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OPTIMIZING CONJUNCTIVE USE UNDER SUSTAINED YIELD CONSTRAINTS

by

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SUMMARY: Distributed parameter groundwater management model utilizing quadratic programming to develop a steady-state potentiometric surface is presented. Minimization of cost of meeting water needs from groundwater and alternative water sources is achieved. Drawdowns, groundwater withdrawal and recharge are all constrained. Applicable for assuring a regional sustained yield of groundwater.

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INTRODUCTION

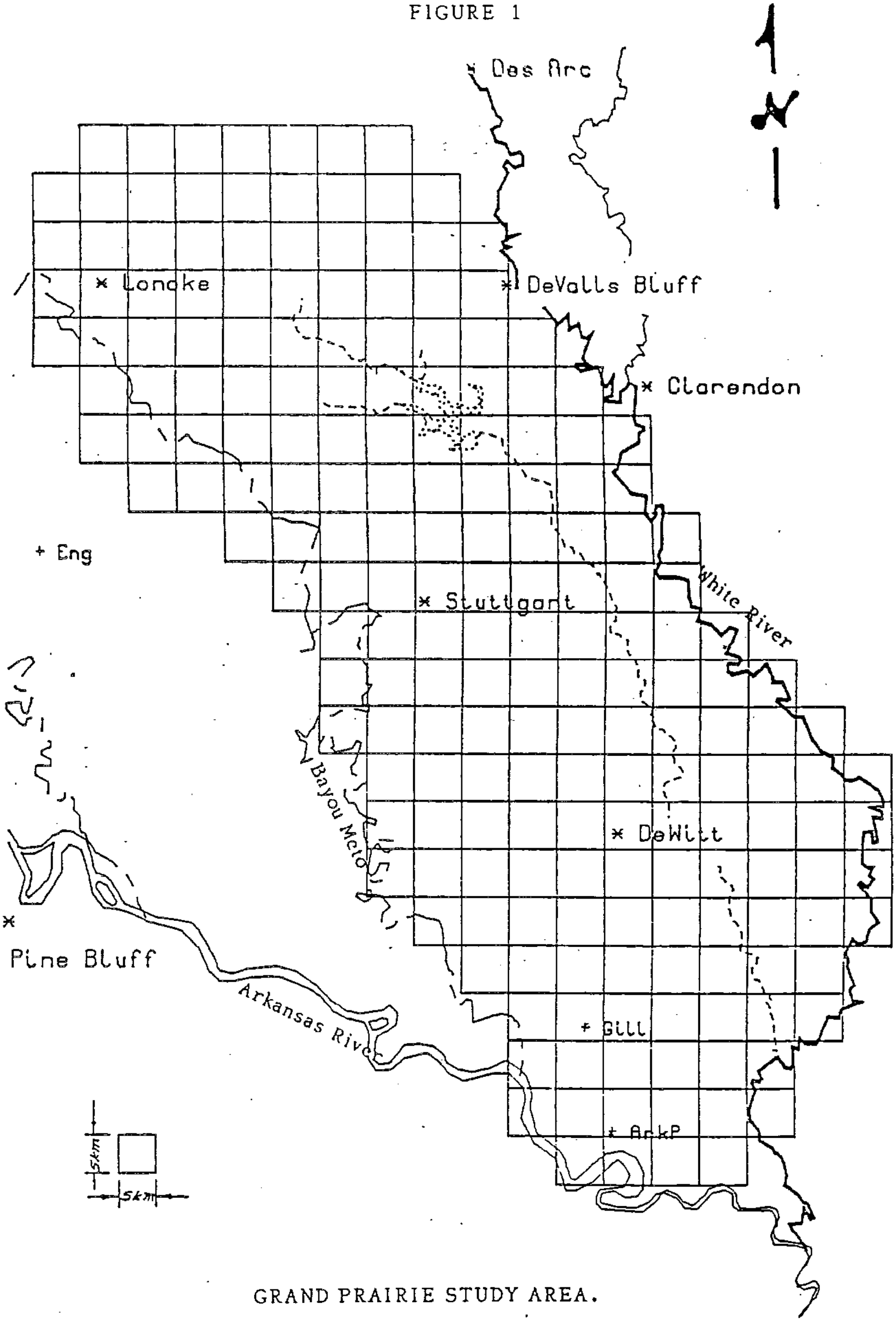
The Grand Prairie of Arkansas is an important rice producing region. Most of the irrigation water needs for rice and its rotation crop, soybeans, have historically been met by groundwater from an unconsolidated Quaternary alluvium. This extensive formation, the Mississippi alluvial aquifer, underlies much of eastern Arkansas, as well as parts of neighboring states. In Figure 1 the Grand Prairie region is shown divided into cells 5 km by 5 km in size. It has historically been assumed that an impermeable clay layer prevents recharge of the aquifer except for at some locations along the area's periphery where streams penetrate to the permeable material (Engler et al, 1945; Griffis, 1972, Peralta et al, 1984b). As a result, recharge has not kept pace with groundwater pumping and the potentiometric surface has been declining. Saturated thicknesses are decreasing and in some locations Quaternary groundwater cannot be obtained at useful discharge rates. This trend is projected to continue if current groundwater usage continues (Peralta et al, 1984b). The use of alternative sources of water is a prerequisite for meeting current water needs if stable groundwater levels are to be achieved and maintained.

The Grand Prairie Water Supply Project was initiated to determine how best to physically and legally coordinate the use of available water resources to meet long-term water needs and to develop the technical/institutional tools necessary to implement the resulting water management strategy, should that be desired. An overview of the different sub-projects, funding agencies and critical path approach to the effort is described by Peralta et al (1984a).

