ABSTRACT

Platooning Safety and Capacity in Automated Electric Transportation

by

James Fishelson, Master of Science
Utah State University, 2013

Major Professor: Dr. Kevin Heaslip
Department: Civil and Environmental Engineering

Automated Electric Transportation (AET) proposes a system of automated platooning vehicles electrically powered by the roadway via wireless inductive power transfer. This has the potential to provide roadway transportation that is less congested, more flexible, cleaner, safer, and faster than the current system. The focus of this research is to show how platooning can be accomplished in a safe manner and what capacities such an automated platooning system can achieve. To accomplish this, first two collision models are developed to show the performance of automated platoons during an emergency braking scenario: a stochastic model coded in Matlab/Simulink and a deterministic model with closed-form solutions. The necessary parameters for safe platooning are then defined: brake variances, communication delays, and maximum acceptable collision speeds. The two collision models are compared using the Student’s t-test to show their equivalence. It is shown that while the two do not yield identical results, in most cases the results of the deterministic model are more conservative than and reasonably close to the results of the deterministic model.

The deterministic model is then used to develop a capacity model describing automated platooning flow as a function of speed and platoon size. For conditions where platooning is initially unsafe, three amelioration protocols are evaluated: brake derating, collaborative braking, and increasing the maximum acceptable collision speed. Automated platooning
flow is evaluated for all of these scenarios, compared both with each other and with traditional roadway flow patterns.

The results of these models show that when platooning is initially safe, very high vehicle flows are possible: for example, over 12,000 veh/hr for initial speeds of 30 m/s and 10 vehicle platoons. Varying system parameters can have large ramifications for overall capacity. For example, autonomous (non-platooning) vehicles do not promise anywhere near this level, and in many cases struggle to approach the capacity of traditional roadways. Additionally, ensuring safety under an emergency braking standard requires very small communication delays and, most importantly, tight braking variances between the vehicles within a platoon. As proposed by AET, a single type of electric vehicle, combined with modern wireless communications, can make platooning safer than was previously possible without requiring amelioration. Both brake derating and collaborative braking can make platooning safer, but they reduce capacity and may not be practical for real-world implementation. Stricter versions of these, cumulative brake derating and exponential collaborative braking, are also evaluated. Both can degrade capacity to near current roadway levels, especially if a large degree of amelioration is required. Increasing maximum acceptable collision speed, such as through designing vehicles to better withstand rear-end collisions, shows more promise in enabling safe intraplatoon interactions, especially for scenarios with small communication delays (i.e. under 50ms).
PUBLIC ABSTRACT

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This thesis examines the theoretical traffic flow of Automated Electric Transportation (AET). This is a system of self-driving cars (the car drives itself with no human input) which act together and receive electricity directly and wirelessly from the roadway. It promises roadway transportation that is less congested, more flexible, cleaner, safer, and faster than the current system. The vehicles in AET can be arranged in platoons, which is where a number of vehicles travel together with very small separations between them (i.e. under 3 feet). The closest equivalence to platooning in the modern world is on railroads; a 10 car train equates to a 10 car platoon. However, unlike trains, the vehicles in a platoon are not physically connected. Instead, the cars themselves maintain the necessary separations using sensing, computers, and wireless communications. The primarily goals of this thesis are to identify how platooning can be accomplished safely and to evaluate the performance of an AET system. The obtained results show that by utilizing modern technology, including rapid wireless communications and the use of a single type of electric vehicle, AET can allow for far greater capacities than were previously possible. Current roadways can support only about 2,000 vehicles per hour in each lane, but AET has the ability to raise that figure to over 10,000.

Three models are developed here: two collision models and one capacity model. The collision models show how an AET system would respond in an emergency braking scenario,
where one vehicle unexpectedly brakes as hard as it can. The models can therefore define what vehicle separations are safe for AET and platooning. Different parameters, such as the braking ability of the vehicles, are varied to show what effect they would have on system safety. The simpler collision model is used to define the parameters that are necessary for safe platooning, and it is compared with the more complicated model to show that they are equivalent.

The simpler collision model is then used as an input for the capacity model; the more complex collision model could not do this. The capacity model gives the flow of platooning vehicles (in vehicles per hour) as a function of both speed and number of vehicles in a platoon. For situations where platooning would be initially unsafe, three different techniques are considered that could improve safety: allowing for worse collisions to be considered acceptable, having the vehicles act in unison to brake together, and having earlier vehicles brake weaker than their followers. The effects that each of these techniques could have on capacity are discussed and calculated, and the potential problems with each of the techniques are also addressed.

Overall, the results presented here show that when platooning initially safe, very high vehicle flows are possible: for example, over 12,000 veh/hr for initial speeds of 30 m/s and 10 vehicle platoons. However, different systems can have very different maximum vehicle flows. For example, self-driving vehicles without platooning cannot offer anywhere near this level, and in many cases struggle to approach the flows of traditional roadways. Additionally, ensuring safety under the emergency braking standard used here requires very small communication delays (how long it takes for one vehicle to send a command to another and for that vehicle to respond), and more importantly, the vehicles braking performance must be very similar (one vehicle cannot brake much stronger or weaker than another). However, as proposed by AET, a single type of electric vehicle, combined with modern wireless communications, can help meet these requirements and make automated platooning safer and with higher vehicle flows than was previously possible.
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James Fishelson
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CHAPTER 1
INTRODUCTION

Automated Electric Transportation (AET) represents a new approach for surface transportation that addresses major challenges associated with automobile dependency: energy, capacity, safety, and emissions. In dense urban environments, integrated multi-modal systems have made measurable improvements on the cost of transportation and the quality of life. Similar solutions are needed in less dense places that will rely on automobiles for the foreseeable future. AET does not seek to eliminate the automobile, but rather to improve it, proposing a system of automated (self-driving and coordinated) vehicles communicating with each other and the infrastructure and electrically powered by the roadway via in-motion inductive power transfer (IPT), as shown in Figure 1.1.

AET also calls for the vehicles to be arranged in platoons, which traditionally refer to the linear vehicle groupings that form at signalized intersections. Here, automated vehicle platoons are defined as linear groups of vehicles acting in unison and traveling in close-following formation, with very small separations between them, as shown in Figure 1.2. \( Z \) is the number of vehicles in each platoon, \( S_{iw} \) is the intraplatoon separation (distance between vehicles within a platoon) and \( S_{ip} \) is the interplatoon separation (distance between platoons). These separations are measured from the rear bumper of a lead vehicle to the front bumper of the follower.

This research borrows from traditional traffic flow theory and car-following models, applying them to systems of automated platooning vehicles, and examines the resulting similarities and differences between flow on existing roadways and in an AET system. It shows the ability of platooning and automation to offer highway lane-capacities of greater than 10,000 vehicles per hour (veh/hr), far in excess of the 1,800-2,200 veh/hr of traditional highway lanes [2]. A necessary condition for achieving these capacities is ensuring safe platooning. The collision model developed in this research addresses not only interplatoon interactions, but also fills a gap in the literature by examining and quantifying the necessary parameters for safe intraplatoon interactions during emergency braking scenarios. The
Fig. 1.1: AET Conceptual Schematic

[1]

Fig. 1.2: Sample Platoon

\[ S_{iv} \quad S_{ip} \quad Z = 5 \]
collision model is used to develop a capacity model, and this research quantifies the effects of varying different parameters (such as communication delay, braking rate, and platoon size) on both the safety and vehicle capacity of an AET system.

1.1 Importance of AET

This section examines the state of the existing road network. It then introduces and explains the benefits that automation can provide by reducing driver and vehicle variability.

