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Modelling for Optimal Conjunctive Water Management: Irrigated Crop Production Versus NPS Pollution Prevention

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MODELLING FOR OPTIMAL CONJUNCTIVE WATER MANAGEMENT:
IRRIGATED CROP PRODUCTION VERSUS NPS POLLUTION PREVENTION

O.H. Daza¹ and R.C. Peralta²

ABSTRACT

Conjunctive water management (CWM) involves coordinating use of ground and surface water sources. Agricultural (A) and nonagricultural (NA) users compete for available water of adequate quality. A Simulation/Optimization (S/O) conjunctive water management model was developed to aid estimating the effects of water and environmental management decisions on crop yield and water quality. Included subsystems are groundwater, surface water, reservoir, delivery system, drainage, and A and NA water users. The nonlinear model addresses flows described by nonsmooth piecewise-linear functions which have discontinuous derivatives. Embedded constraints describe all significant subsystem flows. For example, deep percolation and runoff from surface irrigation are explicitly described as functions of furrow inflow rate. Solution involves quasilinearization and cycling. We apply the model to a study area representative of part of Salt Lake Valley, Utah. We use the E-constraint method to maximize irrigated crop production subject to constraint on leaching to groundwater. Tested scenarios demonstrate model capabilities for transient management.

INTRODUCTION

Government agencies seek to assure the long-term availability of sufficient water of adequate quality. They commonly use simulation models (termed S models here) to predict the consequences of implementing different water management strategies. To compute management strategies they are also using more models that couple simulation with optimization algorithms (S/O models). Most models presented in the literature are somewhat specialized in applicability. There is a need for models that incorporate all significant flow processes and are broadly applicable.

Water for irrigated agriculture (A) and nonagricultural (NA or municipal and industrial) use is obtained from groundwater and/or surface water sources. In return, water quality is frequently degraded by use. For example, fertilizers and pesticides are common nonpoint source (NPS) pollutants. NPS pollution often results in response to rainfall and irrigation when chemicals

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move overland with runoff or percolate through the soil profile. Runoff results when water is applied to the soil surface at a rate greater than it can infiltrate into the soil. Deep percolation results when more water infiltrates than can be held in the root zone. The more efficient an irrigation method (technology), the less runoff or leaching results from irrigation.

Where urban water demand is high, water supplies are scarce, and irrigation-caused contamination threatens the major water source, water quality and quantity conservation practices are important. The Salt Lake Valley, where two-thirds of the provided water is groundwater, is such an area.

A groundwater flow simulation model by McDonald and Harbaugh [1988] has been calibrated and applied to the area [Waddell et al., 1987]. A simulation/optimization (S/O) groundwater management model has also been applied there [Gharbi and Peralta, 1994]. As is generally done in regional models, the above models assumed constant values for boundary recharges, including deep percolation losses from irrigation.

In other words, optimization of field level water management has not previously been considered. The presented model improves on that by:

1. distinguishing between and quantifies the effects of A and NA management changes and
2. including optimization of field water management.

Here we include within an S/O model, simulation and optimization of furrow irrigation and all other flows important for irrigation water management. The model relates furrow inflow rate to deep percolation and runoff losses. The resulting ability to simultaneously optimize regional and field conjunctive use should be useful to decision makers.

**OBJECTIVES**

The main objectives of this paper are to:

1. Describe a new simulation/optimization model that simultaneously optimizes regional conjunctive water management (CWM) and field-level water
management. Simulated processes are depicted in Figure 1. In essence this involves enhancing the Utah State/Embedding Model (US/EM), a S/O groundwater management model, to include transient modeling of: (a) Use of water for distinct A and NA activities; (b) Management of subsystem Unit Command Areas (UCAs) consisting of one to several irrigated cells; (c) Surface water diversion and delivery to UCAs and cells; (d) Surface and subsurface drainage collection from cells to UCAs and to rivers; (e) Reuse of drainage water and runoff by diversion from receding waters; (f) Injection of excess surface water; (g) Reservoir storage; (h) Soil moisture storage as a function of water application and losses; (i) Crop evapotranspiration as a function of water availability; (j) River stage dependency on inflows, diversions, groundwater pumping and seepage; and (k) Deep percolation and runoff losses described explicitly as functions of field-level (furrow irrigation) management.

2. Demonstrate model application by computing CWM strategies that maximize total crop yield subject to water quality constraints, for a one year planning horizon.

REVIEW OF LITERATURE

Developed groundwater management models have ranged from simulation (S) models [Trescott, 1976; Trescott et al., 1976; Morel-Seytoux et al., 1980; Illangasekare and Morel-Seytoux, 1982; Illangasekare et al., 1984; McDonald and Harkaugh, 1988] to simulation/optimization (S/O) models [Aguado and Remson, 1974; Maddock and Haines, 1975; Morel-Seytoux, 1975; Heidari, 1982; Willis and Liu, 1984; Danskin and Gorelick, 1985; Willis and Yeh, 1987; Mahon et al., 1987; Cantiller et al., 1988; Peralta and Datta, 1990; Yazicioglu, 1990; Gharbi and Peralta, 1994]. Although applications have addressed a wide range of hydrogeologic and management situations, none of the previously presented models have addressed the diversity of flows of the model presented here.

The S models consist of a set of equations that represent the physical
system. These models compute system response to assumed input values and an assumed water management strategy (Spatially and temporally distributed set of groundwater pumping rates). Developing a strategy acceptable for particular management goals can involve a tedious trial and error effort. Computing an optimal management strategy for a complex situation/problem is usually impossible when using an S model alone.

On the other hand, S/O models have an objective function, a set of constraint equations, and imposed limits on acceptable values for decision and state variables. Physical system response to management is represented via constraint equations. The model computes the optimal management strategy directly.

Gorelick [1983] classified groundwater management S/O distributed parameter models into two main categories, 1) hydraulic management models and 2) policy evaluation and allocation models. Some hydraulic management S/O models, have been used for contaminant plume management [Willis, 1976; Willis, 1979; Remson and Gorelick, 1980; Colarullo et al., 1984; Atwood and Gorelick, 1985; Lefkoff and Gorelick, 1986; Datta and Peralta, 1986; Heidari et al., 1987; Willis and Yeh, 1987; Peralta and Ward, 1991]. Others have been used for regional planning or both functions [Morel-Seytoux, 1975; Heidari, 1982; Willis and Yeh, 1987; Yaziciogil et al., 1987; Peralta and Kowalski, 1988; Peralta and Datta, 1990; Yaziciogil, 1990; Gharbi and Peralta, 1994].

Objective functions have included maximization of groundwater extraction or conjunctive use [Morel-Seytoux, 1975; Heidari, 1982; Yaziciogil et al., 1987; Peralta and Kowalski, 1988; Peralta and Datta, 1990; Yaziciogil, 1990; Gharbi and Peralta, 1994]; minimization of pumping costs [Maddock III, 1972; Morel-Seytoux, 1975; Willis and Newman, 1977; Remson and Gorelick, 1980; Peralta and Killian, 1985]; minimization of pumping (Remson and Gorelick, 1980); minimization of drawdowns [Willis and Liu, 1984; Yaziciogil and Rasheeduddin, 1987; Peralta and Datta, 1990; Yaziciogil, 1990]; maximization of net economic returns [Casola et al., 1986; Willis and Yeh, 1987; Peralta and
Kowalski, 1988]; and maximization of pumping in farm irrigation [Peralta et al., 1990]. Other applications include multiobjective optimization [Yazdanian and Peralta, 1986; Peralta and Killian, 1987; Yazicigil and Rasheeduddin, 1987] and goal programming [Peralta and Kowalski, 1986; Yazdanian and Peralta, 1986].

Policy evaluation and allocation models include hydraulic-economic response models and linked S-S/O models [Bredehoeft and Young, 1970; Young and Bredehoeft, 1972; Maddock III and Haimes, 1975; Daubert and Young, 1982; Bredehoeft and Young, 1983; Willis and Liu, 1984; Danskin and Gorelick, 1985; Mahon et al., 1987; Reichard, 1987; Willis and Yeh, 1987; Cantiller et al., 1988; Peralta et al., 1988; Hatchett et al., 1991; Peralta et al., 1991; Matsukawa et al., 1992].

