MAXIMIZING RELIABLE PRODUCTION
IN A
DYNAMIC STREAM/AQUIFER SYSTEM

by

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SUMMARY:
Reliable production of a dynamic stream/aquifer system is
determined through an implicitly stochastic optimization model.
Adequate representation of the inflow process and dynamic modeling of
the stream/aquifer system results in optimum crop yield at specified
reliability levels. Results include optimal spatial and temporal
allocation of groundwater and diverted river water use. These can be
used for planning cropping patterns and evaluating potential diversion
systems.

KEYWORDS:
Groundwater, Stochasticity, Reliability, Modeling,
Optimization, Operations research, Management, Conjunctive

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INTRODUCTION

The stochastic nature of streamflow is generally accepted and has led to the widespread use of synthetic hydrologic modeling in surface water studies. The random nature of streamflow is an important consideration in an area where crop yield is dependent on the applied surface water as well as groundwater. However, the vast majority of modeling efforts that involve systems with stream/aquifer interaction components do not incorporate this stochasticity.

This paper describes an implicitly stochastic optimization (ISO) procedure that couples inflow information (having an associated level of reliability) with a stream/aquifer system model. The purpose of the modeling effort is to develop strategies for groundwater pumping and river water diversion that minimize the reduction in crop yield. Such strategies provide valuable guidelines for cropping pattern selection and water management in an irrigation district.

Application of the methodology has two stages: a) inflow modeling, and b) system modeling. In the first stage, the statistical characteristics of the inflow process and prespecified probability levels establish influent magnitudes for which optimal strategies are to be developed. In the second stage, the best conjunctive use strategy is determined by an optimization model that adequately represents the dynamic nature of the stream/aquifer system. The resulting strategies are used as guides in cropping pattern selection. The methodology is applied to a hypothetical area for illustrative purposes.

PREVIOUS WORK

The estimation of the inflow model from available surface water data has led to a distinct discipline of hydrologic modeling. Jackson (1975) provides a comprehensive and critical discussion of the models developed before 1970. Of the numerous models that are available, linear stochastic models of the inflow process have gained acceptance. Salas et al. (1980) is an excellent reference of a detailed and instructive discussion of this group of models. Thus, methods for finding a process that adequately represents the stochasticity of inflow is well documented. No attempt to rigorously discuss the estimation procedure is included in this paper.

Many stream/aquifer simulation models have been reported. Maddock (1974), Morel-Seytoux (1975), Illangasekare and Morel-Seytoux (1982) and Danskin and Gorelick (1985) are a few examples. Gorelick (1983) provides a review of models oriented toward facilitating water management decision-making. Very few of the models address the reliability of the surface water resource and its consequences on irrigated agricultural planning.

MODEL DEVELOPMENT AND ASSUMPTIONS

Governing Equations

The following theory is appropriate for a scenario in which the objective is to maximize crop yield in an irrigation or water management district (Figure 1). Assume that crop yield is a function of the timed availability of water and that the water supply is inadequate to meet total irrigation requirements. Let the result of having unsatisfied water requirements be expressed as a reduction in yield from that which would be obtained if irrigation water needs were completely satisfied. Thus, the objective can be
simply restated as: minimizing the reduction in crop yield caused by inadequate water supply.

\[
\text{max Yield} = \text{Potential Yield} - \text{min Reduction in Yield} \quad \ldots \ldots 1
\]

The minimum reduction in yield caused by inadequate water availability during \(K\) time steps in a system consisting of \(J\) cells is expressed as:

\[
\text{min Reduction in Yield} = \sum_{i=1}^{J} \sum_{k=1}^{K} c_{i,k} u_{i,k} / w_{i,k} \quad \ldots \ldots 2
\]

where

\(y_i\) is the maximum potential annual crop yield from a cell \(i\) assuming that irrigation water needs are completely satisfied throughout the growing season, known, \((M)\);

\(u_{i,k}\) is the volume of unsatisfied water needs in cell \(i\) in time step \(k\), unknown, \((L)\);

\(w_{i,k}\) is the volume of water (including irrigation and effective precipitation) required in cell \(i\) in time step \(k\) in order to produce the maximum potential yield, known, \((L)\);

\(c_{i,k}\) is a dimensionless crop loss coefficient. It equals the proportional reduction in the annual potential yield in cell \(i\) that results from a proportional lack of adequate irrigation water in time step \(k\), known;

\(K\) is the number of time steps in the planning period, known;

\(u_{i,k} / w_{i,k}\) is the proportion of water needs in cell \(i\) in time step \(k\) that are unsatisfied.