One significant result of the project has been acknowledgement, within the state water plan, of the physical and legal feasibility of attempting to achieve a steady state potentiometric surface in areas with critical groundwater problems by conjunctive use of ground and surface water (Peralta and Peralta, 1984). In that report, a "steady state" potentiometric surface is a set of <u>spring</u> groundwater elevations (target levels) which are maintained year after year. A finite difference form of the Boussinesq equation is used to determine the annual volume of groundwater which must be withdrawn from the aquifer in each cell in order to maintain a particular set of target levels. The authors presented an example, using dynamic simulation, in which spring target levels were maintained for at least ten years (even though water needs and hence groundwater usage were not constant throughout the year), as long as each cell's total annual groundwater withdrawal did not exceed its annual withdrawal volume calculated by the steady state equation.

There are many possible sets of steady state (target) groundwater levels for any area, each one corresponding to a particular strategy of sustained groundwater withdrawal. In this paper the term "pumping" is used to refer to groundwater withdrawal. Some sets of target levels and groundwater pumping strategies are more desireable than others. An important step in

FIGURE 1



GRAND PRAIRIE STUDY AREA.

the total Grand Prairie project is development of a management model for determining optimal target levels. The model discussed in this paper is a computer program which uses a quadratic programming subroutine (Liefsson et al, 1981) for optimization and embedded equations describing porous media flow as constraints. The objective of this paper is to describe an application of this model in calculating the set of regional potentiometric surface elevations which result in the least expenditure for meeting water needs from ground and alternative water resources, subject to sustained yield and other constraints.

MODEL FORMULATION AND APPLICATION

In this paper the term "water needs" refers to current groundwater usage. It is assumed that actual current needs being met by other means will continue to be met by those means. The problem which the managment model addresses is how best to meet "water needs" either from Quaternary groundwater or from some new alternative source under steady-state conditions. The new sources are either real--surface water diverted from the Arkansas or White Rivers or pseudo--water saved by reduction in water needs. Thus, it is assumed that in each cell there are two sources of water. One is groundwater, the other is the most inexpensive alternative source. For purposes of this paper, diverted surface water is the alternative source in all cells where it is assumed to be available. In other cells reduction in needs (ie by reducing irrigated or aquacultural acreage) is the alternative. A state or local water management agency may consider other alternative sources in using the model, such as increased use of on-farm reservoirs, reduced water needs due to conservation measures, etc. Preliminary assessment of the cost per unit volume and the potential quantitative availability of these other "sources" for the Grand Prairie has been accomplished (Harper, 1983).

Figure 2 shows the cost of an appropriate alternative source of water for each cell in $/dam3. Cells showing a price of 15 or 16 $/dam3 are those to which the Corps of Engineers feels it can deliver water diverted from the Arkansas River. Preliminary Corps estimates are that White River water can probably be diverted to other cells at a price of 51 $/dam3. Recent reconnaisance level evaluation has indicated that legally and physically available Arkansas and White River water is adequate to meet "water needs" in the cells serviceable by those rivers, assuming average climatological and hydrologic conditions (Dixon and Peralta, 1984). The lost opportunity cost of not being able to meet the water needs of aquacultural production of fish or minnows is assumed to be 114 $/dam3, whereas that for rice production is assumed to be 195 $/dam3. These costs are shown where appropriate in cells to which diverted surface water is unavailable. Cells showing a 0 value in Figure 2 are used as constant-head cells in the management model. Water needs, groundwater withdrawal and water use are assumed to be zero in constant-head cells. Validation of an unsteady state groundwater simulation model AQUISIM, developed by Verdin et al (1981), verified that the study area could be treated as a groundwater system surrounded by

FIGURE 2

cells marked with 52 are from Eastern ~~Arte~~ G.P.I Study

In general cells marked with 15 are from Bay on Meto Study

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
24																		
23																		
22		0	0	0	0	0	0	0										
21	0	114	114	114	114	114	185	185	0									
20	0	16	16	16	114	114	195	195	195	0								
19	0	16	16	16	16	52	52	52	52	0								
18	0	15	15	15	16	52	52	52	52	52	0							
17		0	15	15	15	52	52	52	52	52	185	0						
16		0	15	15	15	15	52	52	52	52	52	185	0					
15			0	0	15	15	52	52	52	52	52	185	0					
14					0	15	15	52	52	52	52	52	185	0				
13						0	15	15	52	52	52	52	52	0				
12							0	15	15	52	52	52	52	185	0			
11							0	15	15	52	52	52	52	185	185	0		
10							0	15	15	52	52	52	52	52	185	185	0	
9								0	15	15	52	52	52	52	195	195	195	0
8								0	15	15	52	52	52	52	195	195	195	0
7								0	15	15	15	52	52	52	52	185	195	0
6									0	15	15	52	52	52	52	185	185	0
5										0	15	15	52	52	52	52	0	
4											0	15	52	52	52	52	0	
3											0	15	15	15	15	0		
2											0	15	15	15	15	0		
1												0	0	0	0			
0																		
-1																		

COST OF "ALTERNATIVE" WATER. ($\$/dam^3$)

Cell #'s on this page are wrong.

constant-head cells whose elevation reflected average groundwater elevations (Peralta et al, 1984b).

Because the sum of groundwater and alternative sources, including reduction in needs, must equal the water needs in each cell, the following expression is used.

$$WAN_{i,j} = P_{i,j} + SW_{i,j} \quad \ldots\ldots 1$$

where, $WAN_{i,j}$ is the annual water needs of cell i,j (L^3/T).

$P_{i,j}$ and $SW_{i,j}$ are the annual volumes of groundwater and alternative water, respectively, which are used to meet needs under a developed conjunctive use strategy (L^3/T).

Development of a regional steady state set of target groundwater levels requires the use of a steady state equation for each cell. The following has been developed for two-dimensional steady flow in a heterogeneous isotropic aquifer from both the linearized Boussinesq equation (Illangasekare and Morel-Seytoux, 1980) and the Darcy equation (Peralta and Peralta, 1984).

$$\begin{aligned} Q^{ss}_{i,j} = & - T_{i-1/2,j} S_{i-1,j} - T_{i+1/2,j} S_{i+1,j} \\ & + [T_{i-1/2,j} + T_{i+1/2,j} + T_{i,j-1/2} + T_{i,j+1/2}] S_{i,j} \\ & - T_{i,j-1/2} S_{i,j-1} - T_{i,j+1/2} S_{i,j+1} \quad \ldots\ldots 2 \end{aligned}$$

where $Q^{ss}_{i,j}$ = the net vertical accretion of groundwater moving into or out of the aquifer in a cell. It is positive when flow is into the aquifer, negative when flow is out of the aquifer (L^3/T).