Depending on computational capabilities and site conditions, these S/O models have incorporated either the embedding method or the response matrix method. The response matrix method has been frequently preferred over the embedding method for transient problems because of its numerical stability and computer memory requirements [Gorelick, 1983; Tung and Koltermann, 1985]. However, the embedding technique has been recently used in large models with success [Cantiller et al., 1988; Peralta and Datta, 1990; Takahashi and Peralta, In Press; Gharbi and Peralta, 1994].

The effects of irrigation return flows on groundwater quality have been widely documented [Fausey et al., 1990; Food and Agriculture Organization of the United Nations, 1979; National Research Council, 1989; Rail, 1989]. Measures to prevent contamination from agricultural practices and irrigation have also been well addressed [Page, 1987; Rail, 1989].

Best Management Practices (BMPs) have been developed for reducing the amount of contaminants reaching streams or groundwater [Duttweiler and Nicholson, 1983]. BMPs affect the hydrologic, ecologic, agronomic, and economic subsystems. Applying BMPs can involve adjusting agronomic practices (the source of pollution) and the hydrologic subsystem (the regulator of the
rate of delivery) within economic constraints to affect the ecologic system (via the amount of contaminants reaching water bodies).

Several researchers have evaluated the effect of irrigation technology on groundwater quality and quantity conservation [Ranjha et al., 1992a,b]. This same issue has been approached economically [Letey et al., 1983; Dinar et al., 1989; Hanson, 1989; Wichelns and Nelson, 1989; Knapp et al., 1990; Tsur, 1991; Dinar and Zilberman, 1991; Wichelns, 1991].

Groundwater management S/O models reported in the literature assume that deep percolation losses are a fixed fraction of the amount of irrigation water applied [Young and Bredehoeft, 1972; Morel-Seytoux, 1980; Reichard, 1987] or are a fixed amount [Gharbi and Peralta, 1994]. However, an approach to consider them as a variable dependent on irrigation technology has not yet been reported.

No reported S/O model coupled the processes important to both irrigation district management and conjunctive water use. Irrigation simulation models usually assume adequate groundwater exists to supplement surface water supply, and do not explicitly model hydrologic interaction. Groundwater or conjunctive use models use the assumptions mentioned above [Peralta et al 1990].

Keller [1987] suggested the need for linked groundwater and irrigation district simulation capabilities. The resulting model should incorporate the concept of Unit Command Area (UCA), an irrigated area subject to identical water management [Keller 1987]. A UCA is the smallest irrigated area usually addressed within surface water distribution systems [Merkley, 1993]. However, no reported model has this feature.

**S/O MODEL FORMULATION**

Described is a conjunctive water management S/O model that integrates discretized A and NA uses of water. Mathematical formulation of the model
uses assumptions detailed by Daza [1993]. Included are volume balances of all flow systems (Fig. 1).

Objective Function

The primary model objective function is to maximize total crop yield over the managed system. Solution is constrained by physical and managerial constraints discussed below.

\[
\text{Max } Z = \sum_{i \in X} \sum_{c \in C} Y_{i,c} A_{i,c}^{ai}
\]

where \( Z \) = objective variable [M]; \( i \) = index denoting cell \((i,j)\); \( Y_{i,c} \) = actual crop yield per unit area \([\text{ML}^{-2}]\); \( A_{i,c}^{ai} \) = area of cell that is irrigated \([\text{L}^2]\); \( c \) = index denoting crop; \( C \) = set of crops; \( N_c \) = number of crops; \( X, N_X \) = set and number of cells requiring water for \( A \) use, respectively.

Constraints in the Water Supply System

Groundwater

Flow equation. Flow simulation is based on an implicit 3-D finite difference approximation of the flow equation [McDonald and Harbaugh, 1988]. Assuming cells located by row \( i \), column \( j \), and layer \( l \), saturated groundwater flow is represented by

\[
f(h, \Delta x, \Delta y, T) = \frac{S_{o} \Delta x \Delta y}{\Delta t_{k}} (h_{b,k} - h_{b,k-1}) + q_{b,k} + q_{b,k} + q_{b,k} + q_{b,k}^{bo}
\]

where \( \delta \) = index denoting cell \((i,j,l)\); \( M \) = set of cells in the study area; \( K \) = set of stress periods; \( S_{o} \) = storage coefficient for cell \( \delta \); \( \Delta x, \Delta y \) = cell size in the x and y direction, of cell \( \delta \) located in row \( i \), column \( j \) \([\text{L}]\); \( \Delta t_{k} \) = duration of stress period \( k \) \([\text{T}]\); \( h_{b,k} \) = average potentiometric head \([\text{L}]\); \( q_{b,k} \) = known flows across the boundaries of the study area \([\text{L}^3\text{T}^{-1}]\); \( q_{b,k} \) = flow components that depend directly on water management \([\text{L}^3\text{T}^{-1}]\); \( q_{b,k}^{bo} \) = reduction in vertical flow between cells in layer \( l \) and the lower layer \( l+1 \) due to drop in head below the top of layer \( l+1 \) \([\text{L}^3\text{T}^{-1}]\); \( q_{b,k}^{bo} \) = boundary recharges that
result from A and NA water use on the ground surface \([L^3T^{-1}]\); \(T\) = transmissivity \([L^2T^{-1}]\). The left hand side term is similar to that described by McDonald and Harbaugh [1988]. However, the sign convention is (+) for discharge from and (-) for recharge into the aquifer.

The following equations group: the components of flow that are known across the boundary of the study area; the flow components that depend on water management; and boundary recharge resulting from A and NA water use on the ground surface.

\[
q_{b,k}^b = q_{b,k}^{\text{sp}} + q_{b,k}^{\text{be}} + q_{b,k}^{\text{rf}}
\]

\[
q_{b,k}^r = q_{b,k}^b + q_{b,k}^d + q_{b,k}^x + q_{b,k}^s + q_{b,k}^r + q_{b,k}^n
\]

\[
q_{b,k}^{bo} = q_{b,k}^p + q_{b,k}^{bo} + q_{b,k}^{pe} + q_{b,k}^{pc} + q_{b,k}^{pa} + q_{b,k}^n
\]

where \(q_{b,k}^{\text{sp}}\) = known discharge through springs \([L^3T^{-1}]\); \(q_{b,k}^{\text{be}}\) = known recharge through bedrock \([L^3T^{-1}]\); \(q_{b,k}^{\text{rf}}\) = groundwater recharge in noncropped and cropped nonirrigated areas resulting from precipitation \([L^3T^{-1}]\); \(q_{b,k}\) = groundwater pumping (+) \([L^3T^{-1}]\); \(q_{b,k}^d\) = capillary rise from groundwater table into the crop root zone \([L^3T^{-1}]\); \(q_{b,k}^x\) = horizontal flow across a boundary \([L^3T^{-1}]\); \(q_{b,k}^s\) = flow from the aquifer to the drains \([L^3T^{-1}]\); \(q_{b,k}^r\) = flow between the aquifer and reservoir facilities \([L^3T^{-1}]\); \(q_{b,k}^n\) = saturated flow between the aquifer and general head boundary cells \([L^3T^{-1}]\); \(q_{b,k}^p\) = flow between the aquifer and streams \([L^3T^{-1}]\); \(q_{b,k}^{\text{sp}}\) = deep percolation losses due to irrigation inefficiency \([L^3T^{-1}]\); \(q_{b,k}^{\text{be}}\) = deep percolation from excess water in the crop root zone \([L^3T^{-1}]\); \(q_{b,k}^{\text{rf}}\) = seepage losses from the primary and secondary irrigation delivery system, respectively \([L^3T^{-1}]\); \(q_{b,k}^{\text{sp}}\) = seepage from NA use of water \([L^3T^{-1}]\); \(q_{b,k}^{\text{rf}}\) = surface water rate delivered for artificial recharge \([L^3T^{-1}]\).

Expressions for reduction in vertical flow between layers due to drop in head \((q_{b,k}^{\text{rf}})\) and saturated flow between the aquifer and general head cells \((q_{b,k}^n)\) are defined by McDonald and Harbaugh [1988].
Surface Water

Surface water can be diverted from rivers, and conveyed through the primary distribution system to A and NA users. There can be one or more diversion points within a single cell (that contains a river). Each diversion can supply water to one or more UCAs.