1.1.1 Existing Road Network

Successful economies are dependent on transportation to connect people and goods, and improvements in transportation can create substantial benefits for the economy as a whole; the construction of the Interstate Highway System has been credited with approximately one-quarter of the national productivity gains realized during the ensuing four decades [3]. However, America’s transportation infrastructure is aging, and its increasing inefficiency costs the economy over $1 trillion annually. This comprehensive value includes losses from vehicle crashes, wasted fuel, time delays, and other miscellaneous factors [4]. Roadway capacity limitations combined with the need to import oil threaten the status quo of the US transportation system. America’s road vehicles account for over half its petroleum consumption and one-third of its CO$_2$ emissions (45% of the world’s vehicular CO$_2$ output) [5]. Beyond CO$_2$, internal combustion engines (ICEs) also contribute to air-quality problems through particulate and NO$_x$ emissions. In 2007, traffic congestion alone was responsible for losses of $87.2 billion dollars in excess fuel costs in the United States and nearly the equivalent of a 40-hour work week per commuter in lost time [6].

Safety is another pressing concern. In the US alone, approximately 30-40,000 die annually from vehicle collisions [7,8]. However, the number of both deaths and injuries per vehicle mile traveled (VMT) has been steadily dropping for the past half-century. As of 1998, 1.27 fatalities and 80 injuries occur every 100 million VMT [8]. Freeways, the focus of this research, are even safer [9]. Therefore, roadways can be considered safer than they were in the past, but there is ample room for improvement.
1.1.2 Current Difficulties in Addressing Congestion

Too many vehicles overwhelming insufficient roadway infrastructure is a global problem. This is especially true in the rapidly developing world, where road building and improvements cannot keep pace with rapidly increasing vehicle ownership rates. The recurring rush-hour traffic jams in modern megacities such as Beijing, Mexico City, São Paulo, and Lagos make the roads in Houston and Los Angeles appear free-flowing by comparison [10]. Mass transit can help relieve this congestion, offering substantially higher person-capacities. Heavy-rail transit (HRT), such as subways, offer the potential to carry over 100,000 passengers per hour [11]. Transit is very attractive in dense urban environments, but in less dense, sprawling areas, it is prone to the “last-mile” problem. This occurs where trip destinations (home, work, shopping, etc.) are relatively dispersed, so that the user has no reliable way to get from the transit stop to his or her final destination. By contrast, while traditional automobiles cannot offer the capacity of transit, they provide far greater flexibility. The driver can depart whenever he or she wants and go to any destination that is road-accessible.

1.1.3 AET: A Unique Congestion Solution

AET could help solve this problem by offering the capacity of mass transit with the flexibility of the automobile to go anywhere, anytime [4]. Users could travel on high-capacity automated lanes during the highway portion of the trip [12], and then take manual control of the vehicle on non-automated roadways to reach their final destination [13].

Recent work on automation and platooning generally focuses on platooning within mixed traffic scenarios (automated and non-automated vehicles in the same lane), such as Europe’s KONVOI system [14] and Safe Road Trains for the Environment (SARTRE) project [15], autonomous, non-platooning vehicles, such as the Google Car, or lower-speed urban transit projects, such as CityMobil [16]. Though these efforts claim some flow improvements, most share a focus with SARTRE to offer “safety, comfort, and environmental benefits” [15]. Safety and comfort improvements are evident for vehicle automation, and energy savings could be obtained both due to reduced unnecessary accelerations and the “drafting effect” (reduced aerodynamic drag) allowed by platooning. Wind-tunnel testing
has shown this drafting reduction to be as great as 50% for traditional passenger sedans [17], though other work indicated lesser values of approximately 25% [18].

The work presented in this thesis is an extension of the capacity and flow models of the Automated Highway Systems (AHS) project, with corresponding research performed mostly between 1970 and 2003. AET and AHS are unique in promising the additional benefit of transformative capacity improvements due to platoons of automated vehicles primarily traveling on dedicated lanes [19–22]. This research seeks to exhibit how automated vehicle platooning can be safely achieved and how modern technology, especially electric controls and wireless communications, can offer superior performance to AHS.

1.2 Research Question

The main research question that this thesis seeks to address is: “What capacities are AET capable of achieving?” It evaluates the corresponding hypothesis: “Platoons of AET vehicles can provide capacities of over 10,000 veh/hr.” A necessary condition of reaching these capacities is that safety must be maintained. To test this condition, and thereby show that these high capacities are achievable, two collision models are developed and compared, one stochastic and one deterministic. A statistical analysis shows that the deterministic model is reasonably equivalent to the come complex stochastic model, and is therefore acceptable to use as an input for the capacity model developed here.

A roadway’s lane-capacity \( q_{\text{max}} \), in veh/ha, represents the maximum flow \( q \) on that lane, which can be described by the fundamental flow relationship:

\[
q = kv
\]  

(1.1)

where \( v \) is the vehicle speed and \( k \) is density (vehicles per unit distance). Vehicle separations can be quantified using car-following models, which explicitly or not consider what drivers deem to be the safe following distance. Therefore, the issues of capacity and safety cannot be separated from each other. Since AET does not rely on human drivers, this research disregards psychological factors and the actual safe following distances are used: separations
necessary to avoid an unsafe collision during an emergency brake.

1.3 Research Challenge

One notable challenge in evaluating any hypothetical network is that by definition, no system exists to provide data for analysis. While microsimulation tools can potentially be used to help model AET, substantial changes in both the input parameters and the coding itself may be needed. Therefore, while this research aims to provide an overview of how automated vehicle flow should differ from traditional flow, physical demonstrations (both table-top and full-scale) would be useful in validating the claims made here and allowing for model calibration. Additionally, while this research considers the intraplatoon interactions during emergency scenarios, it does not evaluate steady-state intraplatoon vehicle performance, which is governed by platooning control strategies. This research identifies the maximum allowable intraplatoon separations, and the control strategies will quantify the minimum achievable separations. Together, these efforts will help identify an AET system where the former is smaller than the latter, as required for platooning to be safe.

1.4 Major Theories and Anticipated Contributions

Traditional traffic flow theory and related car-following models contain universal patterns for vehicle flow, and are used as a base for the collision and flow models developed in this research. However, the consideration of platooning, combined with the inclusion of automation that provides far higher performance and predictability over human drivers, produces sizable deviations in flow from traditional roadways. This research will identify and evaluate both the similarities and the differences in flow between traditional and automated platooning networks. Hopefully this will provide a needed framework for transportation professionals to evaluate the speed-density-flow relationships for systems of automated platooning vehicles.

Additionally, these AET collision and flow models will help contribute to the development of AET microsimulation models in VISSIM and Paramics, including the changes that must be made to the softwares’ underlying car-following models in order to represent
automated platooning vehicles. The models developed within are intended as a general, widely-applicable framework, numerous assumptions are made. Each assumption is explained and justified when it is presented.

More specifically, this research, via the development of a collision model, seeks to define the necessary parameters for safe platooning, both for intraplatoon and interplatoon interactions. The input parameters include maximum acceptable collision speed ($\Delta v_{safe}$), initial vehicle speed ($v_0$), communication delay ($T$), brake actuation constant ($\tau$), and vehicle braking rates: $a_l$ and $a_f$ for leader and follower braking rates respectively, both absolute values and variances. The collision model is tested against an emergency braking scenario, and the output gives the maximum safe intraplatoon separation ($S_{iv,max}$) and the minimum safe interplatoon separation ($S_{ip,min}$). These together provide the Unsafe Separation Zone (USZ): the vehicle separations that are unsafe for automated vehicle platooning. These results are then used to develop a capacity model for automated platooning capacity model also examines the effects of varying the number of vehicles in a platoon. For situations where platooning is initially unsafe, different amelioration techniques are considered, such as brake derating: the lead vehicle is instructed to brake weaker than the follower.