$S_{i,j}$ = the vertical distance between a horizontal datum located somewhere above the ground surface, and the potentiometric surface. In this paper this is a steady state drawdown (L).

$T_{i-1/2,j}$ = the geometric average of the transmissivities of cells (i,j) and (i-1,j) (L^2/T).

For an entire groundwater flow system of n cells, this equation is expressed in matrix form as:

$$(Q) = [TT](S) \qquad \ldots\ldots 3$$

where (Q) is an n x 1 column vector of net steady-state accretion values (L3/T).
[TT] is an n x n symmetric diagonal matrix of finite difference transmissivities (L2/T).
(S) is a column vector of steady state drawdowns (L).

Because vertical recharge of the aquifer in the Grand Prairie is negligible for all interior (non-constant head cells), the net annual vertical accretion for each of those cells equals its groundwater withdrawal volume. The value in the (Q) vector corresponding to a constant head cell is the volume of water entering (-) or leaving (+) the system at that cell. Since no pumping is considered at constant-head cells in this paper, this is the volume of recharge to or discharge from the aquifer and the outside system. The transmissivities in this paper are based on a hydraulic conductivity of 82.3 meters per day (Engler et al, 1945; Griffis, 1972; Peralta et al, 1984b).

The objective of the management model is to determine the set of steady-state drawdowns (S) which minimize the total cost of meeting water needs. For a system of m internal cells:

$$\min Z = \sum_{k=1}^{m} [TGCk + TSCk] \qquad \ldots\ldots 4$$

where TGC_k is the total cost of groundwater used in a year in cell k (\$/T).

TSC_k is the total cost of "alternative" water used in a year in cell k (\$/T).

$$TGCk = CGk \times TDHk \times \frac{(S_k - GRSUR_k)}{(SI_k - GRSUR_k)} \times Pk \qquad \ldots\ldots 5$$

where CG_k is the cost of raising a unit volume of groundwater one unit of distance at cell k based on maintenance, labor and current energy costs (\$/L^4).

TDH_k is an initial total dynamic drawdown assumed for cell k based on representative well sizes, pumping rates corresponding to irrigation scheduling, initial assumed static lift and aquifer saturated thickness (L)

S_k is the optimal static drawdown (distance between the datum elevation and the optimal potentiometric surface) for cell k (L).

$GRSUR_k$ is the distance between the datum elevation and the ground surface elevation at cell k (L).

SI_k is an initially assumed static drawdown (L).

P_k is the same as $P_{i,j}$ in equation 1 (L^3/T).

The difference between a static drawdown and GRSUR is a static lift. The static lift is the major contributor to the total dynamic drawdown. Therefore the ratio (S-GRSUR)/(SI-GRSUR) serves to increase or decrease the initial estimate of the TDH to provide an estimate of total dynamic head which is appropriate for the optimal drawdown S. It does so in a linear manner which can be readily incorporated in the objective function. Thus Equation 5 is expressible as:

$$TGC_k = CGO_k \, (S_k - GRSUR_k) \, P_k \quad \ldots\ldots 6$$

where $CGO_k = CG_k \times TDH_k / (SI_k - GRSUR_k)$ ($\$/L^4$)

The cost of alternative water in a cell is:

$$TSC_k = CS_k \times (WAN_k - P_k) \quad \ldots\ldots 7$$

where CS_k is the cost of the alternative water source ($\$/L^3$).

The quadratic programming subroutine which is used in this model is based on the general differential algorithm (Wilde and Beightler, 1967). In order to insure that a local minimum is also a global minimum, the objective function must be convex. Convexity is assured if the matrix of coefficients of the quadratic terms in the objective function (Hessian matrix) is positive definite. To obtain an appropriate Hessian matrix, all of the net pumping volumes (P) of equations 6 and 7 must be expressed in terms of static drawdown. Recalling that the net accretion equals the pumping volume for all interior cells, Equation 2 is used to redefine (P) as a function of transmissivities and drawdowns. Matrix [TI] is defined as the m x m transmissivity matrix for the m number of interior cells of the system. Each cell is labeled with a unique integer value k. Thus the objective function is expressed in matrix form as:

$$\text{Min } Z = (SI)^t\ [TI_a]\ (SI) - [TI_b]\ (SI) + (d) \qquad 8$$

subject to

$$(L_q) \leq [TT]\ (S) = (Q) \leq (U_q) \qquad 9$$

$$(L_{si}) \leq (SI)_t \leq (U_{si}) \qquad 10$$

where $(SI)_t$ is the 1 x m transpose of the column vector of

drawdowns for the interior cells (L).

$[TI_a]$ is the Hessian matrix which results when each column k of [TI] is multiplied by CGO_k ($\$/L^2T$).

$[TI_b]$ is the matrix which results when each column k of [TI] is multiplied by $((CGO_k)(GRSUR_k)+CS_k)$ (\$/LT).

(d) is a vector of constants which is therefore not included in the optimization, but is added to the output value to determine the actual least cost (\$/T).

(L_q) is a n x 1 column vector of lower bounds on the net accretion in each cell. It's value is zero for most internal cells since no recharge occurs internally. (For demonstration purposes the lower and upper bounds on accretion were made equal in a few cells.) At constant-head cells it is a negative number representing the maximum physically or legally feasible recharge at constant-head cells (L3/T).

(Q) is a column vector representing the optimal steady-state accretion values, ie optimal sustained yield pumping values for all internal cells and the optimal volume fluxes for all constant-head cells (L3/T).