**Total surface water diversion to a UCA.** Surface water diversion rate to a UCA is given by

\[
q_{\mu,k}^W = \sum_{i \in \delta(UCA)} \left( q_{i,k}^{pl} + q_{i,k}^{pr} \right) \sum_{h=0}^{n_l} \left( q_{i,h,k}^R + S_{i,h,k}^S \right)
\]

\[\forall \delta, \delta \in \mu, \Lambda \in \mu, k \in K\] (6)

where \( \mu \) = index denoting a UCA; \( q_{\mu,k}^W \) = total surface water diversion \([L^3T^{-1}]\); \( q_{k,i}^{pl}, q_{k,i}^{pr} \) = overflow spillage and seepage loss rates from the primary delivery system, respectively \([L^3T^{-1}]\); \( \Lambda \) = set of UCAs; \( I(\mu) \) = set of cells in UCA \( \mu \); \( N_l \) = number of cells in UCA \( \mu \); \( l \) = index denoting layer number; \( n_l \) = index denoting the number of layers in the aquifer system. Note that for clarity and convenience, \( i \) denotes cell location \((i,j)\) whereas \( \delta \) denotes cell location \((i,j,1)\) including aquifer layer.

**Total surface water diverted at a diversion point.** This equals the sum of surface water diversions to all UCAs attached to that diversion point.

**Total surface water diverted from a river or canal cell.** This equals the sum of all surface water diversions occurring at all diversion points within that single cell.

**Volume balance in a river or canal cell.** The surface water volume balance in a river or canal cell is defined by:

\[
V_{\tilde{\alpha},k}^s + V_{\tilde{\alpha},k}^s \left( q_{\tilde{\alpha},k}^I + q_{\tilde{\alpha},k}^O + \sum_{l=1}^{n_l} q_{\tilde{\alpha},h,k}^R - q_{\tilde{\alpha},h,k}^S \right) \Delta t_k
\]

\[\forall \tilde{\alpha}, \delta \in \mathcal{Z}, k \in K\] (7)

where \( V_{\tilde{\alpha},k}^s \) = storage in river cell \([L^3T^{-1}]\); \( q_{\tilde{\alpha},k}^I, q_{\tilde{\alpha},k}^O \) = inflow and outflow rates in the upstream and downstream side of the river cell, respectively \([L^3T^{-1}]\); \( q_{\tilde{\alpha},k}^R \) = total drainage water disposal rate \([L^3T^{-1}]\); \( q_{\tilde{\alpha},k}^S \) = stream-aquifer interflow rate \([L^3T^{-1}]\); \( \mathcal{Z} \) = set of river or canal cells.

**River discharge-storage and stage-discharge relationships.** Discharge-
storage is represented using the Muskingum method [Chow et al., 1988]. Flow depth in a river or canal cell is represented by a linearized function of the average inflow and outflow rates at the upstream and downstream ends of the river cell.

River-aquifer interflow. This constraint describes the flow between aquifer and river. The equations simulate both the saturated and unsaturated flow conditions and are expressed in nonlinear form by

\[
q_{b,k}^s = I_0^s \max \left( b_{b,k} - a_{b,k}, B_b - C_{b,k} \right) \quad \forall \xi \in \mathbb{Z}, k \in K
\]

where \( q_{b,k}^s \) = flow between the aquifer and streams \([L^3 T^{-1}]\); \( I_0^s \) = hydraulic conductance of that portion of the cell subject to stream-aquifer interconnection \([L^2 T^{-1}]\); \( a_{b,k}^s \) = elevation of the free water surface in the stream cell \([L]\). \( a_{b,k}^s \) equals the sum of the bottom elevation of the stream \( B_b \), plus the average flow depth in the river or canal cell \( d_{b,k} \) \([L]\).

Constraints in the Delivery System

The delivery system of an irrigated area is assumed to consist of primary and the secondary delivery systems (Fig. 2 and 3). The primary delivery system is composed of main and distribution and minor canals; the secondary delivery system is composed of lateral canals and field irrigation ditches within a UCA.

Performance of the delivery system is determined by the water losses that occur along the different reaches in an irrigation project. Such losses can be due to overflow spillage and seepage. Overflow spillage losses are eventually collected by the drainage collection system. Seepage losses eventually recharge the groundwater system.

Overflow spillage and seepage losses are assumed to be some fixed proportion of the total surface water diverted to a UCA. Seepage and spillage loss coefficients can differ by UCA. These losses in a cell are assumed to be known fractions of the total...
irrigation water delivered to the cell. Loss coefficients are based on field conditions.

**Constraints Relating Water Users to Water Sources**

Ground and surface water are both available for A and NA use. The total ground water pumped in a cell (Eq 9) and the total surface water delivered to a cell (Eq 10) equal the sum of the amounts of water provided for A and NA use in that cell.

\[
\sum_{a} g_{s,k} - \sum_{a} g_{n,k} = g_{s,k} + g_{n,k} \quad \forall \alpha \in \Omega \subset \Phi, k \in K
\]

\[
g_{s,k} = g_{s,k} + g_{n,k} \quad \forall \alpha \in \Omega \subset \Phi, k \in K
\]

where \( g_{s,k} \) = total groundwater pumping from cell \( a \) during stress period \( k \) \([L^3 T^{-1}]\); \( g_{n,k} \), \( g_{n,k}^a \) = groundwater pumping for A and NA use \([L^3 T^{-1}]\); \( s_{s,k} \) = surface water delivered \([L^3 T^{-1}]\); \( s_{n,k}^a \) = surface water delivered for NA use \([L^3 T^{-1}]\). The amounts of water for A and NA use \( \{g_{s,k}, g_{n,k}^a, s_{s,k}, s_{n,k}^a\} \) are all decision variables and are not a fixed ratio of each other.

**Reservoir Facilities**

A reservoir can store surface water surplus for future use. The volume stored in the reservoir is represented by:

\[
V_{s,k} - V_{s,k-1} = \left( s_{s,k} + g_{s,k} - \sum_{a} g_{s,a,k} - \sum_{a} g_{n,a,k} - g_{s,k}^d - g_{n,k}^d - g_{s,k}^v - g_{n,k}^v \right) \Delta t_k
\]

\[
V_{s,k} - V_{s,k-1} = \left( s_{s,k} + g_{s,k} - \sum_{a} g_{s,a,k} - \sum_{a} g_{n,a,k} - g_{s,k}^d - g_{n,k}^d - g_{s,k}^v - g_{n,k}^v \right) \Delta t_k
\]

\[
\forall \alpha \in \Omega \subset \Phi, k \in K
\]

where \( V_{s,k} \) = volume in the reservoir facility \([L^3]\); \( q_{s,k}^p \) = precipitation contribution to the reservoir storage \([L^3 T^{-1}]\); \( q_{s,k}^d \) = spillage water losses from the reservoir facility \([L^3 T^{-1}]\); \( q_{s,k}^v \) = evaporation losses rate from the reservoir facility \([L^3 T^{-1}]\).

Spillage from the reservoir is:
\[ q_{i,k} = \max \left[ V_{i,k-1} + \left( g_{i,k} + q_{i,k} - \sum_{l=1}^{n} q_{i,k} + q_{i,k} + q_{i,k} \right) \Delta t_k \right] \]

\[ \forall \, \bar{a} \in N, \, k \in K \]

where \((V_i)^u\) = upper limit of the capacity in the reservoir facility \([L^3]\).

The reservoir storage-stage relationship is:

\[ V_{i,k} = C_1 h_{r,i,k}^2 + C_2 h_{r,i,k} \quad \forall \, \bar{a} \in N, \, k \in K \]

where \(C_1, \, C_2\) = coefficients of the reservoir storage-stage relationship; \(h_{r,i,k}\) = water depth in the reservoir facility \([L]\).

Reservoir-aquifer interflow is represented by an expression analogous to equation 8. Additional equations and terms include the reservoir water surface area-stage relationship, the contribution from precipitation, and the evaporation losses from the reservoir.

**Water for Agricultural Use**

Soil moisture parameters. These are required in the crop root zone volume balance equation. They are calculated from soil moisture characteristics and include \(W_{\text{max}}\) = net maximum depth of soil water that should be depleted between irrigations in crop \(c\) \([L]\); \(MAD_c\) = management allowed depletion level [%]; \(R_c^{\text{wg}}\) = average rooting depth \([L]\); \(AW_c\) = average value of available water in the root zone of the soil profile [%]; \(\theta_c^{\text{sc}}, \theta_c^{\text{wp}}\) = water content on a volume basis at field capacity and wilting point, respectively [%]; \(Z_c^{\text{sc}}, Z_c^{\text{wp}}\) = soil moisture depth at field capacity and wilting point, respectively.

