planning perennial groundwater yield in areas where most cells are pumped and many heads must be constrained.\textsuperscript{26}

'Perennial yield' is defined as the maximum quantity of water that can be continuously withdrawn from a groundwater basin without adverse effect.\textsuperscript{5} A 'perennial-yield pumping strategy' is a spatially distributed pumping pattern that causes the evolution and subsequent maintenance of an appropriate potentiometric surface. Barring unforeseen changes in boundary conditions and climatic variability, such a strategy assures that a certain amount of water will be available over a long period. The strategy can be computed using a steady-state S/O model. Knapp and Feinerman\textsuperscript{8} endorsed the usefulness of computing optimal steady-state solutions.

Gharbi and Peralta\textsuperscript{11} used the Utah State University Embedding Model (USUEM) to deal with the 1,086 cell, two-layer (unconfined/confined), aquifer underlying Utah's Salt Lake Valley (south of the East Shore Area). Nonlinearities of unconfined flow, evapotranspiration, and aquifer-stream interflow, are solved by cycling and using linear and nonlinear versions of the piecewise flow expressions in tandem. The nonlinear version makes it possible to obtain a feasible solution when the linear version could not, or would not oscillate. Others have also used cycling approaches.\textsuperscript{8,24,33,37} As described below, an enhanced
cycling approach can avoid reliance on nonlinear constraints.

The RM approach is most commonly used for situations involving transient pumping or relatively few pumping sites and control locations.\textsuperscript{26} The use of superposition to compute heads is fully appropriate for linear systems.\textsuperscript{25} An RM model calculates and constrains aquifer response only at specified locations, thus potentially requiring less computer memory than the EM approach. However, preliminary (one simulation per pumping cell) simulations using a separate simulation module or model are needed to generate influence coefficients. Any changes in assumed aquifer parameter values can require performing many simulations anew, regeneration many influence coefficients, and reoptimizing — i.e. cycling.\textsuperscript{8,12} Influence coefficients are also termed discrete kernels,\textsuperscript{15,23} technological functions,\textsuperscript{3} algebraic technological functions,\textsuperscript{9} and response functions.\textsuperscript{33,36}

The equation for saturated groundwater flow is linear for a confined aquifer but is nonlinear for an unconfined aquifer in which saturated thickness varies significantly with head. In linear systems, it is valid to derive a composite response by the superposition of system responses to individual stimuli. Such an approach generally cannot be applied to nonlinear systems without adaptive measures or assumptions. Several researchers have addressed this problem with RM models,\textsuperscript{8,10,14,16,20,37} some with cycling or MIP approaches,\textsuperscript{8,28} but none addressed situations with external flows (described by nonsmooth functions such as drainage) that interacted significantly with pumping. Such flows are commonly assumed to be insignificant or known (fixed); or their nonsmooth nature is ignored or irrelevant. For example, the conventional RM approach is suitable where all nonsmooth flows have ceased due to significant water table declines.

The new methods were tested in a study area that contained more significant external flows described using nonsmooth functions than are considered in previous studies. Nonsmooth flows were about half of total aquifer discharges, and there were tradeoffs between discharge from flowing wells and groundwater pumping. Under these conditions, other linear cycling

\textbf{Fig. 1. Map of the East Shore Area, Utah.}\textsuperscript{7}
approaches diverged or oscillated. Cycling with nonlinear versions of the piecewise flow equations would result in a large number of nonlinear constraints. Applying a MIP approach would require large numbers of constraints, and each constraint would add at least two integer variables.

Here we present improved EM and RM approaches and use them to compute optimal perennial groundwater yield planning strategies. The presented approaches address the nonlinearity of unconfined flow and flows described by nonsmooth functions better than previous approaches.