A complete management model requires, in addition to an objective function (Equation 2), the inclusion of pertinent bounds on variables and constraints to assure that physical and institutional limits are appropriately
considered and that the hydrologic system is modelled adequately. Assume a study area underlain by an aquifer that is in hydraulic connection with a stream passing through the region. If there are practical or legal limits on how much groundwater and diverted river water can be used to attempt to satisfy water demand, a simple statement of bounds to be considered (assuming discharge to be positive in sign and recharge to be negative) is:

\[
\begin{align*}
0 & \leq u_{i,k} \leq u_{i,k} \\
0 & \leq g_{i,k} \leq u_{i,k} \\
0 & \leq r_{i,k} \leq u_{i,k} \\
U & \leq s_{i,k} \leq s_{i,k} \\
L & \leq e_{i,k} < e_{i,k} \\
L & \leq \sigma \leq \sigma \\
& \text{for } i = 1 \ldots J, k = 1 \ldots K \\
& \text{for } i = 1 \ldots J, k = 1 \ldots K \\
& \text{for } i = 1 \ldots J, k = 1 \ldots K \\
& \text{for } i = 1 \ldots J, k = 1 \ldots K \\
& \text{for } i = 1 \ldots J, k = 1 \ldots K \\
& \text{for } m \in \mathbb{R}, k = 1 \ldots K \\
\end{align*}
\]

where

- \(g_{i,k}\) is the groundwater that is pumped from the aquifer and used for irrigation in cell \(i\) in time step \(k\), unknown, \((L)\);

- \(r_{i,k}\) is the river water that is delivered to cell \(i\) in time step \(k\) and used for irrigation, unknown, \((L)\);

- \(s_{i,k}\) is the difference in groundwater level at the center of cell \(i\) between the initial level and the level at the end of time step \(k\), unknown, \((L)\). It is a positive valued drawdown if the level has declined;

- \(U\) is the upper bound on acceptable drawdown in cell \(i\) by the end of period \(k\), known, \((L)\);
e is the volume of groundwater that will enter the study area aquifer in cell i and time step k from extensions of the aquifer outside the study area, unknown, \( L \). For interior cells, \( i, k \), e equals zero;

\( e \) and \( e \) are lower and upper bounds on the volume of groundwater flowing between the aquifer underlying cell i and extensions of the aquifer outside the study area in time step k, known, \( L \);

\( \sigma \) is the stage of water flowing in the stream in cell m in time step k, unknown, \( L \). It is measured from a datum located beneath the aquifer;

\( \sigma \) and \( \sigma \) are lower and upper bounds on acceptable stream stage elevations, known, \( L \);

\( R \) is a set of cell numbers containing river reaches.

In the model presented in this paper, \( u(i, k) \) is a constant and \( u(i, k) \), \( r(i, k) \) and \( \sigma(m, k) \) are actual variables, permitting Equations 3, 5 and 8 to be included within the model as shown above.

If one assumes that groundwater and diverted river water are the only sources of water, the relationship between groundwater use, water needs, river water use and unmet needs at any cell is:

\[
g + r + u = w \quad \ldots \ldots 9
\]

\( g, r, u, w \) i, k, i, k, i, k, i, k

Equation 9 maintains the water volume balance at the ground surface (field).

The bounding conditions specified by Equations 5 and 7 can be satisfied simultaneously by: 1) replacing the left-hand side (LHS) of Equation 5 with a function that describes aquifer response to the hydraulic stimuli of pumping and flow in the river, and 2) converting the recharge bounds specified by Equation 7 into drawdown bounds that can be included within the RHS of Equation 5. The following equation (Peralta et al. 1986), is used in the first step. (This expression of head response to pumping and stream-aquifer interflow is similar to an approach taken by Illangasekare and Morel-Seytoux in 1982).
\[ s = \sum_{i,N} \sum_{j=1}^{N} \left( B_{i,j,N-k+1}^{j,k} (g - q_{j,k}) - v_{i,j,N-k+1}^{j,k} (\sigma - h_{j,k}) \right) \]

where

- \( B_{i,j,N-k+1}^{j,k} \) is a nonnegative-valued linear influence coefficient that describes the effect on the hydraulic head at cell \( i \) in time step \( N \) caused by \( (q_{j,k} - q_{j,k}) \). The temporal subscript \( N-k+1 \) is used merely to insure that the proper \( B \) is utilized in each time step, known, \((T/L)\);