Rainfall runoff and infiltration. Computation of precipitation that contributes to surface runoff and precipitation that infiltrates into the soil is performed using the SCS method [Chow et al., 1988]. Daza (1994) describes the details.

Crop water requirements. These equal the sum of the water evaporated from the soil surface plus the water transpired by the plant. They are
expressed as crop evapotranspiration during a stress period [Jensen et al., 1989].

Actual crop evapotranspiration is expressed in nonlinear form as a function of the soil moisture content by

\[
D^e_{c,k} = \frac{D^{stp}_{c,k}}{Z_c^{FC} - \max_c - Z_c^{FC}} \min \left[ Z_c^{FC} - \max_c, \frac{Z_c^{rs} - Z_c^{FC}}{Z_c^{FC}} \right]
\]

\( \forall a \in X, c \in C, k \in K \)

where \( D^{stp}_{c,k} \) = potential evapotranspiration [L]; \( D^e_{c,k} \) = actual evapotranspiration [L].

Relative crop yield reduction for each crop in each cell is related to the relative crop evapotranspiration deficit [Doorenbos and Kassam, 1979] by

\[
\left( \frac{1 - Y_c}{Y_c^p} \right) = K_y \left( 1 - \frac{\sum_{i=1}^{nk} D^e_{k,i} - \sum_{i=1}^{nk} D^{stp}_{k,i}}{\sum_{i=1}^{nk} D^{stp}_{k,i}} \right)
\]

\( \forall a \in X, c \in C, k \in K \)

where \( Y_c^p \) = unit potential crop yield [ML^{-2}]; \( K_y \) = yield response factor; \( nk \) = number of stress periods.

**Volume balance in the crop root zone.** This is maintained only for the cropped-irrigated areas. The soil moisture storage in the crop root zone is defined at the end of each stress period.

\[
Z_{c,k}^{rs} = Z_{c,k}^{rs,0} + \sum_{i=1}^{nk} D^{ir}_{i,c,k} E_{a,c} - D^e_{c,k} + \sum_{i=1}^{nk} D^{stp}_{c,k} - D^e_{c,k} - D^{cap}_{d,c,k} - D^{cap}_{d,c,k}
\]

\( \forall a \in X, c \in C, k \in K \)

where \( Z_{c,k}^{rs} \) = soil moisture storage [L]; \( D^{ir}_{i,c,k} \) = equivalent depth of irrigation water applied [L]; \( E_{a,c} \) = application efficiency [fraction]; \( D^{cap}_{d,c,k} \) = depth of capillary rise from groundwater table into the root zone [L].

**Excess water from the root zone.** This is also maintained only for the cropped irrigated areas. Excess water is water beyond that which can be held at field capacity.
$D_{i,c,k}^{\text{ex}} = \max \left( Z_{s,c,k} + D_{i,c,k}^{\text{lw}} + D_{i,c,k}^{\text{po}} + \sum_{l=1}^{n_l} D_{i,c,k}^{\text{al}} - D_{i,c,k}^{\text{st}} - Z_{c,0}^{\text{pc}}, 0 \right) \quad \forall \, \bar{a} \in X, \, c \in C, \, k \in K$ \hfill (17)

where $D_{i,c,k}^{\text{ex}} =$ excess water from crop root zone [L]. This excess does not include the amount of deep percolation losses that result from an irrigation event due to the irrigation method itself.

Application of eq. 17 over the cropped irrigated area during a stress period results in a flow rate.

$$ q_{s,k}^{\text{ex}} = \frac{\sum_{l=1}^{n_l} D_{i,c,k}^{\text{lw}} A_{i,c}}{A t_k} \quad \forall \, \bar{a}, \bar{c} \in X, \, k \in K $$ \hfill (18)

where $q_{s,k}^{\text{ex}} =$ deep percolation from excess water in the crop root zone [L$^3T^{-1}$].

Irrigation water delivered to a cell. The following equation represents the actual amount of irrigation water delivered to a cell whether it does or does not have a reservoir.

$$ q_{s,k}^{\text{lw}} = q_{s,k}^{\text{rd}} + q_{s,k}^{\text{rd}} + q_{s,k}^{\text{di}} + q_{s,k}^{\text{al}} - \sum_{l=1}^{n_l} q_{s,k}^{\text{ss}} \quad \forall \, q_{s,k}^{\text{rd}} \geq 0, \, \bar{a} \in X; \, q_{s,k}^{\text{rd}} \leq 0, \, \bar{a} \in X; \, k \in K $$ \hfill (19)

where $q_{s,k}^{\text{lw}} =$ irrigation water delivered [L$^3T^{-1}$]; $q_{s,k}^{\text{rd}} =$ surface water released from the reservoir facility [L$^3T^{-1}$]; $q_{s,k}^{\text{di}} =$ drainage water reused in irrigation [L$^3T^{-1}$]; the latter term is drainage return flow that is returned to the secondary irrigation delivery system.

Volume balance of water delivered for agricultural use in a cell. The amount of irrigation water delivered to a cell is:

$$ q_{s,k}^{\text{for}} = \frac{\sum_{l=1}^{n_l} D_{i,c,k}^{\text{lw}} A_{i,c}}{A t_k} \quad \forall \, \bar{a} \in X, \, k \in K $$ \hfill (20)

Descriptors indicating the performance of irrigation. The following describe irrigation performance during single irrigation event [Walker and
Skogerboe, 1987]. These descriptors are defined on a cell and crop basis.

1. **Application efficiency**

   \[
   Ea_{i,c} = \frac{W_{\text{max}_c}}{W_{\text{max}_c} + D_{i,c}^{\text{dp}}} + D_{i,c}^{\text{ro}}}
   \]  
   \(\forall \alpha \in X, c \in C\)  
   \hspace{1cm} (21)

2. **Deep percolation ratio**

   \[
   \text{DPR}_{i,c} = \frac{D_{i,c}^{\text{dp}}}{W_{\text{max}_c} + D_{i,c}^{\text{dp}}} + D_{i,c}^{\text{ro}}}
   \]  
   \(\forall \alpha \in X, c \in C\)  
   \hspace{1cm} (22)

3. **Tailwater ratio**

   \[
   \text{TWR}_{i,c} = \frac{D_{i,c}^{\text{ro}}}{W_{\text{max}_c} + D_{i,c}^{\text{dp}}} + D_{i,c}^{\text{ro}}}
   \]  
   \(\forall \alpha \in X, c \in C\)  
   \hspace{1cm} (23)

where \(Ea_{i,c}\) = application efficiency \([\text{fraction}]\); \(\text{DPR}_{i,c}\) = deep percolation ratio \([\text{fraction}]\); \((\text{TWR}_{i,c})\) = tailwater ratio \([\text{fraction}]\); \(D_{i,c}^{\text{dp}}\) = depth of deep percolation losses per irrigation event \([\text{L}]\); \(D_{i,c}^{\text{ro}}\) = depth of tailwater runoff losses per irrigation event \([\text{L}]\).

Deep percolation losses per irrigation event are represented as a power function of inflow rate per furrow for each crop-soil combination and are represented by

\[
D_{i,c}^{\text{dp}} = BDF_{i,c} Q_{i,c}^{MDP_{i,c}}
\]  
\(\forall \alpha \in X, c \in C\)  
\hspace{1cm} (24)

where \(Q_{i,c}\) = inflow size per furrow \([\text{L}^3\text{T}^{-1}]\); \(BDF_{i,c}, MDP_{i,c}\) = regression coefficients.

The magnitude of runoff per irrigation event is also furrow inflow rate dependent. Changes in inflow rate for different crops and soils affect application efficiency and the amount of runoff that returns to the drainage collection system.
Tailwater runoff losses per irrigation event are expressed as a linear function of the inflow rate per furrow for each crop-soil combination and are given by

\[ D_{\text{Rw,c}}^{\text{f}} = \text{MRO}_{\text{Rw,c}} \cdot \text{Qw,c} + \text{BRO}_{\text{Rw,c}} \quad \forall \text{X}, \text{c} \in \text{C} \]  

(25)

where \( \text{MRO}_{\text{Rw,c}} \) =, \( \text{BRO}_{\text{Rw,c}} \) = regression coefficients.