1.5 Document Organization

This thesis document provides a report on the research conducted over the period of January 2011 to April 2013. The remaining chapters contained in this thesis document are organized as follows:

- Chapter 2: Literature Review
- Chapter 3: Methodology
- Chapter 4: Results and Analysis
- Chapter 5: Conclusions and Future Research
CHAPTER 2
LITERATURE REVIEW

This chapter first evaluates traditional car-following and traffic flow models, then provides an overview of the history of automated vehicles. It concludes with a description of automated vehicle modeling research, both platooning (control) models and flow/capacity models. It identifies the advances made by this past work, but also points out deficiencies in the literature that this thesis hopes to address.

2.1 Car-Following Models

Car-following models take a microscopic approach to traffic, determining the links between individual vehicles and quantifying the way that one vehicle follows another. The outputs of these models are usually a vehicle’s position, speed, and acceleration. By providing the density characteristics of a roadway as a function of speed, the microscopic car-following models can be extrapolated to yield macroscopic flow models. This section evaluates a range of car-following models and describes their importance for AET.

Car-following behavior is approximate in nature [23]. As described by Herman [24], the goal of any traditional model is to represent “a theoretical basis for describing the dynamical behavior of vehicles in terms of human behavior.” Therefore, the success of traditional models can be gauged by how closely they represent actual roadway conditions. Any potential automated platooning car-following model does not have existing flow data against which to test and validate, so their success cannot be measured as with the traditional models. However, as AET seeks improvements over driver behavior, it is useful to consider existing car-following models as a starting point. Identifying effective automated models and seeing how they differ and can be modified from traditional models is a topic of interest for automated vehicle research.

2.1.1 Early Car-Following Models

Car-following theories were first seriously developed in the 1950s. Pipes’ theory is one
of the earliest, and is an expression of the safety heuristic of “one vehicle length separation for every ten miles per hour of speed” [25]. Where vehicle length is given as 20 feet, this can be written as:

\[ h_{MIN} = 1.36 |\dot{x}_{n+1}(t)| + 20 \]  

(2.1)

where:
- \( h_{MIN} \) The minimum safe following distance (space-headway) measured from the front bumper of one vehicle to the front bumper of the other (ft)
- \( \dot{x}(t) \) The vehicle’s velocity at time \( t \) (ft/s)
- \( n \) and \( n + 1 \) Mark the follower and leader vehicle, respectively.

As seen here, the distinction between space-headway (\( h \)) and separation (\( S \)) is that the former includes the vehicle length (\( L \)) and the latter does not. The two can be related as:

\[ h = L + S \]  

(2.2)

Despite its simplicity, Pipe’s theory provides reasonably close agreement with field studies in middle-speed ranges (not very congested nor very free-flowing), especially if the model is calibrated to consider vehicle length as being 21 or 22 feet. Forbes’ theory, a related model that considers driver reaction time, has a similar agreement pattern [26].

\[ h_{MIN} = \Delta t [\dot{x}_n(t)] + 20 \]  

(2.3)

where \( \Delta t \) is the driver’s reaction time (s).

### 2.1.2 GHR Family of Equations

During the late 1950s and early 1960s, General Motors (GM) developed five separate generations of car-following theories [27]. These five related equations form the basis for the
Gazis-Heman-Rothery (GHR) family of equations, given by Brackstone and Macdonald [28]:

\[
\ddot{x}_{n+1}(t + \Delta t) = a \cdot \dot{x}_{n+1}^m(t + \Delta t) \cdot \frac{[\dot{x}_n(t) - \dot{x}_{n+1}(t)]}{[x_n(t) - x_{n+1}(t)]^l}
\]  \hspace{1cm} (2.4)

where:

- \( m \) and \( l \) Constants determining the type of theory being used
- \( a \) Constant representing sensitivity
- \( \ddot{x} \) The vehicle’s acceleration (\( m/s^2 \))

These models were major steps forward for a number of reasons. Different GHR/GM models were far better at representing very congested or very free-flowing conditions in field tests (which were extensive) than Pipe’s or Forbes’ theories. Additionally, it was GM that first recognized the link between the microscopic and macroscopic flow models [29]. For example, using the GHR model where \( m = 0, l = 2 \), one can derive the Greenshields traffic flow model described in the following section and given in Equation 2.6. Lastly, GM codified the “perception-reaction” protocol to code for a driver’s response to a lead vehicle, which can be considered a three-step process [30]. First the driver perceives his surroundings. Then, using this information he/she decides what action to take. Lastly, he/she acts (also known as the control step), implementing the decision to the best of their ability. Automated vehicles should be able to achieve superior perception-reaction responses, taking less time to perform an action that is closer to the ideal.

### 2.1.3 Microsimulation Models

The above research laid the foundation for the more elaborate car-following models used in traffic microsimulation softwares. These are important tools for current transportation planners, helping them to model road networks and predict the effects of modifications such as greater vehicle demand, lane additions, and new stop lights. These softwares all use a time-step modeling approach; time is discretely considered, and space is continuous. At each time interval, a simulation is run, the vehicles’ position, speed, and acceleration values updated, and the process is then repeated. Different softwares use varying meth-
ods to accurately model vehicle behavior, and the car-following models/equations used are sometimes proprietary. The models used in the traffic simulation software TransModeler is not released to the public. The traffic simulation software Paramics based on the Fritzsche psycho-physical model [28], and each vehicle unit seeks to maintain its desirable separation target [31]. The traffic simulation software VISSIM uses the Wiedemann psycho-physical model, developed in 1974 and modified since then.

Psycho-physical models consider the driver’s expected behavior and characteristics, as well as the surrounding conditions, and use threshold limits where the driver changes behavior. Drivers are considered to react to outside stimuli, such as changes in relative spacing, only when the threshold is reached [32]. These thresholds define the regimes of different driver behavior as a function of relative spacing and velocity. For example, Wiedemann considers four separate regimes: free driving, following, closing in, and emergency. Fritzsche is similar, but the placements of the regimes and the defining thresholds are different; it also considers two separate following regimes [28].

The traffic simulation software AIMSUM uses the open-source Gipps’ model, a safety distance-based model. The original Gipps’ equation considers only close-following conditions, defining a follower vehicles speed as a factor of driver’s reaction time $\Delta t$, maximum desirable acceleration ($a_{max} > 0$), and minimum desired deceleration ($a_{min} < 0$), as well as the position and speed of both leader and follower. This is a deterministic model, though the equation is not reproduced here due to its complexity. AIMSUM uses a two-regime approach: the original Gipps’ equation to describe impeded flow, and a modified version for unimpeded travel [33]. This unimpeded regime does not consider any lead vehicle, and is the functional equivalent of the psycho-physical models’ free-driving regime.

2.1.4 Applications for AET Modeling

This research considers the massive improvements in both reaction time and consistency in braking rates promised by AET, and using these values in traditional car-following models can yield superior system performance and flow. However, this does not inherently model a fully-automated system, which will have different vehicle, driver/controller, and
system characteristics. In particular, the collaborative nature of automated vehicles allows for platooning, which is described in greater detail in Sections 2.3 and 2.4. Additionally, psychological considerations must be considered and revised for when a computer is in charge of control, rather than a driver. For example, what would be an automated system’s “desirable minimum separation at zero velocity,” which is one of the input parameters for both Paramics and VISSIM.

This thesis does not consider such psychological factors, instead seeking to define the “best-case” where only safety concerns are the limiting factor on capacity. Therefore, it borrows more heavily from Forbes’ and Gipps’ equations than from psycho-physical models. Similar to AIMSUM’s impeded case, the safety/collision model describes the minimum allowable distance separation as a function of speed. An AET flow/capacity model (macroscopic) can then be derived from the safety/collision model (microscopic). The development of both models is described in Chapter 3.

2.2 Traffic Flow Theory

This section provides an introduction to traditional traffic flow theory, using May’s Traffic Flow Fundamentals as the basis [27]. It also addresses more recent work performed since the book was published in 1990, and concludes with a discussion of capacity-specific modeling.