THE STUDY AREA AND SIMULATED FEATURES

The 450-square-mile East Shore Area is bounded by the Wasatch Front to the East, the Great Salt Lake to the West, and Salt Lake Valley to the South (Fig. 1). The area population has tripled during the last 40 years.27

Groundwater is utilized for M&I, irrigation, stock, watering, and domestic purposes. Irrigated agriculture is the main water user and is mainly supplied from the Weber River. Groundwater supplies about 70% of M&I water use from a three-layer aquifer system (Fig. 2). Near the mountains are large M&I wells.5 Near the shore, potentiometric heads of the middle and lower aquifers are above the ground surface, and many natural artesian wells provide water for agriculture, wetlands, and biota.

Groundwater levels have declined for more than 40 years, and exceed 15.25 m (50 ft) near Hill Air Force Base (HAFB) (Fig. 1). Users hope that the aquifer can satisfy much of the expected increased demand for water. However, unless groundwater is managed properly, several problems could result, including: (i) increases in pumping cost or numbers of inoperable wells due to declining water levels; (ii) well discharges inadequate for agriculture, wetlands, and wildlife; (iii) conflict among water users; (iv) salt or brackish water intrusion from the Great Salt Lake; and (v) contamination of groundwater.9

To describe aquifer system response to management, Clark et al.7 used MODFLOW,21 a quasi-3D flow simulation model (Figs 3–5). The upper shallow, unconfined layer 1 has 1274 cells. There, discharge from drains and flowing wells, evapotranspiration, and upward inflow from the underlying aquifer to the Great Salt Lake are all functions of head. The partially unconfined layer 2 has 1644 cells. The 1962–cell Layer 3 is unconfined near the mountains and confined elsewhere. Takahashi29 modified MODFLOW so that discharge from free flowing artesian wells is a linear function of head above the ground surface, rather than an input parameter. He calibrated those linear expressions for each pertinent cell of the East Shore system.

EMBEDDING S/O MODELING APPROACH USING A MODIFIED VERSION OF USUEM

Model formulation

These EM S/O model was used to compute the maximum perennial groundwater pumping yield,
subject to the embedded constraints describing the same flow types that Clark et al. simulated using MODFLOW. Our model also included an objective function, bounds and other constraints related to additional management goals. It was written in the General Algebraic Modelling System, GAMS, language. A modified primal simplex method in MINOS (version 5.1) was used for optimization.

USUEM organizes all the coefficients of the optimization model equations into the proper rows and columns so they can be read by MINOS. The user simply prepares data (aquifer parameters, bounds on variables) in tabular format (by row, column, and layer of the study area). No part of MODFLOW is used in USUEM. Equations perform the same pre-simulation functions as beginning routines in MODFLOW. It also includes equations to perform all necessary pre-optimization computations.

The utilized USUEM objective function is

\[ \text{maximize } z = \sum_{\Delta} q^p \]

where \( q^p \) = groundwater pumping extraction in cell \( \Delta \), (L³/T); \( M^p \) = total number of cells with potential pumping wells.

Constraints include the steady-state, finite-difference form of the quasi-three-dimensional groundwater flow equation for every cell and layer. The right hand side (RHS) of the flow equation is the sum of external flows, \( \sum_{x=1}^{X} q_i^x \), where \( i, j, \ell = \) cell layer, row, and column indices; and \( q_i^x \) = the \( x \)th external flow, (L³/T). External flows include known constant recharge \( q' \), groundwater pumping \( q^p \), discharge from flowing artesian wells \( q^a \), flow through a general head boundary \( q' \), evapotranspiration \( q^e \), drain discharge \( q^d \), and vertical interlayer flow reduction due to desaturation of a confined aquifer \( q^b \). All of these are defined as in MODFLOW, with the addition of

\[ q_{i,j} = \Gamma_{i,j} (h_{i,j} - h_{i,j}') \quad \text{for } h_{i,j} > h_{i,j}' \]  

(2a)

\[ = 0 \quad \text{for } h_{i,j} < h_{i,j}' \]  

(2b)

where \( h_{i,j} \) = potentiometric head, (L); \( \Gamma = \) coefficient describing naturally flowing well discharge as a
function of head, \((L^2/T)\); \(h^g\) = ground surface elevation, \((L)\).