Deep percolation losses due to irrigation. Deep percolation losses for a particular irrigated area can be expressed as a computed proportion of the total depth of irrigation water applied during a stress period. Therefore, the total depth of water lost by deep percolation is given by

\[ D_{\text{dp,c,k}}^{\text{f}} = \text{DPR}_{\text{dp,c}} \cdot D_{\text{aw},c,k}^{\text{f}} \quad \forall \text{a}, \text{c} \in \text{X}, \text{c} \in \text{C}, \text{k} \in \text{K} \]  

(26)

where \( D_{\text{dp,c,k}}^{\text{f}} \) = depth of water lost as deep percolation from irrigation [L].

Integrating the depth of water lost as deep percolation due to irrigation inefficiency during a stress period yields:

\[ q_{\text{d,k}}^{\text{f}} = \frac{\sum_{\text{a}} D_{\text{dp,c,k}}^{\text{f}} \cdot A_{\text{a},\text{c}}^{\text{f}}}{\Delta t_k} \quad \forall \text{a}, \text{c} \in \text{X}, \text{k} \in \text{K} \]  

(27)

where \( q_{\text{d,k}}^{\text{f}} \) = deep percolation losses from irrigation [L^3T^{-1}].

Recall that the groundwater volume balance equation (5) contains two deep percolation terms, \( q_{\text{dp}}^{\text{f}} \) and \( q_{\text{wx}}^{\text{f}} \). The first, defined above, reflects irrigation inefficiency. The second is defined by conversion from \( D_{\text{aw}}^{\text{c}} \) of the root zone volume balance expression (Eq. 17). It describes the result of all other root zone inflows and outflows.
Tailwater runoff losses. Tailwater runoff losses resulting from irrigation inefficiency can also be expressed as proportion of the total depth of irrigation water applied during a stress period:

\[ D_{o,c,k}^{ro} = TWR_{o,c} D_{o,c,k}^{SW} \quad \forall a \in X, c \in C, k \in K \] (28)

where \( D_{o,c,k}^{ro} \) = depth of water lost as tailwater runoff from irrigation [L].

Integrating the depth of water lost as tailwater runoff over the irrigated area in cell during a stress period results in:

\[ q_{o,k}^{ro} = \sum_{c=1}^{nC} D_{o,c,k}^{ro} A_{s,c}^{al} / \Delta t_k \quad \forall a \in X, k \in K \] (29)

where \( q_{o,k}^{ro} \) = tailwater runoff losses from irrigation events [L'T⁻¹].

Thus, runoff is expressed to occur in two ways within the model: tailwater runoff due to the operation of the irrigation method itself; and runoff as overland flow resulting from excess rainfall.

Capillary rise from groundwater table into the root zone. Irrigated agriculture can benefit from water entering the root zone by capillary rise. However, capillary rise also occurs when the crop is not irrigated or the land is not cropped. Its magnitude is dependent on the groundwater table elevation and is expressed in piecewise-linear form.

\[ D_{o,k}^{gm} = \frac{E_o}{ds_o} [\min (h_{s_o}, h_{o,k}) - \min (h_{s_o} - ds_o, h_{o,k})] \quad \forall o \in O, k \in K \] (30)

where \( E_o \) = maximum capillary rise from groundwater table into the root zone [LT⁻¹]; \( ds_o \) = extinction depth (depth below which there is no capillary rise) [L]; \( h_{s_o} \) = potentiometric surface elevation below which capillary rise begins to decrease [L]; \( D_{o,k}^{gm} \) = water moving from groundwater table into the rootzone [L]; \( O \) = set of cells where groundwater moves upward into the root zone.

Application of eq. 30 for the depth of capillary rise over a given area.
results in a flow rate.

\[
g_{b,k} = \frac{D_{b,k} A_k}{\Delta t_k} \quad \forall b \in B, k \in K \tag{31}
\]

where \( g_{b,k} \) = total rate of capillary rise from groundwater table into the root zone \([L^3 T^{-1}]\); \( A_k \) = area of cell devoted to agricultural use \([L^2]\).

**Constraints describing the Drainage Collection System**

Each river cell can have at least one drainage exit disposing of water from at least one UCA (Fig. 2).

**Drain-Aquifer Interflow**

This constraint simulates drainage under saturated flow conditions. Drainage occurs when the water table in the aquifer is above the water level in the drain. It is expressed in piecewise-linear form.

**Volume Balance in a Cell**

The surface drainage collection system is assumed to receive all forms of drainage water that occur in the managed system. The collected water can follow different paths in the system: 1) return to the river for downstream allocation; 2) return to the irrigation system for reuse; and 3) depart from the study area.

\[
g_{r,e,k} + g_{s,k} + g_{s,k} + g_{s,k} + g_{s,k} + g_{s,k} + g_{s,k} + \sum_{s \in K} q_{b,s} - q_{b,s} + q_{b,s} + q_{b,s} + q_{b,s} + q_{b,s} + q_{b,s} + q_{b,s} + q_{b,s} = 0 \tag{32}
\]

where \( g_{r,e,k} \) = collected drainage water that returns to the river \([L^3 T^{-1}]\); \( g_{s,k} \) = return flow as surface drainage from NA use \([L^3 T^{-1}]\); \( g_{s,k} \) = drainage water that leaves the boundary of the study area \([L^3 T^{-1}]\); \( q^{dr}, q^{dl}, q^{do} \) = set of cells where drainage water is collected and returned to the river, reused in irrigation, and departs from the study area, respectively.

**Total Drainage Water Released to a River or Canal Cell**
This is the sum of drainage water collected at all drainage exits existing in a single cell.

\[ q^{\text{dr}}_{k} = \sum_{i=1}^{N_{c}} q^{\text{e}}_{i,k} \quad \forall i \in \mathbb{Z}, k \in \mathbb{K} \]  

(33)

where \( q^{\text{dr}}_{k} \) = total drainage water rate disposed to river or canal cell [L^3T^{-1}]; 
\( q^{\text{e}}_{i,k} \) = total drainage water collected in drainage exit [L^3T^{-1}]; \( i \) = index denoting drainage exit; \( N_{c} \) = number of drainage exits served by a river or canal cell. This includes: 1) total drainage water from a UCA that returns to the river; 2) total drainage water collected in a drainage exit.

**Bounds**

Upper and lower limits can be placed on values of: groundwater extracted from the aquifer, artificial recharge, aquifer potentiometric head, flow entering or leaving through constant head cells, stream-aquifer interflow for each river reach, surface water delivered, reservoir capacity and water depth, soil moisture content, irrigation application, furrow inflow rate, total deep percolation and total tailwater runoff losses from irrigation, streamflow, and total surface water diversion.

**MODEL NONLINEARITY, AND CYCLING**

**Model Nonlinearity**

The described S/O model poses a nonlinear programming problem having discontinuous derivatives (DNLP). The formulation includes linear equations, and three types of nonlinearities:

1. The groundwater flow equation in an unconfined aquifer is nonlinear in the transmissivity terms. Transmissivity is a function of the saturated thickness which is head dependent. This nonlinearity is addressed via a quasi-linearization approach described in the next section.

2. Max/Min functions are used in the model to define: capillary rise
from the groundwater table into the crop root zone, subsurface drainage, stream-aquifer interflow, soil moisture volume balance in the crop root zone, excess water from the crop root zone, and spillage water losses from the reservoir facilities.

(3) Power and quadratic equations defining some system relations. These are, reservoir storage-stage relationship, reservoir surface water area-stage relationship, and unit deep percolation losses from irrigation.

The Cyclical Solution Procedure

In an unconfined aquifer, transmissivity is a function of head. That means a nonlinear flow equation is most appropriate. However, the resulting nonlinear models are difficult to solve. To permit using linear surrogates, a cycling procedure is followed.

Transmissivity in the groundwater flow equation is first approximated using assumed head values. Model solution proceeds using the assumed transmissivity values to calculate new head values. The initially assumed head values are then replaced with the new head values. The process of assuming-calculating-replacing is termed cycling. This process continues until the difference between the head values computed in two consecutive cycles is insignificant.

MODEL APPLICATION AND RESULTS

To highlight model features, the CWM model is demonstrated using different multi-objective scenarios. It is applied to a representative 38-cell study area (Figure 4). Input data, representative of Salt Lake Valley - Utah, are detailed by Daza (1994).

Pareto Optimum

Maximizing crop yield versus minimizing deep percolation. These two specified goals conflict because maximizing crop yield requires much irrigation. Unless water logging or nutrient leaching become problematic, the more one irrigates, the greater the crop yield, until potential yield is attained. However, no irrigation system is completely efficient. As
irrigation increases deep percolation increases. Deep percolation carries pollutants which can contaminate groundwater.