(U_q) is a column vector of upper bounds on the net accretion in the cells. In this paper the upper bound for a particular internal cell is that cell's current annual groundwater pumping. The upper bound for constant-head cells equals a large positive number. This is our standard procedure since the total recharge for the entire system is limited to the total discharge. Limiting discharge from one system boundary may result in unnecessarily limiting recharge along a different boundary where recharge is needed (L3/T).

(L_{si}) is a column vector of lower bounds on the optimal static drawdown for internal cells, ie the ground surface elevation (L).

(U_{si}) is a column vector of upper bounds on the static drawdown for internal cells (L).

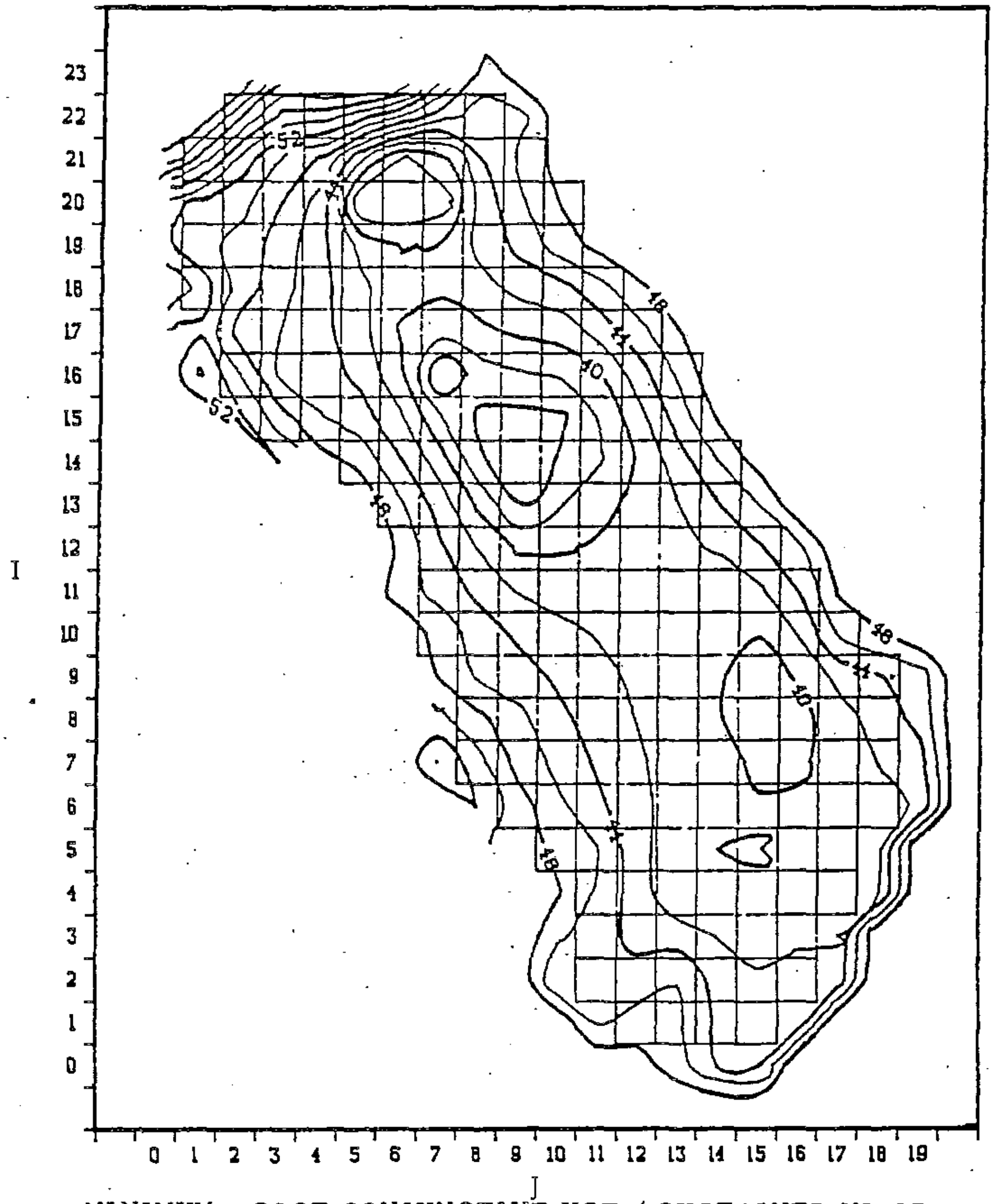
In this example, the upper bound for all but one cell equals the drawdown which leaves 20 feet of saturated thickness in the aquifer in those cells. In the exception cell (i,j=10,9) the limiting saturated thickness is 4 meters, the observed value in 1982. Peralta et al (1984) show 6 meters to be the saturated thickness at which representative wells pumping to meet the scheduled irrigation needs of rice in the Grand Prairie will begin to go dry during the irrigation season (assuming one well per 50 acres of rice, non-interference between wells, no hydraulic gradient other than that of the cone of depression and average climatologic conditions). Dutram and Peralta (1984) show that 4 meters is adequate for cell 10,9 under average climatological conditions and assuming current acreages and wells are maintained.

RESULTS AND DISCUSSION

The objective function of equation 8 uses transmissivities which are based upon some initial saturated thicknesses. As the optimization procedure changes the groundwater levels, the saturated thicknesses and transmissivities also change. To accomodate this fact without introducing non-linear constraints, sequential optimizations are performed: the second uses as input the transmissivities corresponding to the optimal potentiometric surface of the first optimization, the third uses ...etc, until the resulting optimal drawdowns are within an acceptable tolerance of the input drawdowns. In this example, after three successive optimizations, convergence to within 2 feet was achieved for all cells and within 1 foot for all but three cells. At the same time that the transmissivities are modified, the SI_k and TDH_k of equation 5 and CGO_k of equation 6 are changed to reflect groundwater costs appropriate for the optimal drawdowns.