Upper and lower bounds are employed on pumping and head for all cells. Other variables are constrained as needed.

Solution procedure

Embedded groundwater flow equations can contain nonlinearity; (i) in unconfined aquifers, where transmissivity is a function of head; and (ii) in nonsmooth functions of head \(- q^i, q^e, q^d, \text{ and } q^\text{rd} \). Here, both types of nonlinearities are addressed using the cycling approach of Fig. 6(a). In overview, assumed heads are input in Step 1. In Step 2, the transmissivity is computed based on those heads (or heads from the previous cycle). Also, for each cell, one segment of each piecewise expression is selected for use during the cycle. In Step 3 the optimal solution is computed by an optimization algorithm, based upon the assumed parameters and selected segments. In Step 4 the results of the optimal strategy are compared with those of the previous strategy. If the computed strategy and heads differ from those of the previous cycle, a new cycle begins with Step 5. In Step 5, the optimal solutions of the previous cycle are stored to provide the heads for the next Step 2. Cycling halts when computed optimal strategies and assumed values and segments cease to change with cycle. The optimal solution has converged.

More discussion of Step 2 is appropriate. Described are the details that permit this modified version of USUEM to converge properly when applied to the East Shore area. the same general process is applied to all nonsmooth functions. Illustrated is application to the two-segment equation describing groundwater flow to drains, \(q^d\),

\[
q^d_{i,j} = \Gamma^d_{i,j}(H^i_{i,j} - B^d_{i,j}) \quad \text{for } H^i_{i,j} > B^d_{i,j} \quad (3a)
\]

\[
= 0 \quad \text{for } H^i_{i,j} < B^d_{i,j} \quad (3b)
\]

where \(\Gamma^d\) = drain/aquifer conductivity \((L^2/T)\); \(H^i\) = unknown head in the current \((n)\) cycle; \(H^{i-1}\) = head known from the previous \((n-1)\) cycle \((L)\); and \(B^d\) = elevation of base of drain, \((L)\).

Figure 7 illustrates the segment selection process. Figure 7(a) shows the piecewise nature of eqn (3). Assuming a physical system containing a number of drain cells, in Step 2 (Fig. 6(a)), based on the head from Step 1 or 5, USUEM will select either the equation shown in Fig. 7(b1) (eqn 3(b1)) or in 7(b2) (eqn 3(a)).

Figure 7(c1) shows that the initial guesses of head \((H^0)\) are above the drain bottom. Therefore, the S/O model uses an equation represented by Fig. 7(b1) in the first cycle. During computation in that cycle, some heads fall below the drain bottoms, and their drain discharges become improbable recharges (Fig. 7(c2)). In this case, optimal regional pumping is greater than the true optimal pumping because the model behaves as if recharge is occurring from those drains.

Based on the head resulting from Cycle 1, the segment of Fig. 7(b1) is selected for Cycle 2 (Fig. 7(c2)). During Cycle 2, the \(q^d\)'s are zero at these cells. The improbable drain flows disappear. Drain discharge is allowed to be unrealistic temporarily during cycling, but becomes either zero or a positive value as subsequent cycles converge.

RESPONSE MATRIX S/O MODELLING APPROACH

Model formulation

The RM S/O model uses the same objective function and bounds on pumping as the EM model. However, bounds on head are imposed only at selected cells. Superposition is used as a constraint to compute heads at those cells.

\[
h^s = h^s^m + \sum_{k=1}^{M_s} \delta_{k,a} q^p_k
\]  

\((4)\)
Groundwater yield planning for aquifers

where \( h_b \) = average potentiometric head in cell \( b \), (L); \( h^{un}_{s} \) = unmanaged steady-state head resulting in response to known stresses (bedrock recharge, precipitation, etc.) which do not include pumping rates being optimized (L); and \( \delta_{b,a} \) = influence coefficient describing head response in cell \( b \) to a unit pumping in cell \( a \), (T/L²).