In addition to the priority objective function (Eq 1) the model can use the objective of: minimizing total deep percolation. Below we discuss the non-inferior solutions developed for this bi-objective problem. A 200 m furrow length is assumed. Lower bounds on head permit a maximum drawdown of 18.3 m below the initial potentiometric surface.

Results. Figure 5 shows the developed set of noninferior solutions. Extreme values show the results for Scenario 1 (Maximization of total crop yield) and Scenario 2 (Minimization of total deep percolation). Intermediate values were calculated maximizing crop yield subject to different upper bounds in total deep percolation from the study area. Table 1 summarizes results for Scenarios 1 - 3. Scenario 2 results show system response to crop yield when no irrigation is practiced. Crop yield is reduced by 63.2%, and total deep percolation is 0.004 m$^3$/s. The Scenario 3 strategy results from forcing furrow inflow to be the optimum value from a field perspective alone. Note that it is only one of many potential compromise strategies and is not necessarily regionally the best. The low slope of this curve above 0.113 m$^3$/s indicates that total crop yield is not strongly affected for a large reduction in deep percolation. For instance, reducing total deep percolation by 0.163 m$^3$/s reduces total crop yield only by 5.8%. The reduction in deep percolation corresponds to 60% of the total deep percolation expected in the system.

The flatness of this curve is due in part to the type of production function used; crop yield is a function of evapotranspiration, which at the same time is a function of the soil moisture content in the crop root zone. From the irrigation management perspective, a 55% allowable depletion (MAD) was used. Maximum potential crop evapotranspiration and crop yield are assumed to result from keeping soil moisture above this threshold value.

Figure 5 also shows the change in groundwater pumping per each unit change in deep percolation for scenarios 1 to 3. Groundwater pumping
decreases as deep percolation decreases below 0.113 m³/s. Notice how crop yield and groundwater pumping vary per unit change in deep percolation. Reduction of deep percolation below 0.113 m³/s can seriously affect crop yield because the amount of available groundwater is insufficient to satisfy crop water needs (deep percolation is a source of groundwater). This condition is relevant when developing water management policies for groundwater quality and quantity conservation.

Maximizing Crop Yield Using Groundwater and Reusing Drainage Water for Irrigation

Scenario 4. Maximize total crop yield using only groundwater for irrigation and constraining the maximum drawdown in the aquifer system to 3.66 m. No reuse of drainage water is allowed for irrigation.

Scenario 5. Maximize total crop yield using only groundwater for irrigation and constraining the maximum drawdown in the aquifer system to 3.66 m. Drainage water reuse is allowed for irrigation. The initial value for soil moisture storage is field capacity.

Scenario 6. Maximize total crop yield using only groundwater for irrigation and constraining the maximum drawdown in the aquifer system to 3.66 m. Drainage water reuse is allowed for irrigation. The initial value for soil moisture storage equals the MAD level.

Results. Table 1 summarizes results for scenarios 4 to 6. Comparison of scenarios 1 and 4 shows the effect of the lower bound of head on crop yield; this bound limits the amount of groundwater that can be used in irrigation, thus, reducing crop yield. As a result, a reduction of groundwater of 26.1% causes a yield decrease of 14.2%. Deep percolation and tailwater runoff are also reduced accordingly.

Comparison of scenarios 4 and 5 illustrate the effect of drainage water reuse when the drawdown in the aquifer is constrained. In this case, crop yield and groundwater pumping are reduced 5.4% and 36.7%, respectively.

Comparison of scenarios 4, 5 and 6 regarding irrigation application efficiency indicate a relatively constant value slightly below the maximum
application efficiency (see Scenario 3). Scenario 5 shows the model effort to promote a lower irrigation application efficiency by increasing the inflow rate per furrow. The reduction in application efficiency is reflected in higher tailwater runoff, which is finally reused for irrigation. The increase in tailwater runoff is at least twice as much between scenarios 4 and 5.

Results from Scenario 6 are comparable to Scenario 5 and show the effect of a different initial value for soil moisture content.

Maximize Crop Yield Using Groundwater, Surface Water, and Reservoir Facilities

Scenario 7. Maximize total crop yield using groundwater and surface water for irrigation; maximum drawdown in the aquifer system is constrained to 3.66 m. Drainage water reuse is not allowed for irrigation. The initial value for soil moisture storage is field capacity.

Scenario 8. Maximize total crop yield using only surface water for irrigation. Drainage water reuse is not allowed for irrigation. The option for reservoir facilities is included. The initial value for soil moisture storage is field capacity.

Scenario 9. Maximize total crop yield using groundwater and surface water for irrigation. Maximum drawdown in the aquifer system is constrained to 3.66 m. Drainage water reuse is not allowed for irrigation. Reservoir facilities are used. The initial value for soil moisture storage is field capacity.

Results. Table 1 includes results from scenarios 7 - 9. Scenarios 4 and 7 cause yield reductions of 14.2% and 3.9% respectively, by comparison to Scenario 1. Notice in Scenario 7 that groundwater pumping decreases after surface water is made available as an alternate source of water. The total water used for Scenario 7 is 1.431 m³/s. The greater total water used in Scenario 7 with respect to Scenario 4 is due to the lower irrigation application efficiency.

Scenario 8 has a 2.4% yield reduction because no groundwater is used for irrigation. Total delivered surface water equals 1.961 m³/s whereas total
surface water released from reservoir facilities equals 1.476 m$^3$/s. The latter value equals the water used for irrigation, and is about the same as the total amount of water used in Scenario 7. The difference in total surface water delivered and total surface water released from the reservoir facilities is due to aquifer-storage interflow.

For Scenario 9 yield reduction is 2.4%. The total rate of water used for irrigation is 1.534 m$^3$/s. This rate slightly exceeds the total used in Scenario 7. The difference in total surface water delivered and total surface water released from the reservoir facilities is due to aquifer-storage interflow.

Summary

A computer model is presented that can simulate system response to conjunctive water management and compute optimal management strategies. Incorporated flow processes include those of the following subsystems: multi-layer groundwater aquifer; surface water distribution through rivers and canals; reservoir facilities; irrigation delivery system within unit command areas; agricultural and nonagricultural use of water; irrigation technology; and drainage and reuse systems. The presented S/O model includes an objective function (maximizing crop yield), variable bounds and linear, piecewise-linear and nonlinear constraint equations. Constraints include volume balance equations describing flows and relationships between subsystems, reservoir storage, spill and reservoir-aquifer interflow; irrigation distribution system conveyance, spillage and seepage losses; root zone storage, crop evapotranspiration and yield; relation between furrow length, inflow rate, deep percolation and runoff; drainage-aquifer interflow, drainage collection, and drain water reuse and disposal.

The ability to compute the trade off between maximizing crop yield and minimizing leaching is an important model attribute. Model application is demonstrated by computing optional water management strategies for selected scenarios.
Scenarios include groundwater and surface water use with or without drainage water reuse and with or without surface water reservoirs. An irrigation technology is explicitly incorporated within the model. The model may be helpful to water managers and policy makers in assessing water management strategies for groundwater quality and quality conservation.

REFERENCES


Fausey, N.R., Ward, A.D. and Logan, T.J., 1990. "Influence of Water Table Elevation on Water Quality", International Summer Meeting, Columbus, Ohio,


Keller, A.A. 1987. "The USU Unit Command Area Model", *WMS Report 71*, United States Agency for International Development AID.DAN, Utah State University,


"Ground-water Conditions in Salt Lake Valley, Utah, 1969-83, and Predicted Effects of Increased Withdrawals from Wells", Technical Publication No. 87, State of Utah, Department of Natural Resources, Salt Lake City, Utah.