The optimal steady-state potentiometric surface elevations based on the stated assumptions are shown in Figure 3. The cost per unit volume of groundwater based on this surface is shown in

FIGURE 3



MINIMUM - COST CONJUNCTIVE USE / SUSTAINED YIELD
POTENTIOMETRIC SURFACE. (m above sea level)

Figure 4. Comparison of Figures 2 and 4 indicates that for the optimal potentiometric surface, groundwater is the less expensive source of water in all cells. Subject to the groundwater flow equation and constraints limiting recharge, it is therefore used wherever possible. Figure 5 shows the percentage of the water needs of each cell which are met by groundwater in the optimal strategy. Comparing the cells serviced by surface water in Figure 2 with the cells utilizing groundwater in Figure 5, one realizes that the flow and recharge constraints are somewhat limiting and that, under sustained yield conditions, it is regionally less expensive to use surface water in most cells where it is available.

Figure 6 shows cells in which water needs are unmet by either groundwater or diverted surface water. In these cells the opportunity costs of lost aquacultural or rice production are used in the determination of the least cost solution. Comparison of Figures 2 and 6 reveals that it is only in cells without diverted surface water that water needs are unmet. The volume of unmet water needs in these cells can be divided by 2.2 m or 0.6 m to determine the acres of aquacultural or rice production respectively which cannot be supplied under the optimal strategy.

Figure 7 shows the percent of the maximum feasible recharge in constant head cells which is utilized in the optimal strategy. Scanning the top (northern) edge of the study area, one notes in cell 22,4 that 100 percent of the recharge is utilized. Despite this, Figure 6 shows that there is unmet water demand in the cell directly down-gradient of that cell (21,4). From Figures 2 and 5 respectively, one sees that no surface water is available in cell 21,4 and that only 58 percent of its water needs are met by groundwater. From Figure 5 one sees that the cells to its right and left meet 100 percent of their needs. In order for more groundwater to be able to pass from those two cells to cell 21,4, the groundwater withdrawal of cells 21,3 and 21,5 must be increased. This fact is made clear by examination of Figure 8, which contains the constrained derivatives of the objective function with respect to the net accretion decision variables. For internal cells these non-zero coeficients explain the effect on the total optimal cost of a unit change in groundwater pumping. The constrained derivative for cell 21,3 is -64 $/dam3. Up to a point, for each dam3 of increased groundwater pumping in cell 21,3 the total cost of water, including lost opportunity costs, decreases $64. One can assume that some of the recharge induced at cell 22,3 by increased groundwater pumping at cell 21,3 would find its way to cell 21,4 to help reduce its unmet demand. One sees that all internal cells containing negative constrained derivatives in Figure 9 show 100 percent of their needs met by groundwater in Figure 5. These are all cells in which increasing the permitted volume of pumping decreases the total cost, most probably by increasing recharge. Cells 11,14; 11,15; 12,15; 12,16; 13,15; and 13,16 meet 100 percent of their needs by groundwater, but have positive constrained derivatives in Figure 8. These are the cells in which, for the sake of demonstration, the upper and lower bounds on pumping are equal to each other, forcing needs to be met by groundwater.

Positive constrained derivatives for constant head cells in

FIGURE 4

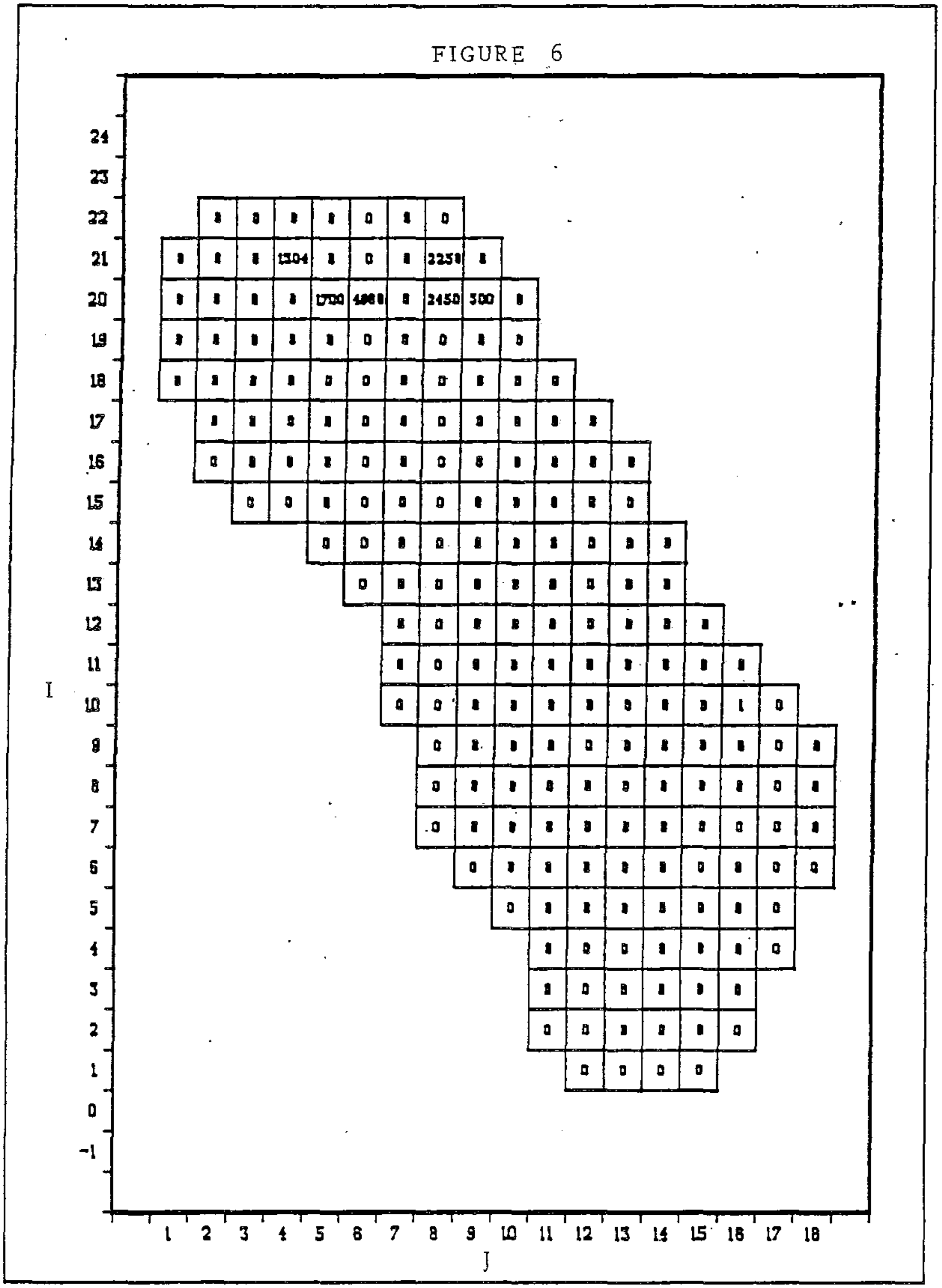
I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
24																		
23																		
22		0	0	0	0	0	0	0										
21	0	10	10	9	12	12	10	7	0									
20	0	9	11	11	15	15	15	10	5	0								
19	0	10	9	9	12	13	14	11	10	0								
18	0	10	10	10	11	12	11	11	11	8	0							
17		0	8	10	10	11	12	11	10	11	9	0						
16		0	8	8	9	9	12	13	11	10	10	9	0					
15			0	0	7	8	10	14	12	12	10	10	0					
14					0	7	8	11	13	12	10	10	8	0				
13						0	7	9	12	10	10	9	9	0				
12							0	8	11	10	9	9	6	8	0			
11							0	8	9	9	9	8	9	8	8	0		
10							0	6	8	8	9	8	8	8	7	7	0	
9								0	7	7	7	7	5	4	7	7	2	0
8								0	6	8	5	7	7	6	8	6	3	0
7								0	5	6	5	7	8	7	6	6	3	0
6									0	5	5	6	7	7	7	3	3	0
5										0	4	6	6	7	3	4	0	
4											0	6	7	4	5	4	0	
3											0	5	6	5	5	0		
2											0	4	4	4	3	0		
1												0	0	0	0			
0																		
-1																		