Applying superposition to unconfined aquifers should be done with care, since the governing groundwater flow equation is nonlinear. The assumption of linearity can also be violated if the physical system contains significant external flows described by nonsmooth functions, such as drain discharge. Violation occurs if the linear equation segment that should be utilized changes between \( \delta \) computation and its use in a tight constraint of an optimal pumping strategy. To address the significant external flows in the East Shore area, the following procedure was developed.

Solution procedure

The approach for addressing these nonlinearities is conceptually similar to that for the EM model, but superposition, influence coefficients, and more steps are involved. Again, the flow equation and constraints describing nonsmooth functions are assumed linear during a cycle. A modified cycling procedure (Fig. 6(b)) is used to ensure that, within one cycle, exactly the same areal set of equation segments and transmissivities are used for computing all influence coefficients.

Step 1 (Fig. 6(b)) is analogous to that for the embedding model. Step 2 involves running an influence coefficient generator (ICG). During one cycle the ICG will employ precisely the same linear segments and transmissivities when computing all influence coefficients.

Fig. 6. Enhanced cycling procedures for: (a) embedding model, and (b) response matrix model.
coefficients. MODFLOW can be used as the ICG for a linear system, because transmissivities and segments would not change, regardless of which influence coefficients were being computed. It can be used advisedly for some nonlinear systems. It should to be used directly as the ICG for a nonlinear system such as is addressed here, because resulting optimal strategies might not converge. The reason is as follows.

A normal simulation model, such as MODFLOW, iterates within a time step (or a steady-state solution) to converge to a correct answer. In MODFLOW, heads known from the former \((m-1)\)th iteration, are used to compute saturated thickness and transmissivity and to select the linear segments of the piecewise equations to be used in the \(m\)th iteration. These values and segments are kept constant during iteration. Equations considered for drainage are analogous to eqn (3), except that \(H^m\) and \(H^{m-1}\) are used instead of \(H^n\) and \(H^{n-1}\) (L). Thus, based on assumed or previous iteration heads, \(q^d\) is described as either a simple linear equation or zero in each iteration. Then, the MODFLOW solver solves the linear flow equation. The solver will iterate, computing new transmissivities and selecting new segments (as needed) with each iteration, until convergence criteria are satisfied. Many iterations are usually required to converge to a solution.

Here, MODFLOW is modified into an appropriate ICG by preventing it from iterating when used as Step 2 of a cycle (Fig. 6(b)). Changes made to MODFLOW and the cycling procedure cause the cycling and iteration processes to be analogous. A cycle in the development of influence coefficients and computation of an optimal strategy is made to be similar to the effect of a single iteration in MODFLOW. During a cycle, precisely the same transmissivities and linear segments are used in computing each and every influence coefficient and the optimal strategy. Some of the assumed equation segments of nonsmooth functions might be wrong during a particular cycle. However, they will be corrected by cycling just as MODFLOW normally assumes and corrects these equations by iteration. In other words, MODFLOW is converted into an ICG by not permitting it to change selected segments and
transmissivities during a cycle, regardless of which influence coefficient is being computed.

In Steps 3 and 4 influence coefficients are read and placed within the s/o model's superposition equations. Optimization is performed in Step 5. In Step 6, the computed optimal strategy and system responses from the current cycle are compared with those read in Step 1, or resulting from the previous cycle. If convergence has been achieved, one can cease cycling. Otherwise, one goes to Step 7(a).

Within Steps 2–6 there is no change in utilized transmissivities and segment equations for \( q^1, q^2, q^6, \) and \( q^{rd}. \) Corrections of segment selection is accomplished in Step 7(a), using a MODFLOW-like 'pre-ICG'. The pre-ICG iterates while performing steady-state simulation (using optimal pumping rates computed in Step 3), and appropriately changes transmissivities and selected segments. A convergence criterion of at least \( 0·3 \) cm (0·01 ft) is used for iterations. Then another cycle begins.

As a result of the new cycling approach, the composite effect (on heads and transmissivities, for example) of all optimal pumping of the previous cycle is considered when computing influence coefficients for the new cycle. Gradually the correct segments are chosen and a converged optimal strategy is computed. A convergence criterion of at least \( 0·01 \) is used for cycling.