### List of Symbols and Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>((V^r_i)^u)</td>
<td>upper limit of capacity in the storage facility</td>
<td>([L^1])</td>
</tr>
<tr>
<td>(a)</td>
<td>index denoting cell ((i,j))</td>
<td></td>
</tr>
<tr>
<td>(A^c)</td>
<td>area of cell devoted to agricultural use</td>
<td>([L^2])</td>
</tr>
<tr>
<td>(A_{ai}^c)</td>
<td>area of cell that is irrigated</td>
<td>([L^2])</td>
</tr>
<tr>
<td>(A_W^c)</td>
<td>average value of available water in the crop root zone</td>
<td>([%])</td>
</tr>
<tr>
<td>(BDP_{c,M,D}^c)</td>
<td>regression coefficients for deep percolation losses</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>index denoting crop</td>
<td></td>
</tr>
<tr>
<td>(C)</td>
<td>set of crops</td>
<td></td>
</tr>
<tr>
<td>(C1_a, C2_a)</td>
<td>coefficients of the reservoir storage-stage relationship</td>
<td></td>
</tr>
<tr>
<td>(D_{1a,c}^{a})</td>
<td>depth of tailwater runoff losses per irrigation event</td>
<td>([L])</td>
</tr>
<tr>
<td>(D_{1a,c}^{d})</td>
<td>depth of deep percolation losses per irrigation event</td>
<td>([L])</td>
</tr>
<tr>
<td>(D_{a,c,k}^{a})</td>
<td>depth of water lost as deep percolation from irrigation</td>
<td>([L])</td>
</tr>
<tr>
<td>(D_{a,c,k}^{a})</td>
<td>actual evapotranspiration</td>
<td>([L T^{-1}])</td>
</tr>
<tr>
<td>(D_{a,c,k}^{p})</td>
<td>potential evapotranspiration</td>
<td>([L T^{-1}])</td>
</tr>
<tr>
<td>(D_{a,c,k}^{e})</td>
<td>depth of capillary rise from groundwater table into the root zone</td>
<td>([L])</td>
</tr>
<tr>
<td>(D_{a,c,k}^{w})</td>
<td>equivalent depth of irrigation water applied</td>
<td>([L])</td>
</tr>
<tr>
<td>(D_{a,c,k}^{w})</td>
<td>precipitation that infiltrates into the soil and contributes to crop evapotranspiration</td>
<td>([L])</td>
</tr>
<tr>
<td>(DP_{a,c}^{c})</td>
<td>deep percolation ratio</td>
<td>([\text{fraction}])</td>
</tr>
<tr>
<td>(D_{a,c,k}^{o})</td>
<td>depth of water lost as tailwater runoff from irrigation</td>
<td>([L])</td>
</tr>
<tr>
<td>(G_{a,k})</td>
<td>average flow depth in the river or canal cell</td>
<td>([L])</td>
</tr>
<tr>
<td>(d_{a})</td>
<td>extinction depth</td>
<td>([L])</td>
</tr>
<tr>
<td>(D_{a,c,k}^{w})</td>
<td>excess water from root zone</td>
<td>([L])</td>
</tr>
<tr>
<td>(R_{a,c})</td>
<td>application efficiency</td>
<td>([\text{fraction}])</td>
</tr>
<tr>
<td>(R_a)</td>
<td>maximum capillary rise from groundwater table into the root zone</td>
<td>([L T^{-1}])</td>
</tr>
<tr>
<td>(S_{a,k})</td>
<td>groundwater pumping for agricultural use</td>
<td>([L^3 T^{-1}])</td>
</tr>
<tr>
<td>(S_{a,k})</td>
<td>total groundwater pumping</td>
<td>([L^3 T^{-1}])</td>
</tr>
<tr>
<td>(S_{a,n}^{n})</td>
<td>groundwater pumping for nonagricultural use</td>
<td>([L^3 T^{-1}])</td>
</tr>
<tr>
<td>(S_{a,x})</td>
<td>groundwater pumping (+)</td>
<td>([L^3 T^{-1}])</td>
</tr>
<tr>
<td>(h_{a,k})</td>
<td>average potentiometric head</td>
<td>([L])</td>
</tr>
<tr>
<td>(h_{a,k})</td>
<td>water depth in the reservoir facility</td>
<td>([L])</td>
</tr>
<tr>
<td>(h_{a})</td>
<td>potentiometric surface elevation below which capillary rise begins to decrease</td>
<td>([L])</td>
</tr>
<tr>
<td>(i,j,l)</td>
<td>indices denoting row, column and layer</td>
<td></td>
</tr>
<tr>
<td>(k)</td>
<td>index denoting stress period</td>
<td></td>
</tr>
<tr>
<td>(K_y)</td>
<td>yield response factor</td>
<td></td>
</tr>
<tr>
<td>(l)</td>
<td>index denoting layer number</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Units</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>MAD_d</td>
<td>management allowed depletion level</td>
<td>[%]</td>
</tr>
<tr>
<td>MRO_{4,c}, BROL_c</td>
<td>regression coefficients for tailwater runoff losses</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>number of drainage exits served by a river or canal cell</td>
<td></td>
</tr>
<tr>
<td>N_l</td>
<td>number of cells in UCA</td>
<td></td>
</tr>
<tr>
<td>N_X</td>
<td>number of cells with agricultural use of water</td>
<td></td>
</tr>
<tr>
<td>N_c</td>
<td>number of crops</td>
<td></td>
</tr>
<tr>
<td>n_k</td>
<td>number of stress periods</td>
<td></td>
</tr>
<tr>
<td>n_l</td>
<td>index denoting the number of layers in the aquifer system</td>
<td></td>
</tr>
<tr>
<td>o</td>
<td>index denoting cell (i,j,l)</td>
<td></td>
</tr>
<tr>
<td>G^b_{k}</td>
<td>known flow across the boundaries of the study area</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G^o_{k}</td>
<td>boundary recharges (-) that result from agricultural and nonagricultural water use on the ground surface</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G^b_{k}</td>
<td>known recharge through bedrock</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G^d_{k}</td>
<td>saturated flow between the aquifer and general head boundary cells</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G^c_{k}</td>
<td>capillary rise (+) from groundwater table into the crop root zone</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{d_k}</td>
<td>total drainage water collected in drainage exit T</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{d_k}</td>
<td>drainage water reused in irrigation</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{r_k}</td>
<td>drainage water that leaves the boundary of the study area</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G{p_{k}}</td>
<td>deep percolation losses from irrigation</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{d_k}</td>
<td>collected drainage water that returns to the river</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{o_k}</td>
<td>outflow rate in the downstream side of the river cell</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{n_k}</td>
<td>total rate of capillary rise from groundwater table into the root zone</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{i_k}</td>
<td>inflow rate in the upstream side of a river cell</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{i_k}</td>
<td>irrigation water delivered</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{r_k}</td>
<td>return flow as surface drainage water from nonagricultural use</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{s_k}</td>
<td>seepage from nonagricultural use of water</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>Q_{o,c}</td>
<td>inflow size per furrow</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>Q_{s_k}</td>
<td>flow components that depend on water management</td>
<td>[L^3 T^{-1}]</td>
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<tr>
<td>Q_{f_k}</td>
<td>overflow spillage losses from the primary delivery system</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{s_k}</td>
<td>reduction in vertical flow between cells in layer 1 and the lower layer l+1 due to drop in head below the top of layer l+1</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{s_k}</td>
<td>seepage losses from the primary irrigation delivery system</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>G_{d_k}</td>
<td>surface water rate released from the reservoir facility</td>
<td>[L^3 T^{-1}]</td>
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<tr>
<td>G_{r_f}</td>
<td>precipitation that contributes to groundwater recharge in noncropped and cropped-non-irrigated areas</td>
<td>[L^3 T^{-1}]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
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<tr>
<td>$q_k$</td>
<td>spillage water losses from the reservoir facility</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$q_k^{20}$</td>
<td>tailwater runoff water losses from irrigation</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$q_k^{20}$</td>
<td>runoff flow rate from precipitation</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$q_k$</td>
<td>flow between the aquifer and reservoir facilities</td>
<td>[L$^3$T$^{-1}$]</td>
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<td>$q_k^{20}$</td>
<td>precipitation contribution to the reservoir storage</td>
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<tr>
<td>$q_k^{20}$</td>
<td>evaporation losses from the reservoir facility</td>
<td>[L$^3$T$^{-1}$]</td>
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<td>$q_k^{20}$</td>
<td>total drainage water disposed</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$q_k^{20}$</td>
<td>overflow spillage losses from the secondary irrigation delivery system</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$q_k$</td>
<td>flow between the aquifer and streams</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$q_k^{20}$</td>
<td>known discharge (+) through springs</td>
<td>[L$^3$T$^{-1}$]</td>
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<td>seepage losses from the secondary irrigation delivery system</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$q_k^{20}$</td>
<td>total surface water diversion to UCA</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$q_k^{20}$</td>
<td>deep percolation losses from excess water in the crop root zone</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$q_k$</td>
<td>horizontal flow across a boundary</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$R_{c_{avg}}$</td>
<td>average rooting depth</td>
<td>[L]</td>
</tr>
<tr>
<td>$s_k$</td>
<td>surface water delivered for agricultural use</td>
<td>[L$^3$T$^{-1}$]</td>
</tr>
<tr>
<td>$s_k$</td>
<td>surface water delivered</td>
<td>[L$^3$T$^{-1}$]</td>
</tr>
<tr>
<td>$s_k^{20}$</td>
<td>surface water delivered for nonagricultural use</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$s_k$</td>
<td>storage coefficient for cell $i$</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$s_k$</td>
<td>surface water rate delivered for artificial recharge</td>
<td>[L$^3$T$^{-1}$]</td>
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<tr>
<td>$T$</td>
<td>transmissivity</td>
<td>[M$^2$T$^{-1}$]</td>
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<tr>
<td>$TWR_{c_{avg}}$</td>
<td>tailwater ratio</td>
<td>[fraction]</td>
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<tr>
<td>$V_k$</td>
<td>volume in the reservoir facility</td>
<td>[L$^3$]</td>
</tr>
<tr>
<td>$V_k$</td>
<td>storage in river cell</td>
<td>[L$^3$]</td>
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<tr>
<td>$W_{max}$</td>
<td>net maximum depth of soil water that should be depleted between irrigations</td>
<td>[L]</td>
</tr>
<tr>
<td>$Y_c$</td>
<td>unit actual crop yield</td>
<td>[ML$^{-2}$]</td>
</tr>
<tr>
<td>$Y_c$</td>
<td>unit potential crop yield</td>
<td>[ML$^{-2}$]</td>
</tr>
<tr>
<td>$Z$</td>
<td>objective variable</td>
<td>[M]</td>
</tr>
<tr>
<td>$\theta_{c_{\text{rc}}, \theta_{c_{\text{wp}}}}$</td>
<td>soil moisture contents at field capacity and wilting point</td>
<td>[L]</td>
</tr>
<tr>
<td>$s_{c_{\text{rc}}, s_{c_{\text{wp}}}}$</td>
<td>soil moisture storage</td>
<td>[L]</td>
</tr>
<tr>
<td>$I_{c_{\text{rc}}, I_{c_{\text{wp}}}}$</td>
<td>bottom elevation of the stream</td>
<td>[L]</td>
</tr>
<tr>
<td>$I_{c_{\text{rc}}, I_{c_{\text{wp}}}}$</td>
<td>hydraulic conductance of the stream-aquifer interconnection</td>
<td>[L$^2$T$^{-1}$]</td>
</tr>
<tr>
<td>$\Delta t_k$</td>
<td>duration of stress period $k$</td>
<td>[T]</td>
</tr>
<tr>
<td>$\Delta x_{i,j,l}$</td>
<td>cell size in $x$, $y$ and $z$ directions of cell $i$ located in row $i$, column $j$, layer $l$</td>
<td>[L]</td>
</tr>
<tr>
<td>$Z$</td>
<td>set of river or canal cells</td>
<td></td>
</tr>
<tr>
<td>$\theta_{c_{\text{rc}}, \theta_{c_{\text{wp}}}}$</td>
<td>water content on a volume basis at field capacity and wilting point, respectively</td>
<td>[%]</td>
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<tr>
<td>$I(\mu)$</td>
<td>set of cells in UCA $\mu$</td>
<td></td>
</tr>
</tbody>
</table>
\( B_k \)  
bottom elevation of the stream  