COST OF GROUNDWATER UNDER THE MINIMUM – COST STRATEGY.
($\$/dam^3$)

FIGURE 5

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
24																		
23																		
22		0	0	0	0	0	0	0										
21	0	100	100	58	100	100	100	0	0									
20	0	0	0	0	79	27	100	0	0	0								
19	0	0	0	0	0	0	0	0	0	0								
18	0	0	0	0	0	0	0	0	0	100	0							
17		0	0	0	0	30	0	0	0	100	100	0						
16		0	67	0	0	0	100	0	24	100	98	100	0					
15			0	0	0	0	100	52	31	100	100	100	0					
14					0	30	0	100	100	32	100	100	100	0				
13						0	0	0	100	0	0	0	100	0				
12							0	0	0	0	0	0	0	100	0			
11							0	0	0	0	0	0	0	100	100	0		
10							0	0	0	0	0	0	0	0	100	100	0	
9								0	0	0	0	0	0	0	100	100	100	0
8								0	0	0	0	0	0	0	100	100	100	0
7								0	100	0	0	0	0	0	0	100	100	0
6									0	0	0	0	0	0	0	100	100	0
5										0	0	0	100	100	100	100	0	
4											0	0	100	100	100	100	0	
3											0	81	0	0	0	0		
2											0	100	100	100	100	0		
1												0	0	0	0			
0																		
-1																		

PERCENT OF WATER NEEDS MET BY GROUNDWATER UNDER THE MINIMUM - COST STRATEGY.



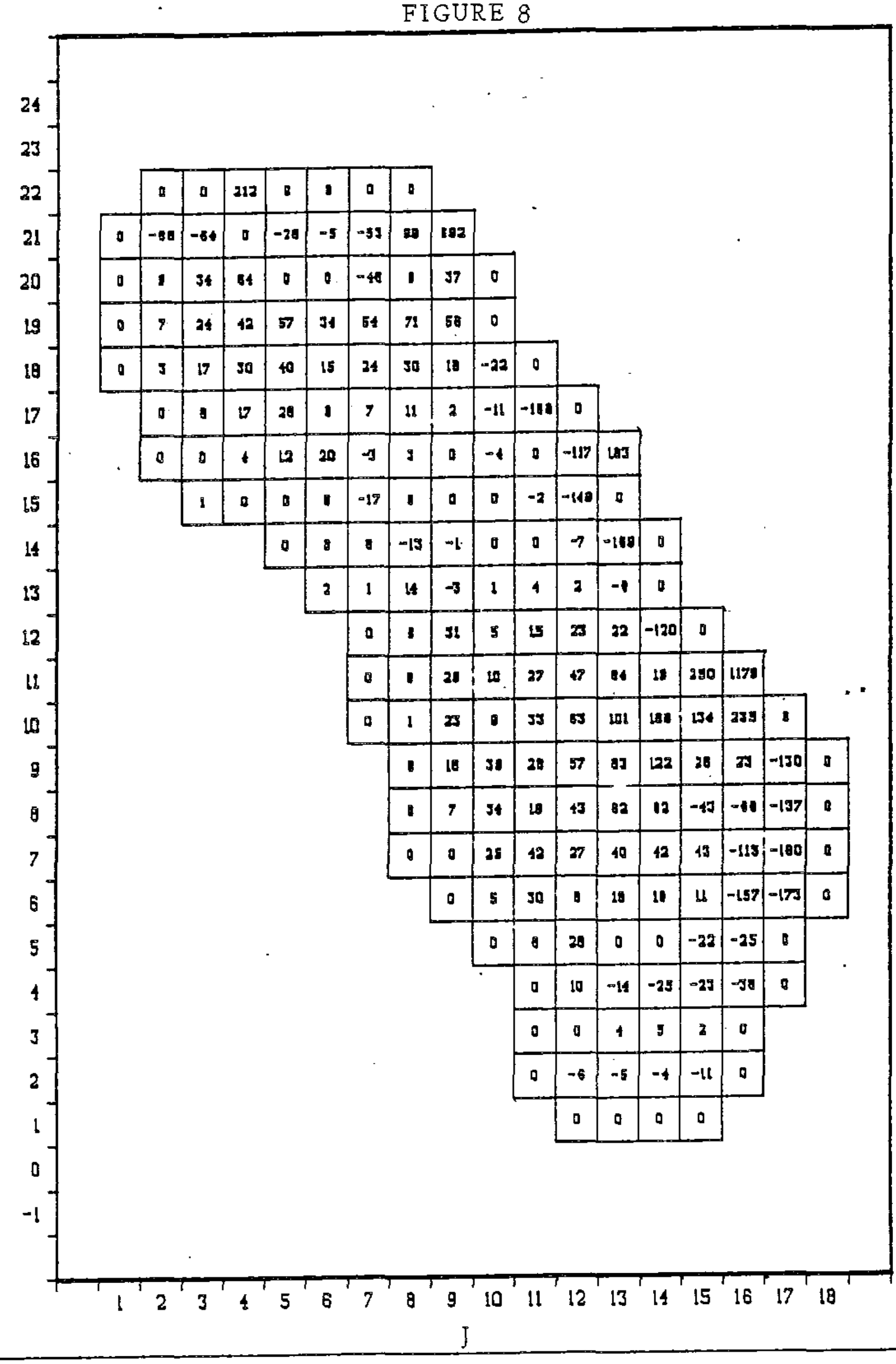
VOLUME OF UNMET WATER NEEDS UNDER THE MINIMUM - COST STRATEGY. (dam^3)