2.2.1 Traditional Traffic Flow Theory: Greenshields Model

As shown in Equation 1.1, lane flow (in veh/hr) is a function of speed and density. Under traditional conditions, a variety of models are used to describe the flow relationship, of which Greenshields is the most common, first proposed in 1934:

\[ v = v_f \times \left(1 - \frac{k}{k_f}\right) \]  \hspace{1cm} (2.5)

where:
Vehicle density at velocity $v$

Free-flow speed: the speed that a vehicle will travel on a roadway if there are no other vehicles

Jam density: the maximum vehicle density that is achieved when $v = 0$

Equations 1.1 and 2.5 can be combined to yield:

$$q = k_j \ast (v - \frac{v^2}{v_f})$$  \hspace{1cm} (2.6)

As discussed earlier, this Greenshields model can also be derived from Equation 2.4 (this occurs when the sensitivity $a$ is set at the free-flow speed $v_f$), and it represents the “traditional fundamental flow relationship.” Flow is zero at $v_f$, and as more vehicles enter the lane, average vehicle speed decreases and density increases. Flow increases until a maximum flow point, which represents the total lane capacity. As more vehicles enter the roadway, density increases, but flow actually decreases. This is considered the congested portion of the curve which transportation planners strenuously try to avoid, as it represents a broken-down network: Level of Service (LOS) F [2]. The uncongested portion of the curve is the obverse; quantitatively, it represents the conditions where an increase in density and a decrease in speed result in an increase in flow. From Equation 2.6, one can derive the optimal speed ($v_o$) and density ($k_o$) that provides lane capacity ($q_{max}$), which occurs when the first derivative of flow ($q'$) equals zero.

$$v_o = \frac{v_f}{2}$$

$$k_o = \frac{k_j}{2}$$  \hspace{1cm} (2.7)

Equation 2.7 can be combined to yield lane capacity:

$$q_{max} = \frac{v_f k_j}{4}$$  \hspace{1cm} (2.8)
2.2.2 Greenshields vs. Field Observations

Greenshields is very effective in describing the general outlines of the speed-density-flow relationships, and for that reason it is still used today. However, its predicted values often deviate from real-world traffic flow, especially when traffic is very congested or very uncongested. Field studies have shown that in relatively uncongested environments, speed does not decrease as rapidly as predicted when flow increases [27]. This is a reasonable observation, as in low-density situations where vehicles are far apart, they will not truly follow the lead vehicle in real-world scenarios. This indicates a flaw in Greenshields; the corresponding GHR car-following equation assumes that a vehicle will always act as a follower no matter how far the leader is ahead.

2.2.3 Variations on Greenshields

All currently used flow models share some characteristics, namely that flow is unimodal and concave with respect to density, reaching a maximum capacity at a given $k_o$, and also that speed is monotonically decreasing with respect to density. However, different models can be used in different situations to more accurately match field observations. Other commonly used single-regime models that can be derived from GHR equations include Greenberg, Underwood, and Northwestern. At high densities, these three predict speed better than Greenshields, but at very low densities Greenberg and Underwood underestimate speed, performing worse than Greenshields (which also underestimates speed, but to a lesser extent). Greenberg and Underwood also underestimate capacity at $\approx 1,600\text{veh/hr}$ as opposed to $\approx 1,800\text{veh/hr}$ for Northwestern and Greenshields [27].

Multi-regime car-following models (such as in Paramics and VISSIM) can provide a more accurate approximation of traffic flow, especially in areas where single-regime models provide poor estimations. To accomplish this, they use different flow models to describe different roadway conditions (regimes). This approach was first proposed in 1961: using Underwood for free-flow conditions and Greenberg for congested conditions [34]. Multi-regime models are particularly effective at addressing vehicles dynamics in low-density scenarios. When the vehicles are very far apart, they effectively act as “free-driving” vehicles, setting
their speed based on the drivers’ preferences. Until traffic flow becomes dense enough that car-following actually occurs (impeded traffic in Fritzsche), they roughly maintain the free-flow speed [35]. This flow pattern has been confirmed in field studies, and is used by the *Highway Capacity Manual* (HCM). There, LOS A corresponds to the density range where average speed is equivalent to the roadway’s design speed [2], and Greenshields is used to define the lower levels of service. Additionally, the traditional axiom of maximum flow occurring at $40 - 45 \text{mi/hr}$ (approximately $v_o$ as given in Equation 2.7) may be inaccurate; University of California Berkeley field studies estimate that maximum flow on I-10 in Los Angeles occurs at around $60 \text{mi/hr}$ [36].

### 2.2.4 Capacity Modeling

As with Greenshields, the capacity for any traffic flow theory with closed-form solutions (deterministic models), whether single or multi-regime, can be calculated by determining the density and speed where $q' = 0$. In stochastic models, such as used in Paramics and VISSIM, the software utilizes random number generation (Monte Carlo simulations) to consider non-discreet values for input parameters, such as distributions of driver reaction times. In these cases, the lane capacity must be derived through multiple simulation runs. Furthermore, these softwares often include lane capacity as an input value for the simulation, rather than the output.

The capacity problem can also be constructed as a function of the speed-spacing relationship, with the capacity given by Rothery’s capacity equation [30].

$$q_{max} = 5280 \frac{v}{L + S}$$

(2.9)

where:

- $L$  Average vehicle length (ft)
- $v$  Speed ($\text{mi/hr}$)
- $S$  Average space separation between vehicles (ft)
As discussed in Section 2.5, most “Automated Highway Systems” (AHS) capacity studies use variations of this equation, which is a different formulation of Equation 1.1 \((q = kv)\) since density can be written as:

\[
k = \frac{1}{L + S}
\]  

(2.10)

Though Equation 2.9 is written to give capacity \((q_{\text{max}})\), it more strictly describes flow as a function of speed: \(q(v)\). \(S\) and \(h\) can be calculated using a variety of input parameters, including driver reaction time, lane width, grade, etc. The following model is provided by the HCM (recall that \(h\) and \(S\) can be converted using Equation 2.2) [2]:

\[
h = \left[ \frac{5280}{3600} \alpha v \right] + \left[ \beta \left( \frac{5280}{3600} v \right)^2 \right]
\]  

(2.11)

where:

\(\alpha\)  Reaction time(s)  
\(\beta\)  Reciprocal of double the maximum average declaration of a following vehicle \((\frac{s^2}{ft})\)

The \(\alpha\) term considers driver reaction time, so will be far reduced in an automated system. The \(\beta\) term considers the necessary spacing to come to a stop in an emergency braking scenario without hitting the lead vehicle [30].

From field data where separations and vehicle lengths are non-constant, density can be obtained by rewriting Equation 2.10:

\[
k = \frac{n}{\sum_{i=1}^{n} S_i + L_i} = \frac{n}{l}
\]  

(2.12)

where:

\(S_i\)  Distance separation behind vehicle \(i\)  
\(L_i\)  Length of vehicle \(i\)  
\(n\)  Number of vehicles in the group under consideration  
\(l\)  Length of the study section of roadway
For its planning tables, the HCM considers capacity to be independent of freeway design speed, with speeds of 60 and 70\text{mi/hr} yielding the same capacity of 2,000\text{veh/hr} at the same $v_0$ (approximately 40-45 \text{mi/hr}) \cite{2}, though as discussed earlier, field observations indicate that maximum flow can occur at higher values. Additionally, the HCM shows how higher design speeds will result in higher flows at densities lower than $k_o$. Non-ideal conditions, including entry/exit ramps, weaving sections, and work zones serve to decrease capacity, though such factors are not considered in this research, which instead examines the capacity of automated platooning systems on a single, isolated lane without merging or exiting.

### 2.3 Automated Vehicles

This section first provides an overview of self-driving vehicle research and introduces the safety concerns of platooning. It then examines the distinction between “automated” and “autonomous” systems, describing how automation and the vehicle-vehicle coordination it implies is important in enabling platooning.