PRELIMINARY APPLICATION SCENARIO FOR EMBEDDING AND RESPONSE MATRICES S/O MODELS

Application

The objective of this section is to compare applicability of the new EM and RM models to the East Shore Area. Both models are formulated to determine the maximum sustained yield from the 61 cells containing existing M&E use wells pumping from the middle and lower layers. Utilized aquifer parameters, and fixed boundary conditions and flows are the same as used previously.\(^1\)

The lower bound on pumping is the current withdrawal rate for all the existing pumping cells (totaling 23,400 ac-ft/year, Figs 4 and 5). For most cells, the upper bound on pumping is twice the current withdrawal rate. Exceptions are the 12 cells containing Weber Basin Water Conservancy District and HAFB wells. There, much pumping occurs and existing well capacities are the upper bounds on pumping.

The same lower bounds on head are imposed for both EM and RM models. As few head bounds as possible were used because RM optimization model memory requirement is based on the number of nonzero elements it contains. Each influence coefficient is a nonzero. In the RM model each head is calculated via eqn (4) — a summation of 61 pumping rates times influence coefficients. In the EM model, since each cell is represented by a separate flow equation, all heads are automatically computed. There is no difference in EM model memory requirement between setting bounds on 1 head or 4,600 heads.

Lower bounds on heads are employed in 13 locations (a location is a particular row, column, and layer). In the 12 major pumping locations, the lower bounds on head in pumped locations are \( 6·1 \) m (20 ft) below 1985 heads. In the upper-layer-cell having the least saturated thickness in 1985 (Layer 1, Row 19, Column 25), the lower bound on head is the base of the aquifer layer.

Results from embedding and response matrix S/O models

Heads in 1985 are used as the initial guesses. Optimal pumping rates and heads computed by both models are almost identical on a VAX 5240. These heads were also compared with those that result from using the optimal pumping strategy as input for MODFLOW simulation. There was insignificant difference between heads computed by the cyclical S/O models and MODFLOW.

The EM model included 12,433 equations, 12,521 variables, 46,533 nonzero elements, and 7 MBytes of memory. (This memory requirement includes preliminary and scratch files needed by MINOS and GAMS.) The RM model had 14 equations, 102 variables, 895 nonzero elements and only required \( 6\% \) of the memory needed by the EM model in every cycle.

The EM model requires 103 min of CPU time for the first cycle but only about 4 min after the second cycle. The RM model needs 8 to 13 min for every cycle, including running two external simulation models. Since both models need 10 cycles to converge, total CPU time is slightly less for the RM model. However, if any new bounds or constraints require new influence coefficients generation, then the RM model could need more total CPU time than the EM model.

Selection of S/O model for subsequent optimizations.

Selection of which modelling approach to use for additional scenarios should consider anticipated computer memory and processing time requirements. For the EM model, memory and processing time requirements do not change as the numbers of potential pumping locations or head control locations increase. For the RM model, these requirements increase exponentially or dramatically as the numbers of locations increase. In essence, RM memory needs to increase greatly as the number of terms \((6q^p\) products) in eqn (4) applied to each head control location increases. RM processing time increases because many more pre-optimization simulations are needed, and the optimization problem formulation becomes more difficult to
solve as the number of nonzero terms increase. The ramifications of expected management scenarios on model selection are explained below.

Many more head control locations will be needed in subsequent scenarios. In the preliminary scenario, it is assumed that the greatest head declines occur at modelled pumping locations, and that few other locations need head constraint. That assumption might be inappropriate here. Legal concerns arise even if water levels drop in unmodelled minor wells. The maximum drawdown actually occurs between modelled wells near the mountains, and the mountains in Layers 2 and 3. We are unable to specify, before optimization, where the maximum drawdown might occur. It is desirable to specify lower bounds on head (maximum drawdown) at more than just pumping cells for subsequent management scenarios (discussed in the next section). The entire urbanized zone is candidate for bounded heads.