FIGURE 7

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
24																		
23																		
22		84	22	100	77	82	30	2										
21	84	0	0	0	0	0	0	0	100									
20	-27	0	0	0	0	0	0	0	0	98								
19	-22	0	0	0	0	0	0	0	0	83								
18	58	0	0	0	0	0	0	0	0	0	85							
17		-50	0	0	0	0	0	0	0	0	0	93						
16		100	0	0	0	0	0	0	0	0	0	0	100					
15			100	-18	0	0	0	0	0	0	0	0	90					
14					88	0	0	0	0	0	0	0	0	86				
13						100	0	0	0	0	0	0	0	53				
12							55	0	0	0	0	0	0	0	90			
11							26	0	0	0	0	0	0	0	0	100		
10							80	0	0	0	0	0	0	0	0	0	81	
9								-27	0	0	0	0	0	0	0	0	0	87
8								33	0	0	0	0	0	0	0	0	0	54
7								75	0	0	0	0	0	0	0	0	0	32
6									35	0	0	0	0	0	0	0	0	32
5										18	0	0	0	0	0	0	15	
4											31	0	0	0	0	0	82	
3											21	0	0	0	0	57		
2											-3	0	0	0	0	19		
1												42	54	-108	0			
0																		
-1																		

(OPTIMAL RECHARGE RATE)/(MAXIMUM FEASIBLE RECHARGE RATE)
FOR CONSTANT HEAD CELLS UNDER THE MINIMUM - COST STRATEGY.

FIGURE 8



∂Z/∂Q, CONSTRAINED DERIVATIVES WITH RESPECT TO NET ACCRETION UNDER THE MINIMUM – COST STRATEGY. ($/dam^3)

Figure 8 show how much the total cost is reduced per unit of increase in recharge volume. This effect is the same as that of negative derivatives for internal cells because recharge is opposite in sign to discharge.

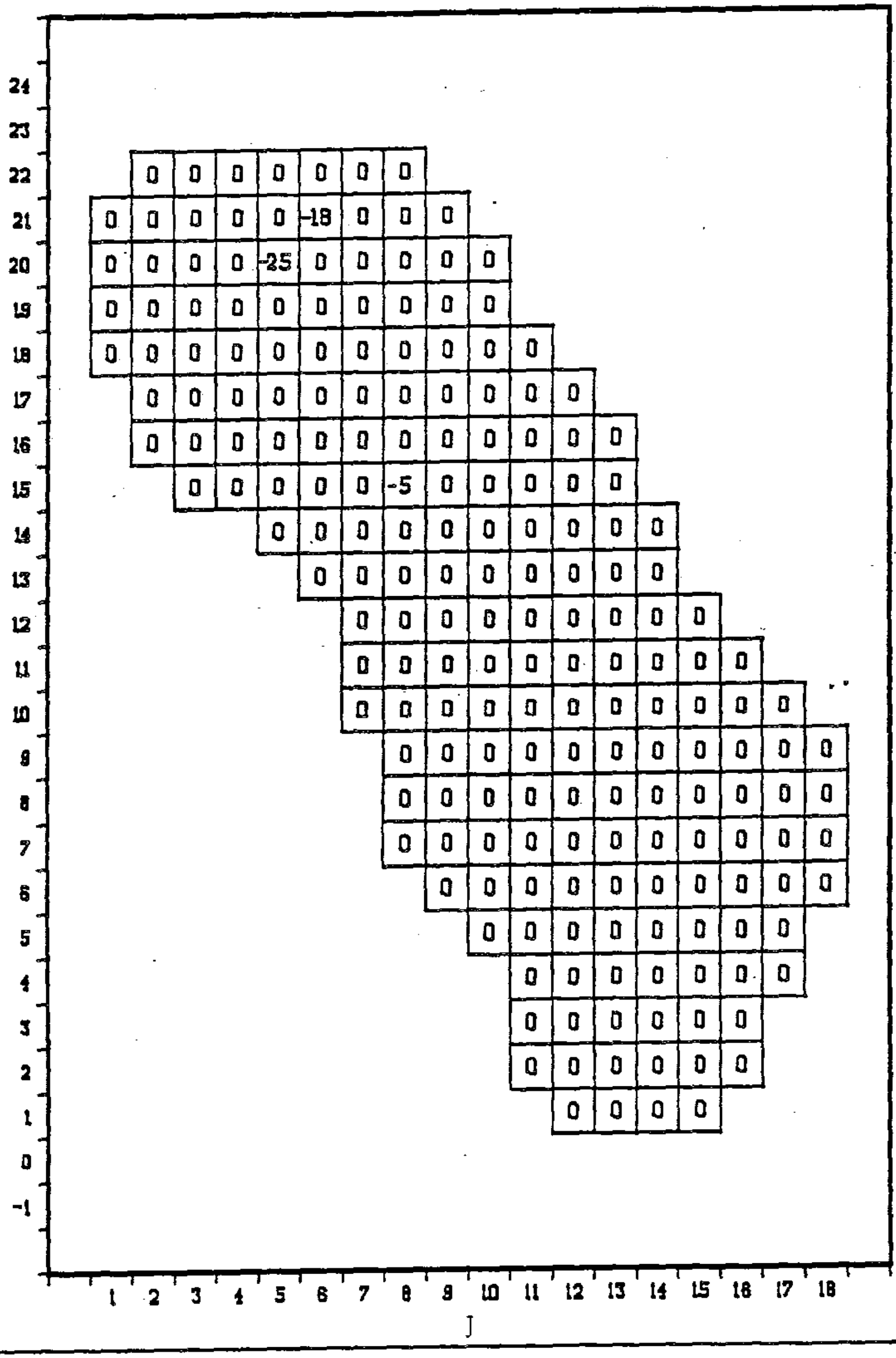
Figure 9 contains constrained derivatives with respect to drawdown. These indicate the reduction in total cost in thousands of dollars per meter increase in the maximum permitted drawdown. Thus these give a measure of the cost of preserving 6 meters of saturated thickness in the cells with non-zero values.