#### 2.3.1 History of Self-Driving Vehicles

GM’s Futurama exhibit at the 1939 World’s Fair was the first definitive expression of the dream for self-driving vehicles, promising automated, elevated highways through dense “cities of tomorrow,” but it was a conceptual rather than technical proposition \cite{37}. Grade-separated freeways were themselves a novel concept, having only been brought to the general public as “Townless Highways” by Benton Makaye in 1930 \cite{38}. AET claims similar goals to Makaye’s; by separating AET lanes from the greater highway and avoiding the poor performance of human drivers, it promises higher speeds, densities, and comfort \cite{39}.

After the World’s Fair, GM started applied autonomous vehicle research in earnest, and was able to produce rudimentary self-driving vehicles that relied on radar sensors and automated mechanical controls in the 1950s \cite{40}. While the efforts faded in America, Japan continued similar programs through the 1960s and 1970s, incidentally considering induction cables as with AET, albeit for lateral guidance rather than power transfer \cite{41,42}. In the 1970s and 1980s, European researchers evaluated systems of “automated guided
vehicles.” While these never entered the roadways, they were the precursors for automated warehousing [43]. Similar special use autonomous vehicles can be seen in present day applications: automation of loading/unloading at ports [44], lateral guidance of snow plows [45], autonomous trains and other people-movers at both airports (i.e. New York City [46] and London [47]) and theme parks [48].

Full autonomy in unpredictable environments is a more recent occurrence. The original Defense Advanced Research Project Agency (DARPA) Grand Challenge of 2005 required vehicles to track a course through a desert environment without any human input [49]. Its successor, the 2007 Urban Challenge, made the vehicles navigate a more complicated city environment, including obeying traffic regulations and responding to obstacles like pedestrians [50]. The “Google Car” is an ongoing project where fully-autonomous vehicles operate on American streets, mixing with regular traffic and using laser-based sensing systems. Technical specifications for this project have not been released. Many car companies have partial and/or full autonomous vehicle projects. Toyota has an “Automatic Vehicle Operation System” program, and Volvo has been very active in both truck platooning and introducing partial-automation technologies, such as adaptive cruise control (ACC) and lane-keeping, to its vehicle fleet [51]. GM remains involved in automation through its EN-V program (see Figure 2.1).

By far the largest effort to date for a fully-automated, grade-separated highway was AHS. The concept unofficially began in 1970, when the capacity benefits of autonomous vehicles were first explicitly considered [52]; platoons were first proposed in 1979 [53]. Work continued in the 1980s, including further studies on AHS capacity [54], heavy vehicles [55], vehicle control [56,57], and even power transfer via electrified rails [58].

In 1994, the National Automated Highway System Consortium (NAHSC) was founded and funded via an act of Congress to work towards implementing the first AHS system [59]. It was a public-private partnership, with core participants as varied as the California PATH research group, Caltrans, General Motors, Lockheed Martin, and the FHWA [60]. The ultimate goal was to bring automated vehicles to the roadway by 2002, and while this never
occurred, they were able to produce a plethora of research and technical papers, as well as putting forth a real-world display of automated platooning at “Demo ’97.” For 4 days, NAHSC ran platoons of passenger sedans (modified versions of readily available vehicles) at highway speeds on a stretch of I-15 outside of San Diego [61,62].

Lateral control was achieved via magnets embedded in the roadway, though vision-based technologies were also tested [63]. Longitudinal control was achieved by a combination of radar/lidar and vision-based systems in the vehicles [21]. These technologies were fundamentally sensor-based. Wireless communications at that time were very rudimentary; Bluetooth did not exist, and the Direct Short Range Communication (DRSC) band for automotive communications was only established in 2008. However, rapid vehicle-to-vehicle communications (V2V) was assumed for AHS capacity modeling, allowing for collaborative braking and time delays as small as 20ms [19, 64, 65]. In 1998, the federal government cut the project’s funding, partially because deployment was hampered by a lack of agreement among the various stakeholders and also due to technological limitations [66].

The remainder of this chapter addresses the findings of past automation research,
describing the control and capacity models used and discussing some of the limitations of AHS that AET hopes to solve.

### 2.3.2 Platooning and the Unsafe Separation Zone

As discussed in Chapter 1, platooning was first introduced as a way to improve safety, fuel consumption, and roadway capacity. The latter is the focus of this work, and is achieved via very small intraplaatoon separations \(S_{iv}\), which allow for higher densities \(k\) and therefore higher flows \(q\) at a given speed. To achieve this, however, it is necessary to show that these small \(S_{iv}\) values can be safely maintained.

For traditional traffic flow and car-following, such as Forbes’s theory, the GHR equations, and the HCM capacity equation, separations are maintained in order to avoid a collision, which generally requires the relatively large vehicle separations seen on roadways, such 112ft at 40mi/hr according to Greenshields, where \(v_f = 80\) mi/hr, \(q_{max} = 2,000\) veh/hr, and \(L = 20\) ft. However, platooning relies on the fact that under certain conditions, very small intervehicle separations can be as safe as very large ones. Vehicles very close to each other may collide, but there is not enough distance to build up a large speed differential before the collision. When this collision speed \(\Delta v\) is small enough to be deemed safe, then platooning is possible. This concept was first qualitatively introduced by Shladover [53] and expanded upon by Tsao et al. [20]. The intermediate, unsafe separations that would provide an unacceptably high \(\Delta v\) value can be defined as the Unsafe Separation Zone (USZ).

This relationship between \(S\) and \(\Delta v\) can be seen in Figure 2.2, which is an output of the collision model developed in Chapter 3. It shows three separate \(S-\Delta v\) curves for a two vehicle scenario, where the following vehicle’s braking rate \(a_f\) is harder, weaker, and equal to that of the lead vehicle \(a_l\). At \(S = 0\), \(\Delta v = 0\) as well, and \(\Delta v\) first increases to a maximum before declining; the point where the curve intersects for a second time at the x-axis represents the minimum separation necessary for the vehicles not to collide.

To determine the USZ, let \(\Delta v_{safe}\) be a threshold corresponding to the maximum acceptable impact velocity for a rear-end collision where there is effectively no risk of serious injury or death. Therefore, the lower bound of the USZ represents the maximum allowable
intraplatoon separation \((S_{iv,max})\), and the upper bound the minimum allowable interplatoon separation \((S_{ip,min})\).

### 2.3.3 Automation vs. Autonomy

Autonomous vehicles can be defined as independent, self-driving vehicles, where a computer and sensors replace the human driver. Automated vehicles are also self-driving, but have high-degrees of connectivity and collaboration; a vehicle is linked to other vehicles via vehicle-to-vehicle communications (V2V) and to the infrastructure via vehicle-to-infrastructure communications (V2I). Therefore, although autonomous and automated are often used interchangeably, they actually refer to two different concepts, a distinction made by Rajamani and Shladover [68] and again by Shladover [69]. Neither driverless control nor vehicle connectivity are binary propositions, but are instead continuums: from fully-manual to fully-driverless for the former, and from fully-independent to fully-connected for the latter. While autonomous technologies have been introduced almost since the beginning of the automobile (i.e. cruise control and automatic transmissions), connectivity improvements, predominantly relying on communication technologies, are more recent occurrences (i.e. cell
phones and the Connected Vehicles project). Connectivity is important for platooning, as vehicles within a platoon must maintain a very small $S_{iv}$, acting collaboratively as part of a greater unit. Past and current platooning control model research addresses this question and lays the groundwork for the requirements demanded of connectivity and autonomy. Acting more collaboratively can help decrease the USZ and enable higher allowable $S_{iv}$ values [64].

2.4 Platooning Control Models

Much effort has been spent describing how vehicles can effectively travel in platoons, maintaining the desired small intraplatoon separations without colliding. Automation is a vital requirement for platooning, as human drivers lack the performance characteristics (i.e. reaction time and perception accuracy) and ability to act collaboratively to maintain these small separations [68, 70]. This section provides an overview of platooning control models used to maintain these separations, describes some limiting factors, including string stability, and addresses some deficiencies in the literature.