- \( g_{j,k} \) is the net vertical hydraulic stimulus in cell \( j \) in time step \( k \), not including stream-aquifer interflow. It is the sum of all vertical discharges from the aquifer and recharges to the aquifer from the ground surface, unknown, \((L/T)\);

- \( q_{j} \) is the net vertical hydraulic stimulus, not including stream-aquifer interflow, that must occur in each time step in cell \( j \) in order for that cell to maintain its initial head. It is calculable using the linearized Boussinesq equation for steady-state two-dimensional flow through porous media (Illangasekare et al, 1984) and does not necessarily represent a steady-state stimulus that is actually occurring initially, \((L/T)\);

- \( v_{i,j,N-k+1}^{j,k} \) is a dimensionless influence coefficient;

- \( \Gamma_{x}^{k} \) is the volumetric reach transmissivity in cell \( x \) for a time step of known duration, \((L^2/T)\);
Before applying this equation to a study area, pertinent hydrogeologic information should be provided. Assume an aquifer system comprised of internal variable-head cells surrounded entirely by constant-head cells. The only discharges from the aquifer that can occur at internal cells are at pumping wells or at the stream that is in hydraulic connection with the aquifer. Recharge to the aquifer at internal cells can occur only at the stream. No other deep percolation through the soil profile is assumed. Thus \( q(j,k) \) replaces \( q(j,k) \) in Equation 10.

The drawdown constraints in the RHS of Equation 6 are useful if it is desirable that groundwater levels in internal cells decline no more than a predetermined distance from initial levels by the end of the planning period. The acceptable decline may be very small, thus assuring that groundwater levels are relatively stable over the long term (a sustained-yield scenario). When the purpose of using the constraint is for water levels to be near initial elevations by the end of the planning period, declines during intermediate steps are generally not constrained. The result may be a strategy that causes excessive decline during the first part of the planning period and water level recovery during the latter part.

The conditions of Equation 7 are important if the aquifer underlying the study area is simulated as being bounded by constant-head cells and if it is necessary that the volume of groundwater entering the study area through the aquifer in these cells must be less than some physically or institutionally-based limit. A physically-based limit is needed for situations in which a "constant-head" cell is not located at a hydrologically infinite source. In such a case, there is a potentially determinable upper limit of groundwater that can enter the study area through such a cell without causing that cell's head to change significantly. An institutionally-based limit is needed if the district is authorized to induce no more than a predetermined rate of recharge along its boundaries. In either situation, the simulated recharge that occurs at a "constant-head" cell in response to a pumping strategy can be calculated from Darcy's Law using the hydraulic gradients between the peripheral cells and adjacent internal cells. Similarly, simulated recharge rates can be forced to adhere to predetermined recharge constraints by imposing limits on groundwater levels in internal cells that are adjacent to constant-head cells (Peralta and Killian, 1985). Such constraints may be imposed during all time steps of the planning period.

In practice, Equation 7 is omitted and the value used for the RHS of Equation 6 (\( s \) ) is the lesser of: 1) the maximum acceptable decline in groundwater levels from initial water table elevations based on the desire for stable water levels 2) the maximum possible decline that will not cause recharge constraints to be violated.

For all internal cells within the study area and each time step assures that the optimal strategy will not cause unacceptable water table declines and that unacceptable recharge will not be induced at peripheral cells. Because the objective function will attempt to induce as much recharge as possible in order to minimize crop yield reduction, it is not necessary to impose the lower bound on recharge that is shown in Equation 7. Through the use of the \( B \) and \( v \) influence coefficients Equation 10 also maintains the volume balance of...
water within the aquifer.