The minimum value of the objective function including (d) is $12,179,000. Of this, $1,940,000 is opportunity cost. The total volume provided is 413,000 dam3 and the average cost per cubic dam is $29. As previously stated, preliminary evaluation indicates that the necessary volume of diverted surface water is physically and legally available from these rivers during "average" summers. A more detailed assessment of streamflow and demand is necessary before hydrologic feasibility can be determined.

One weakness of this steady-state approach is that it does not include consideration of how long it may take for the optimal potentiometric surface to be attained. At the point of greatest difference, the spring 1982 potentiometric surface is about 12 meters lower than the optimal surface. At a different location, the optimal surface is about 14 meters below a natural unstressed potentiometric surface simulated for the area (using the same constant head cell elevations). In very approximate terms, the optimal surface is midway between the unstressed and the current surfaces. It has taken most of this century to dewater the aquifer to its present condition. Assuming that the presented optimal pumping strategy were utilized, beginning in 1982, it would take a number of years for actual levels to approximate target levels. Dynamic simulations using AQUISIM (Verdin et al, 1981) validated for the Grand Prairie (Peralta et al, 1984b) show that 86 percent of the cells would be within 6 meters and 70 percent would be within 3 meters of their target elevation within 10 years. After 30 years 95 percent and 75 percent would be within 6 meters and 3 meters of their target levels respectively. If the optimal strategy were enacted immediately upon development of the aquifer (ie beginning with unstressed conditions), after 10 years of pumping, 85 percent of the cells would be within 6 meters and 63 percent would be within 3 meters of their target elevations. After 30 years, 96 percent and 75 percent would be within 6 and 3 meters respectively.

Assuming a $0.35/dam3m energy cost of raising groundwater (Peralta et al 1984b) the 6 meters represents a $2.14/dam3 difference between actual price and the price assumed in the development of the pumping strategy. Analysis of the significance of such a difference coupled with the long time period necessary to reach an "optimal" potentiometric surface is beyond the scope of this paper. Similarly, analysis of the sensitivity of the solution to variations in energy costs, opportunity costs, surface water costs, etc. is omitted.

FIGURE 9



$\partial Z/\partial S$, CONSTRAINED DERIVATIVES WITH RESPECT TO DRAWDOWNS UNDER THE MINIMUM - COST STRATEGY. ($1000/m)

SUMMARY

This paper presents a procedure for minimizing, in steady-state, the cost of meeting water needs above an aquifer system. It is assumed that groundwater and some alternative source of water is available in each cell (square) of the study area. The total cost of water in each cell is the sum of the cost of groundwater and alternative water for that cell. The cost per unit volume of the "alternative" water, be it diverted surface water, opportunity cost or water saved by conservation measures, is known a priori. The volume of alternative water used in a cell is the difference between that cell's water needs and its groundwater usage.

It is assumed that the cost per unit volume of groundwater is a linear function of the distance between the ground surface and the potentiometric surface in each cell. The steady-state drawdowns (distance between a horizontal datum and the groundwater table) comprising the potentiometric surface are the variables. The total cost of using groundwater is a function of both the volume of groundwater usage and the distance that water must be raised. To insure global optimality, a finite difference form of the Boussinesq equation is used in lieu of the volume of groundwater withdrawal in the objective function and constraint equations. The result is an objective function which is quadratic in the drawdown variables.

The solution space of drawdowns is constrained by lower and upper bounds, the upper bound serving to assure sufficient saturated thickness exists to insure groundwater availability. For internal cells, functional equivalents of groundwater withdrawal are constrained to be non-negative and to be less than water needs. Similarly, for constant head cells (recharge sources), recharge is constrained to be less than an upper physically feasible limit. The constraints thus imbed within the management model the necessary equations describing steady-state two dimensional flow through a porous media. Because steady-state equations are used and recharge is limited to that which is assumed feasible, the groundwater withdrawal (pumping) strategy which will maintain the optimal steady-state potentiometric surface is a sustained yield pumping strategy.

Depending on the difference between a current potentiometric surface and an optimal surface, it may take a number of years of pumping in accordance with an optimal sustained yield pumping strategy before the optimal surface is attained. The optimization does not consider the period of evolving groundwater levels. It develops only the optimal steady-state levels. The sensitivities of the results to the evolutionary era and to the possibility of changing costs of energy, etc. are not presented. The approach's greatest potential lies in situations where a long-term guaranteed supply of groundwater is desired.

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