2.4.1 Longitudinal Control Models

Extensive platooning modeling work began in the 1990s as part of the AHS project. The two primarily concerns for intraplatoon interactions during steady-state conditions where platoon speed is kept relatively constant (as opposed to emergency scenarios) are separation maintenance and string stability. For the former, intraplatoon separations ($S_{iv}$) must be kept between a given set of boundaries around the desirable separation, and the accelerations/decelerations necessary to maintain these separations must not result in driver discomfort [71]. For the latter, a platoon is considered string stable if oscillation errors do not propagate and amplify throughout the platoon; there is no “slinky effect” [72].

For autonomous vehicles, leader navigation can be achieved through an iterative process for each time step using feedback of the velocity error. In response to the error, the vehicle applies a torque to obtain an acceleration to bring the initial speed closer to the desired speed [73, 74]. For maintaining the desired $S_{iv}$ separations, different physical systems have
been used, though spring dynamics is the most common [71]. Here the force applied to any vehicle is a proportional function of the difference between the actual separation and the desired separation ($x$), as according to Hooke’s law where $F$ is force applied and $k$ is a constant:

$$F = -kx$$  \hspace{2cm} (2.13)

Levedahl et al. [71] recommend a critically dampened spring as a means to reduce oscillations. Other work has developed more complicated models that can be used to obtain a stable platoon; Sheikholeslam and Desoer [75] and Swaroop et al. [76] use feedback linearization techniques combined with linear control laws, and Choi and Hedrick [77] and Ferrara et al. [78] use a non-linear slider control.

### 2.4.2 String Stability and Control Saturation

For string stability to be maintained, the follower vehicles in a platoon must have the leader’s position, speed, and acceleration information [79,80]. Swaroop and Hedrick provide mathematical definitions for string stability and indicate that the theoretical formation of very long platoons (i.e. up to 1000 vehicles) is possible if $S_{iv}$ values could be non-constant [81]. NAHSC was able to demonstrate string stability in Demo ’97 at $S_{iv} = 6m$ [21], and their modeling included a “string stability tool,” though the specifics of this tool were not detailed [19]. More work is needed to identify the effects of string stability on AET and if this might limit maximum platoon size.

Similarly, control saturation, where too many communication packets overload the V2V/V2I infrastructure, could also degrade system performance, reducing string stability and limiting platoon size [82]. Layered control has been proposed as a means to reduce this risk [83,84], but more AET-specific work is needed to determine the potential effects of saturation.
2.4.3 Collision Scenarios

Intraplatoon Cooperative Collision Avoidance (CCA) systems emerged from control model research. The goal of CCA is to use vehicle collaborations to eliminate intraplatoon collisions rather than just reduce their severity [85]. However, avoiding collisions entirely requires large vehicle separations (over 15m) and low speeds (under 15m/s) under automated driving conditions with very high levels of collaboration. Gehrig and Stein [86], Araki et al. [87], and Tatchikou et al. [88] have slightly better results due to superior braking rates and communication delays, requiring separations over 10m and speeds around 30m/s to avoid collisions. Gehrig and Stein [86] generally consider obstacle avoidance, rather than an emergency braking scenario, consistent with their goal of allowing for autonomous vehicles in mixed-traffic and urban environments, and it stresses the importance of grouping similar vehicles together to form platoons: for example, no trucks and sports cars in the same platoon. It does present a graph showing the USZ concept, but does not examine it in great detail.

2.5 Automated Vehicle Capacity and Flow Models

This section addresses the history of automated vehicle capacity modeling, starting with the initial free-agent (non-platooning) research of the 1970s and continuing to AHS related research and more recent studies. It stresses the importance of vehicle platooning as a means to substantially increase roadway capacity and explains how AET fits into the research history.

2.5.1 Early Models

Bender and Fenton [52] presented the first capacity model for automated highways in 1970. They evaluate a purely autonomous system, and much of the research focuses on maintaining stable intervehicle separations, which is also a focus of the control models discussed above. They claim possible capacities of up to 3,750veh/hr, but declare this to be a functionally unreasonable value that would demand too low a reaction time and high levels of passenger discomfort and low fuel efficiency due to rapid braking/acceleration. This
work was expanded upon in 1979 by Fenton [89], who discusses, but does not explicitly model, automated platooning vehicles. However, he does model minimum safe separations as a function of stopping distance, which itself is obtained from deterministic input values for leader and follower position, speed, acceleration, and vehicle length. He stresses how ensuring consistent braking between the leader and the follower, so that $a_f \approx a_l$, could provide far superior capacity values, over $6,000 \text{veh/hr}$ as opposed to under $3,800 \text{veh/hr}$, which was the best that could be achieved for differing braking rates even with very small delays.

2.5.2 Automated Highway Systems Design

Tsao et al. [20] provide a design framework for AHS, establishing four “dimensions”: separation of traffic, transitions between manual and automated driving, normal automated driving, and failures and emergency response. They use these dimensions to develop six scenarios for AHS geometry/road design, such as “segregated highways with platoons.” They determine this to be the optimal scenario to maximize capacity, and it was used as a base for the Demo ’97 geometry. The control architecture for emergency scenarios, including equations describing how the vehicles should respond, was expanded upon in 2000 by Lygeros et al. [90] and Horowitz and Varaiya [72]. However, these are all instances of conceptual research, and they do not apply the frameworks to real-world scenarios to derive actual safe separations and capacities.

2.5.3 Platooning Capacity Equations

As first described by Shladover [53], platooning promises a superior vehicle configuration over existing roadways, offering the potential for much higher overall vehicle density, and by extension, higher capacities. Density is a fundamental parameter for traditional traffic flow theory, along with flow and speed, but it must be described slightly differently for platooning vehicles. In tradition traffic flow, separation is often considered as a single negative exponential distribution about a mean (used as a deterministic $S$ in Equation 2.10), but for platooning there are two distinct separation types: interplatoon and intraplatoon. Therefore, assuming deterministic separation values, density can be derived from Equation
Equation 2.14, which was similarly derived by Fernandes and Nunes [91], assumes platoons of constant size (Z) and constant vehicle length. Where v is the vehicle speed, Equation 1.1 can now be rewritten as:

\[ q = 3600v \frac{Z}{S_{ip} + (Z - 1) \cdot S_{iv} + Z \cdot L} \]  (2.15)

Equation 2.15 was used in AHS capacity studies, though was not explicitly derived [65]. Kanaris et al. [64] calculate capacity as a function of time separations, rather than the space separations in Equation 2.15, and show the conversion between the two:

\[ S_{space} = v S_{time} \]  (2.16)

where \( S_{space} \) and \( S_{time} \) are space and time separations, respectively. NAHSC [19] modifies Equation 2.15 to consider different vehicle types: buses, trucks, vehicles etc., with different \( Z \), \( S_{iv} \), and \( S_{ip} \) values. It considers both homogenous and heterogeneous platoons (such as buses and cars with the same platoon), so that for \( x \) vehicle types, there are \( x! \) platoon types. Where \( n_{i,j} \) is the number of platoons of with \( i \) vehicles of type \( j \) in the study section, \( n_t \) is the total number of platoons, and \( q_{i,j} \) is the calculated flow for a given platoon type.
and size according to Equation 2.15, total lane capacity is given as:

\[ q = \sum_i \sum_j n_{i,j}q_{i,j} \]

\[ q = \sum_i \sum_j n_{i,j}q_{i,j} \quad (2.17) \]

### 2.5.4 Capacity and Safety Modeling in AHS

AHS researchers also examined the relationship between capacity and safety in platooning systems. Much work focused on the modeling of safe separations between platoons, but Kanaris et al. [64] most directly consider both intra- and interplatoon separations. They address the USZ concept, though do not give it a name, providing the conceptual framework where “collision intensity” increases from zero at zero vehicle separation to a maximum, and then decreases to zero again as separation increases further. The lower bound of the USZ \( S_{iv,max} \) and upper bound \( S_{ip,min} \) are thereby defined.