Even though Equation 8 may be used directly to assure that optimal primary canal depths are acceptable, insuring physical realism in the river requires use of the continuity equation. In this model, continuity is maintained within the canal reach that exists between the centers of each pair of adjacent main canal cells. The following equation, applied to R-1 such reaches and K time steps, describes the volume of outflow at the downstream end of the reach between cells m and i during time step N.

\[ v = v' - v'' - \Delta v \]

where

- \( v \) is the volume of river water flowing out of the reach and past the center of cell i in time step N. \( (L) \);
- \( v \) is the volume of river water flowing into the reach and past the center of cell m in time step N. \( (L) \);
- \( v' \) is the volume of water that is diverted from main canal between the centers of cells m and i during time step N. \( (L) \);
- \( v'' \) is the volume of water that seeps from main canal to the aquifer between the centers of cells m and i during time step N. \( (L) \);
- \( \Delta v \) is the change in volume of water in storage in the main canal between the centers of cells m and i that occurred during time step N. \( (L) \).

Substituting for the components of Equation 11 term by term, without rearranging, yields:

\[ D (\sigma - b) = D (\sigma - b) - (d + d) / 2 - \]

\[ \begin{align*}
&\left\{ (\tau + \tau') \right\} \left\{ (\sigma - b) - (h - b) + \sigma - (\sigma - b) - (h - b) + \sigma \right\} / 4 \\
&- (\sigma - b) - (\sigma - b) + (\sigma - b) - (\sigma - b) (WY + WY) / 4 \\
&
\end{align*} \]

\[ \begin{align*}
i, N & \quad i, m, N & \quad m, N & \quad i, m, N \\
& \quad i, N & \quad i, N-1 & \quad i, m, N & \quad m, N-1 & \quad m, m
\end{align*} \]
where

\[ D \] is the linear stage-volume ratio for the stream at the center of cell \( x \), known \( (L / L) \);

\[ b \] is the elevation of the bottom of the stream at the center of cell \( x \), \( (L) \). Thus, \( (\sigma - b) \) is the depth of water in the stream at that point;

\[ d \] is the volume of water diverted from the river through canals in cell \( x \) during time step \( N \), unknown;

\( W \) and \( Y \) are the width and length of the stream in cell \( x \), known \( (L) \);

The formulation of the second term in the RHS of Equation 12 shows that we assume that half of the water diverted from the river in a cell is diverted upstream of the cell's center and half is diverted downstream of the center. Note that this ratio may be different for a particular reach, depending on the design of the diversion canal system. The third term in the RHS is simply the average reach transmissivity times the average difference between the river stage and the water table in time step \( N \) between cells \( i \) and \( m \). Note that many of the stream bottom elevations, \( b \), in the third term may be cancelled. Since the volume of river water diverted at a particular location does not explicitly exist as a variable in the model as formulated, it must be defined in terms of delivered river water. Assuming no seepage losses from the lateral diversion canals and an appropriate passage time, the total diverted river water equals the total delivered river water for a particular time step. The following assures that a volume balance is maintained in the diversion canals.

\[
\sum_{i=1}^{J} d_{i,N} = \sum_{j=1}^{J} r_{j,N} \quad \ldots 13
\]

With a priori knowledge for the diversion canal system design, the following can be stated.

\[
d_{i,N} = \sum_{j=1}^{J} f_{i,j,N} r_{j,N} \quad \ldots 14
\]
where

\[ f_{i,j,N} \]

is the proportion of river water diverted to cell \( j \) in time step \( N \) that will come from cell \( i \).

Substituting the RHS of Equation 14 for \( d \) in Equation 12, moving unknowns to the left side and leaving knowns on the right yields:

\[ J_0 = \sum_{j=1}^{J} \left( r_{i,j,N} + f_{i,j,N} \right)/2 \]

\[ + \left( D_{i,m} + c'_{i,m} + c''_{i,m} \right) \sigma_{i,m} \]

\[ + \left( - D_{i,m} + c'_{i,m} + c''_{i,m} \right) \sigma_{i,m} \]

\[ - c'_{i,m} \sigma_{i,m} - c''_{i,m} \sigma_{i,m} + c'_{i,m} \sigma_{i,m} + c''_{i,m} \sigma_{i,m} \]

\[ = c'_{i,m} \left( h_{i,m} + h_{i,m} \right) + D b_{i,m} - D b_{i,m} \]

... 15

where

\[ c'_{i,m} = \frac{\left( I_i + I_m \right)}{4} \]

\[ c''_{i,m} = \frac{\left( W_i Y_i + W_m Y_m \right)}{4} \]

In this formulation it is assumed that the canal water depth at the influent cell is a known constant during a time step. For simplicity, the following assumptions are also made (changing the model to handle different assumptions is not difficult). Rainfall is insignificant, i.e., it will cause no runoff, no deep percolation to the aquifer and no change in yield. No deep percolation or return flow will result from irrigation. Conveyance efficiency of diversion canals is 100 percent.