\( K \)  
set of stress periods  

\( \Lambda \)  
set of Unit Command Areas (UCA)  

\( \mu \)  
index denoting a Unit Command Area (UCA)  

\( M \)  
set of cells in the study area  

\( N \)  
set of cells with reservoir facilities  

\( S \)  
set of drain cells  

\( O \)  
set of cells where capillary rise takes place  

\( \Pi \)  
set of cells that can receive surface water  

\( O^s_{\mu,k} \)  
elevation of the free water surface in the stream cell  

\( \tau \)  
index denoting drainage exit  

\( \Phi \)  
set of cells where agricultural and nonagricultural water use can occur  

\( \phi^{di} \)  
set of cells where drainage water is collected and reused in irrigation  

\( \phi^{do} \)  
set of cells where drainage water is collected and departs the study area  

\( \phi^{dr} \)  
set of cells where drainage water is collected and returned to the river  

\( \chi \)  
set of cells requiring water use for agricultural use  

\( Q \)  
set of pumping cells in the study area

<table>
<thead>
<tr>
<th>S</th>
<th>RYR</th>
<th>EAW</th>
<th>QOW m³/s</th>
<th>QGW m³/s</th>
<th>QSW* m³/s</th>
<th>YLD # 10^6 kg</th>
<th>XDP m³/s</th>
<th>XRO m³/s</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.685</td>
<td>0.002</td>
<td>1.790</td>
<td>16.58/3</td>
<td>0.27/7</td>
<td>0.204</td>
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<tr>
<td>2</td>
<td>0.632</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td>6.021/4</td>
<td>0.00/4</td>
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<tr>
<td>3</td>
<td>0.002</td>
<td>0.711</td>
<td>0.002</td>
<td>1.787</td>
<td>16.55/5</td>
<td>0.26/0</td>
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<tr>
<td>4</td>
<td>0.142</td>
<td>0.691</td>
<td>0.002</td>
<td>1.322</td>
<td>14.23/5</td>
<td>0.20/3</td>
<td>0.128</td>
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<tr>
<td>5</td>
<td>0.054</td>
<td>0.674</td>
<td>0.002</td>
<td>0.837</td>
<td>15.68/6</td>
<td>0.08/6</td>
<td>0.328</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.041</td>
<td>0.684</td>
<td>0.002</td>
<td>0.902</td>
<td>15.89/6</td>
<td>0.13/9</td>
<td>0.274</td>
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<tr>
<td>7</td>
<td>0.039</td>
<td>0.668</td>
<td>0.002</td>
<td>0.250</td>
<td>1.181/4</td>
<td>0.27/6</td>
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<tr>
<td>8</td>
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<td>0.675</td>
<td>0.002</td>
<td>0.000</td>
<td>1.961 (1.476) *</td>
<td>16.18/7</td>
<td>0.31/8</td>
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</tr>
<tr>
<td>9</td>
<td>0.024</td>
<td>0.683</td>
<td>0.002</td>
<td>0.175</td>
<td>1.796 (1.359) *</td>
<td>16.19/0</td>
<td>0.27/6</td>
<td>0.178</td>
</tr>
</tbody>
</table>

S = scenario  
RYR = weighted average yield reduction  
EAW = weighted average irrigation application efficiency  
QOW = weighted average inflow size per furrow  
QGW = total groundwater pumping  
QSW* = total surface water  
(*) = total surface water released from reservoir  
YLD = total crop yield  
XDP = total deep percolation  
XRO = total tailwater runoff  

30.08.2016, 29 November 94
FIGURE 2. Hypothetical Representation of Surface Water Diversion and Surface Drainage Disposal in an Irrigation District Composed of Several Unit Command Areas (UCA).
FIGURE 3. Schematic Representation of a Hypothetical Irrigation District with the Primary and Secondary Delivery Systems Serving a Group of Cells in a 12-cell UCA.
FIGURE 4. Schematic Representation of the Study Area

Cell Attributes

- constant head cell
- variable head cell and UCA number
- river or canal cell and UCA number
- effluent river cell
- Influent river cell

Other attributes may include river grouping, reservoir cell, agricultural use cell, surface water cell, groundwater cell, diversion cell, and drainage cell.
FIGURE 5. Set of Nondominated Solutions: Total Crop Yield and Groundwater Pumping Versus Deep Percolation.
KEYWORDS

Conjunctive water management; groundwater; surface water; irrigation; reservoirs; water pollution; non-point source pollution
1 Dec 1994

Water Resources Research
Dr. George M. Hornberger, Editor
2015 Ivy Road
Terrace Level, Suite 10
Charlottesville, VA 22903

Dear Dr. Hornberger,

I am submitting four copies of 'Modelling for optimal conjunctive water management: irrigated crop production versus NPS pollution prevention'. Please consider it for publication in Water Resources Research. It has not been submitted elsewhere. If you have any questions, please contact me.

Sincerely,

Richard C. Peralta, PhD PE
Professor
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