In one tested scenario discussed below there are 846 potential pumping and 813 potential flowing well locations. There are 602 potential drainage cells and about 1000 cells in the urbanized portion of the study area. Even if heads need to be constrained in only 1000 locations, the RM optimization model could require about 846,000 influence coefficients (846 x 1000). (This results because this is a steady-state optimization, and most concerns about heads in confined layers. Pumping in one lowest-layer cell affects steady heads at most other middle and lowest layer cells.) Each influence coefficient is one nonzero value in the RM model constraint array. This 846k nonzeros is far more than 47k nonzeros required by the EM model (which remain constant in number regardless of how many heads are bounded or pumping values are variables).

For scenarios discussed below, the RM method would require more computer processing time than the EM approach. Scenarios requiring evaluation will permit pumping in up to 846 cells. In the RM approach, for each cycle, one pre-optimization simulation is needed per potential pumping location to develop influence coefficients. Assuming 846 potential pumping locations, 846 simulations of the entire study are needed by the ICG per cycle. In addition to the ICG simulations, the procedure also requires optimal problem solution and one simulation in the pre-ICG.

Solution time for the actual RM optimization problem will also increase as the number of pumping and head control locations increases. This results partially because each added head control location represents an added equation. Perhaps as significant, each head constraint equation (eqn (4)) becomes longer with each additional potential pumping location. Even though it will have fewer constraint equations than the EM model, the RM optimization model will contain many more terms and will take longer to solve.

In summary, both RM and EM models require cycling to address the nonlinear problem. Because it always has one equation per cell and must compute head in each cell, EM approach memory requirement and solution time will be relatively unaffected by increasing numbers of potential pumping and head control locations. The RM approach requires extensive simulations to compute influence coefficients and will require dramatically increasing computer memory and processing time. The RM approach is a viable alternative to the EM approach for steady-state optimizations if constraints and bounds on variables do not need to be specified at too many locations. Because of its flexibility and easy adaptability, the EM model is chosen to compute optimal strategies for the other scenarios evaluated in this study.

USE OF EMBEDDING S/O MODEL FOR PERENNIAL-YIELD PUMPING STRATEGIES

The results of alternative future scenarios are compared. Urbanization during the last 20 years has increased demand for M&I, but demand for irrigation water has increased little. Those trends are expected to continue. Common assumptions for all scenarios are; (i) it is more important to extract water for M&I use than to have flowing wells for agricultural use, and (ii) it is desirable that optimal pumping not be less than current pumping in any cell. Study area cells are divided among 25 water entities (governmental bodies) of Davis, Weber, and Box-Elide counties.

In overview, Scenario (i) is the nonoptimal scenario, and is merely simulated. For the other scenarios, optimal sustainable annual groundwater pumping rates are computed using the modified USU EM. In Scenario (ii), total sustainable pumping is maximized from the 61 cells currently containing M&I pumping wells. If existing wells cannot supply water of sufficient quality, more wells can be installed. In Scenarios (iii) and (iv), the S/O model chooses appropriate pumping locations from among many candidates.

Convergence criteria and solution time

For all scenarios, the EM S/O model is cycled (Fig. 6) until the difference between consecutive optimal pumping rates is less than 0·01%. The difference between heads for two consecutive cycles (DHC) is an indicator of solution stability since the flow equation and all piecewise external flows are functions of head. Except for Scenario (iv), the maximum DHC is 0·3 to 0·6cm (0·01 to 0·02ft) for 4880 cells. For scenario (iv), a 0·9 cm (0·03 ft) DHC occurred in a few cells.

Processing time for each scenario varied depending on proximity of the initial guess to the optimal solution. The longest total processing time (for all cycles needed for convergence) was about 2·5.
ACKNOWLEDGMENTS

The authors are grateful for the financial support of the Nippon Koei Company, the US Geological Survey, the Utah Water Research Laboratory, and the Utah State University Dept. of Biological and Irrigation Engineering and Agricultural Experiment Station. Approved as journal paper no. 4627.

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