In summary, the model consists of the objective function (Equation 2), subject to the bounds of Equations 3 (unsatisfied demand), 4 (groundwater use) 5 (river water use), 8 (canal depth) and the constraints of Equations 9 (field volume balance), 10 (potentiometric head and aquifer volume balance), 13 (diversion canals volume balance) and 15 (primary canal volume balance). Optimization for this study is performed using a code by Liefsson et al. (1981).
APPLICATION AND RESULTS

A hypothetical study area (potential water management district) is shown in Figure 1. It is proposed that water be conveyed in unlined canal through the area and that some water be diverted through lined canals for irrigation. The district is underlain by an unconfined, unconsolidated aquifer that extends beyond the study area in all directions. As is commonly the case, the boundaries of the potential management district do not coincide with hydrologic boundaries.

Decision-makers (DMs) wish to evaluate the desirability of installing the canal system. Particularly, they wish to develop tentative optimal water allocation strategies for alternative stream inflow stages. Resulting information is valuable in identifying areas that will probably have groundwater or diverted river water available for irrigation. This in turn aids in selecting the spatial distribution of crops for planting.

The hydrologic/institutional setting requires that implemented strategies assure that currently existing springtime water levels (Figure 2) are regained by the beginning of the subsequent spring (i.e. a sustained yield scenario). This is assured via a constraint on final water table elevations. In addition, the strategy should not cause a disruption in regional groundwater flow regimes. Thus, constant-head/restrained-flux cells are used for district boundaries. The entire aquifer system that surrounds the study area is in quasi-steady-state. DMs assume that as long as a selected strategy does not induce more than historic groundwater flow across boundaries, existing potentiometric heads will continue to exist over the long-term.

The aquifer is assumed to have an effective porosity of 0.3 and transmissivities computed using saturated thickness and a hydraulic conductivity of 270 ft/day. Discrete kernels are generated using procedures developed by Verdin et al. (1981) and Peralta et al. (1986). Crop loss coefficients for three-month halves of a growing season are assumed to be 0.32 and 0.62. (Such coefficients are site-specific.) All other data required as constants by the model is assumed.

Assume that upstream water managers can guarantee that the influent stream can be maintained at constant stage during the growing season, although they cannot guarantee what that stage will be. (The model can process time varying influent stream stages but that is unnecessary for this paper.) Based on historic management successes, DMs can assume the population of actual influent depths to be normally distributed. Assume a mean depth of 10 ft and depths of 12 ft and 8 ft for alphas of 0.05 and 0.95 respectively (Figure 3). Before looking at how the optimization model may be used in agricultural planning, let's examine representative optimal allocation strategies.

Optimal conjunctive allocation strategies are developed for all three depths using the described optimization model. Figure 4 summarizes optimal production values for each strategy. Production is clearly limited by water availability. Figure 5 displays seasonal field, canal and aquifer volume balances for the strategies.

Figure 5 shows that water needs are the same, regardless of strategy. Since unsatisfied demand is so great, crop production is clearly limited by water availability. As canal depth increases, the volume of unsatisfied demand decreases, diverted canal water and pumped groundwater increase. Pumped groundwater increases because of increased flow from stream to aquifer.

Flow into the system increases linearly with canal depth (in accordance with the linear stage/discharge relation). Because of the 2 foot constraint on minimum acceptable effluent stream depth, the volume of water leaving the system through the canal is the same for all three strategies.
Figure 1. Hypothetical study area.

Figure 2. Initial (springtime) potentiometric surface (ft. above sea level).