Carbaugh et al. [65] model the tradeoffs between capacity and safety, measuring it as done here as a function of \( \Delta v \) at the point of collision. As with other AHS capacity models [19], they consider four cases: autonomous vehicles, low-cooperative individual vehicles (small amounts of V2V), high-cooperative vehicles (high amounts of V2V) and platooning vehicles. The authors assume total delays of 120 – 150ms (brake actuation of 100ms plus a communication delay of 20 or 50ms), compare the performance of these systems with alert and typical manual drivers (the latter has a mean reaction time of 0.9s), and evaluate capacities and safety at 20, 30, and 40 m/s. As with other AHS work, they treat brake actuation (where the braking systems take time to reach full braking level) as a pure delay, so brake actuation and communication delays are lumped into a single variable.

They show that independent autonomous vehicles could achieve slightly improved capacities of 2,500 veh/hr at far lower collision probabilities than manual drivers: 2.8% vs. 11% and 87% for alert and typical drivers, respectively. They evaluate collision probabilities at capacities of 2,500 veh/hr for the three different speeds, and find an increasing collision likelihood as speed increases, indicating the maximum capacity for autonomous systems with the given input parameters occurs at speeds of at least less than 30 m/s. They also model collision severity, in addition to probability, for intraplatoon separations, finding that at
30 m/s, expected collision speeds at $S_{iv} = 1$ m are only $1.71$ m/s, increasing to $2.16$ and $2.72$ m/s for $S_{iv} = 2$ and $3$ m respectively.

Kanaris et al. [64] take a different approach, similar to the technique for this thesis. Rather than calculating safety for a given capacity, they determine the capacities achievable at a given safety level. They claim capacities ranging from $4,116$ veh/hr for free-agent autonomous vehicles to $7,704$ veh/hr for 10 vehicle platoons. Depending on the scenario, 20 car platoon systems have higher or lower capacities than 10 car platoons; this is due to the increased interplatoon separations necessary as platoon sizes grow for many of the cases. James et al. [92] provide similar values, but give capacities in graphs as a function of speed, so precise capacity values cannot be obtained. However, they do provide the $S_{ip}$ values for autonomous (58m) and platooning (86m) systems at speeds of 30 m/s. By using the same basic methodology but stricter inputs, NAHSC [19] gave smaller values of 36m and 56m respectively. Unlike Kanaris et al. [64], NAHSC considers $S_{ip}$ to be independent of platoon size, and assumes $S_{iv} = 2$ m and $L = 5$ m.

Figure 2.3 provides capacity as a function of speed for different platoon sizes obtained by NAHSC, and the speed at which maximum capacity occurs increases as a function of platoon size in what appears to be a linear fashion. Overall, the capacity values in Figure 2.3 roughly agree with those provided by Kanaris et al. [64] and James et al. [92]. All show maximum capacities of approximately $8,000$ veh/hr under ideal conditions: dry pavement and platooning passenger vehicles with no trucks.

All of these safety models use emergency braking as the standard for evaluating system performance, as is done in this work. An emergency brake is defined as where a lead vehicle brakes as hard as it can to a complete stop. In real-world observation, it has been estimated that drivers maintain only 60% of the necessary minimum separations to avoid collisions in such scenarios [19]. Therefore, maintaining the emergency braking minimum separations for automated driving provides a higher level of safety than the observed typical driver behavior [65].
2.5.5 Differences Between Interplatoon and Intraplatoon Collisions

The research presented in the previous section considers each platoon as a single vehicle-unit with different performance characteristics than the individual vehicles. For example, Kanaris et al. [64] assume that the reaction time between platoons is 0.3s and between vehicles within a platoon it is 0.1s. Additionally, in order to obtain relatively safe intraplatoon interactions, it assumes a 30% brake derating factor; the leader brakes at 30% of its initial given braking rate. This initial rate is given as a bounded Gaussian distribution between $-4$ and $-10 \text{m/s}^2$, with a mean of $-7.01 \text{m/s}^2$ and a standard deviation of $1.01 \text{m/s}^2$. A similar concept to brake derating is used in James et al. [92], where it is called the “brake amplification factor.” These AHS studies do not focus on multiple vehicle interactions within platoons and the possibility of intraplatoon chain reaction collisions. They assume that if one vehicle in a platoon has effectively stopped and avoided an unacceptable collision with its leader, all of the following vehicles would act in a collaborative way in order to avoid other collisions.
2.5.6 Factors that Decrease Capacity

The safety-capacity models described above represent the maximum achievable safe capacities for a given set of inputs, rather than the actual expected capacity of automated systems. In addition to non-ideal inputs to the safety model, such as less predictable braking due to wet pavement [19, 64], system design factors could also decrease capacity. Inclusion of mixed vehicle classes is one such factor [93]; even a 5% truck ratio could reduce capacity by 20% [64]. Increasing densities of entry/exit points also serves to reduce capacity over the isolated, free-flowing scenario. Depending on those densities and the entrance strategy used, Hall et al. [94] show that the inclusion of entry/exit ramps could cause the functional capacity to range from 95% of the nominal down to only 30%, which for the values used approximate conventional highway capacities.

2.6 Summary and Conclusions

Overall, AHS research indicated that capacities of approximately 8,000 veh/hr are possible for platoons of automated passenger sedans, and it did much to describe the necessary conditions for platooning. However, gaps in the literature do exist, which this thesis hopes to address. Specifically, there are areas of the modeling work, both on platooning control and flow, that need to be made more explicit and replicable, and some of the assumptions made must be questioned. Additionally, AET flow models must deviate from the AHS work to consider the different design of AET systems: uniform vehicle type vs. mixed vehicles, electric vehicles (EVs) vs. internal combustion engines (ICEs), and faster wireless communications.

2.6.1 Modeling Gaps

As seen in Section 2.4, most platooning control models do not explicitly consider the USZ concept. They show the effective performance of platoons at small separations during steady-state procedures, both in modeling and in real-world demonstrations as in Demo ’97. Collision avoidance work was performed by control research, but it generally required larger separations in order to fully avoid collisions, effectively considering interplatoon inter-
actions [86–88]. The flow models described in Section 2.5 did address this issue, but further investigation is needed. Most capacity and safety models were not made explicit, and their results are not replicable. For example, Kanaris et al. [64] provide the input values as well as the output results in the form of separations, capacities, and collision probabilities and severities, but do not describe how these values were obtained.

Some other flow studies such as Bender and Fenton [52], Fenton [89], and Goto et al. [95] do explicitly determine minimum separations necessary to avoid collisions and use Equation 2.9 to determine capacity. However, as with the control modeling work, they do not consider small intraplauntoon separations that would result in safe (low-velocity) collisions [85]. Therefore, these efforts effectively consider autonomous vehicle flow, rather than automated platooning flow. Furthermore, NAHSC [19] and Kanaris et al. [64] measure collision severity as the “expected” collision speed. It is assumed that this is a mean value, but as its distribution is not given, more severe, unacceptable collisions may occur at non-negligible rates even when the expected collision speed is acceptably low.

Additionally, it is useful to question the real-world reasonableness of some of the assumptions made to ensure safety. For example, brake derating effectively assumes that a vehicle engaging in emergency braking is still very well-functioning, as it must recognize that it is the first vehicle engaging in the emergency brake and apply the derating factor. As shown in Chapter 4, the collision likelihoods and severities would be far higher if brake derating had not been considered. Similarly, NAHSC [19] and Kanaris et al. [64] assume an intraplauntoon delay of only 0.1s, including both brake actuation and pure time delays, and the maximum delay in Carbaugh et al. [65] is only 0.15s. This was technologically unrealistic for the time; more work is needed to identify if this value is even possible today, though the use of electric vehicles should help [96]. Interplatoon delay is assumed to be 0.3s [19,64], still a fairly low perception-reaction response time.