Figure 3. Probability distribution of influent stream depths.
ANNUAL RESULTS OF STRATEGY IMPLEMENTATION (in 10^6 lbs)
for three influent river stages, d

\[
d = 8', 10', 12'
\]

Reduction in crop production due to unsatisfied demand:

\[
\begin{array}{lll}
68.1 & 65.8 & 65.6 \\
\end{array}
\]

Total crop production:

\[
\begin{array}{lll}
138.0 & 139.3 & 140.5 \\
\end{array}
\]

Figure 4. Annual crop production consequences of optimal strategy implementation.

\[
d = 8', 10', 12'
\]

\[
\begin{array}{lll}
51,500 & 50,500 & 49,500 \\
1,300 & 2,200 & 2,100 \\
17,700 & 18,100 & 18,400 \\
4,000 & 5,000 & 6,000 \\
1,500 & 2,200 & 2,000 \\
1,500 & 1,000 & 2,200 \\
1,000 & 1,000 & 1,000 \\
17,700 & 18,100 & 18,400 \\
5,900 & 5,900 & 5,900 \\
1,500 & 1,000 & 2,000 \\
6,300 & 6,400 & 6,400 \\
\end{array}
\]

Figure 5. Total seasonal field, canal and aquifer water balances (in ac-ft) for three influent canal depths, d.
The volume of reduction in storage during the growing season is fairly constant despite stage changes. Recharge through the boundary also changes little.

Assume that DMs would like to use the optimization model to formulate plans for planting crops. Since canal flow depth for the irrigation season is not likely to be known by planting time, the statistical nature of the influent should be used to guide decision-making.

First, cell by cell analysis of optimization model results shows that the annual water volume allocated to each cell never decreases with increasing flow depth. In other words, a cell's combined allocation of groundwater and diverted water is always at least as great for a 10 ft depth as for an 8 ft depth, etc. Exhaustive testing using systematic variation of influent stage is necessary to determine whether this trend is always true for this system. In subsequent discussion, in which we refer to a single cell as if it were a single water user, we assume that the trend is consistent.

Let us accept the previous conclusion and recall the influent probability distribution. Before planting, a user can be 50 percent sure of receiving, during the irrigation season, the amount of water allocated to him as being optimal for a 10 foot influent flow depth. He can be 95 percent sure of receiving the optimal allocation computed for an 8 foot influent depth.

Assuming water is the only limitation on crop production, the user can be 95 percent confident of having the production computed by the model for him, using the 8 foot influent stream depth. Figure 5 contains similar practical guidance for planting practice. It shows the percent confidence users in different cells can have of achieving at least 40 percent of potential production. Analogous tables can be prepared to show the probability of having more or less production. However, since only influent depths with 5, 50 and 95 percent probabilities are tested, these are the only probabilities that can be displayed. Once again, the validity of such tables relies on the assumption that, as influent stage increases, allocation volume never decreases.

The fact that the model considers the time-varying harmful effect of water shortage is illustrated by Figure 7. This figure is analogous to Figure 6, except it displays the confidence a user can have in being allocated at least 40 percent of total water needs. Note that it differs from Figure 6 in having some lower probabilities. This shows that the model is able to time the unavailability of water to when it does the least harm. Detailed analysis shows that the percentage of potential production that is produced is always greater than or equal to the percentage of total demand that is supplied.

Model results can also be used to determine the spatially distributed acreages that can be assured, to some degree, of receiving some irrigation water. Figure 6 shows the rounded cell-by-cell acreages that one can be 95 percent confident will receive at least some irrigation water during the growing season. Acreages increase somewhat with decreasing confidence level. DMs can select seasonal cropping patterns based on their attitudes towards risk.

SUMMARY

The production of a dynamic stream/aquifer system for specific reliability levels is determined through an implicitly stochastic optimization (ISO) model. Conceptually, the ISO model consists of an inflow model and a system model. The inflow model adequately represents the random nature of the inflow process and provides influent stream information to the system model to obtain minimum reduction in crop yield. The system model is characterized by time-varying crop loss coefficients as well as time variant, interdependent
response of stream stages, groundwater levels, and stream aquifer interflow to
groundwater pumping and diversion of river water to nonriparian lands. The
ISO model results in alternative strategies that guarantee optimum spatial
and temporal distribution of groundwater and river water. It is a potentially
valuable tool for evaluating future cropping patterns and irrigation water
distribution systems.

Figure 6. Spatially distributed probabilities of achieving at least 40
percent of crop water needs (in percent).

Figure 7. Spatially distributed probabilities of being allocated at least
40 percent of crop water needs (in percent).
### Figure 8. Area that will probably (95 percent confidence) receive some irrigation water during the growing season (in hundreds-of acres).

Acreages (in hundreds) with 95% probability of receiving some irrigation.

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Additional acreages with 50% probability of being irrigated.

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Additional acreages with 5% probability of being irrigated.

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