Kanaris et al. [64] also consider both coordinated and staggered braking; for the former the vehicles within a platoon would break at exactly the same time, and for the latter each vehicle would break 0.1s before the leader. As with above, it should be determined
if staggered braking and perfect coordination are possible in realistic emergency braking scenarios. In particular, staggered braking would require the leader of a 20 vehicle platoon to delay braking by at least 1.9 seconds (19 following vehicles).

2.6.2 Differences Between AHS and AET

The results presented in Chapter 4 show the effects of the lower brake variance that a system of identical electric vehicles would provide, as well the potential benefits of smaller delays due to rapid wireless communications (V2V) and vehicle-to-infrastructure communications (V2I). The potential benefits of electrification, as proposed by AET, must also be considered. Previous platooning control models typically have been designed for internal combustion engines (ICEs), though there has been some research on autonomous electric vehicles [96]. Electric vehicles (EVs) can offer more precise and rapid control responses than vehicles with ICEs. The ICE control models developed by AHS are very complex, functions of air intake, fuel consumption, throttle response, etc., which could result in larger intervehicle delays [19, 22, 72]. An EV could offer superior performance, requiring only a single torque command for both acceleration and braking [96,97]. For the flow models this should serve to give reduced brake actuation delays (i.e. 10ms instead of 100ms), though more work is needed to quantify this effect.

2.6.3 Goals of this Research

This research seeks to develop a replicable collision model with realistic assumptions that will quantify the outlines of the USZ. Of special interest is identifying $S_{\text{iv, max}}$ and showing how this value could be increased. As with the AHS research, it applies the results of this collision model to a flow model, and identifies how different input parameters affect vehicle flow. It shows the effects of varying braking rates, communication delays, vehicle speeds, and platoon size with higher granularity than was done with AHS. Ultimately, it shows that while AHS required some unrealistic inputs to enable safe platooning, by using a single type of automated EV and modern communication technologies, AET is capable of producing safe platooning and high capacities under a variety of scenarios.
CHAPTER 3
METHODOLOGY

This chapter presents the research plan for modeling and analyzing vehicle collisions and platooning traffic flow in an Automated Electric Transportation (AET) system, showing how the collision model acts as an input for the capacity model. Both a stochastic and a deterministic collision model are developed, evaluated, and compared for a two vehicle scenario under an emergency braking standard. Necessary assumptions are made and justified. The stochastic model was developed with Spencer Jackson; some of this work was presented in his master’s thesis [67], and where appropriate it is cited here.

The collision models are used to define the Unsafe Separation Zone (USZ: separations for which an emergency brake results in an unsafe collision) as a function of initial speed, brake variance, communication delay, and acceptable collision speed. The deterministic model is then used to define the necessary brake variance and communication delay to allow for safe platooning at a given initial intraplatoon separation. For parameter sets where platooning is initially unsafe, three amelioration protocols are evaluated: increasing acceptable collision speed, brake derating, and collaborative braking.

By defining the maximum safe density for platooning vehicles, the collision model is used to develop an AET capacity model that provides the maximum capacity that AET systems are capable of safely achieving. This model describes the speed-density-flow relationships for automated platooning vehicles. The capacity is evaluated under a number of scenarios considering different brake variances, initial speeds, communication delays, and platoon size. The effects of the three amelioration protocols are also considered.

This methodology considers a free-flowing environment of a single isolated straight lane. Other factors, such as exiting/merging procedures, could reduce actual flow, but this safety-based model provides an upper bound for potential AET capacity. It is anticipated that through this methodology, the capacity of free-flowing AET lanes, and the factors that would change it, will be properly understood within the context of maintaining separations necessary to avoid unsafe collisions.
3.1 Research Questions and Hypotheses

The major question that this thesis seeks to answer is: “What capacities are AET capable of achieving?” To answer this question, it is necessary to examine how different factors could affect that capacity, including:

- Absolute Braking Rate
- Variance in Braking Rate Between Leader and Follower
- Communication Delay
- Brake Actuation Delay
- Initial Vehicle Speed
- Amelioration Protocol (Such as Collaborative Braking)

Borrowing on and expanding from previous AHS work, this research takes a safety-based approach, identifying the required inter- and intraplatoon separations for safe platooning. As described in Chapter 2, collision speed decreases at small separations as the vehicles get closer together, so that the USZ defines minimum safe interplatoon separation ($S_{ip,\text{min}}$) and maximum safe intraplatoon separation ($S_{iv,\text{max}}$). Therefore, an important question is: “What different factors can increase $S_{iv,\text{max}}$ as defined by the USZ so that safe separations can be reasonably maintained?” Ultra-small $S_{iv}$ values may not be achievable by the control model governing steady-state (non-emergency) vehicle movements, either due to technological limitations or comfort constraints, such as unacceptably high jerk and acceleration values [19,21].

3.1.1 Hypotheses Related To Collisions

Advances in modern technology, especially the tight brake variances obtainable by using a single type of electric vehicle (EV) and the reduced brake actuation delay of those EVs, should allow for AET to achieve safe platooning in a way that was not previously possible. AHS required strong amelioration techniques in order to achieve acceptable safely levels,
such as a 30% brake derating [19,64,92]. This work adds to the existing research literature by explicitly defining the necessary brake variance and communication delay to enable safe platooning for a given set of system parameters. The results of the two collision models, one stochastic and one deterministic, are compared with the findings of AHS research, and they are also compared with each other using the Student’s t-test. Two separate but related hypothesis are tested here. The first is that the stochastic and deterministic models yield statistically identical results. The second hypothesis is less stringent; that the deterministic model yields results that are similar to (within 10%) and more conservative than those of the stochastic model.

3.1.2 Hypotheses Related to Capacity and Flow

The above analysis will show that the deterministic collision model can be used as an input for the capacity model and provide reasonable and conservative estimates of automated platooning flow for free-flowing AET lanes. It is expected that this capacity model will show that a system of automated platooning electric vehicles can provide higher capacities than achievable by traditional roadways (≈2,000veh/hr) or were promised by AHS (≈8,000veh/hr); the null hypothesis is that AET systems are not capable of achieving capacities of 10,000veh/hr. If platooning is not safe for a given system as according to the collision model, many of the benefits of AET disappear, including capacities more than triple those of traditional roadways [19]. Amelioration protocols can be used to reduce effective brake variance (brake derating) and communication delay (collaborative braking), but both have qualitative concerns and will reduce capacity over what could be achieved if they were not needed.

Additionally, by increasing platoon sizes, AET systems could increase flow without needing to decrease speed as is required by traditional roadways. If no amelioration strategies are used and $S_{ip,min}$ is independent of platoon size, then capacity at a given speed is asymptotically increasing as a function of platoon size. If that is not the case, a single optimal platoon size could exist to maximize capacity. However, as described by AHS research (Figure 2.3), for a given platoon size capacity should first increase then decrease as
a function of speed increasing from zero.

3.2 Collision Model Development

The remainder of this chapter details the methodology for developing, modifying, and analyzing first the collision models and then the capacity model. This section describes the development of two collision models used to describe the kinematics of emergency braking collisions. It first provides equations that define the vehicles’ positions and the collision speed ($\Delta v$), and it then details the construction of the two models. The first collision model is a deterministic model with closed-form solutions, and the second is a more complex, stochastic model developed in Matlab/Simulink.

3.2.1 Collision Model Outline

As with any kinematic car-following model, leader and follower position can be given as a function of time. Collision speed is defined as:

$$\Delta v = \dot{x}_f(t) - \dot{x}_l(t)$$

When: $x_f(t) = x_l(t)$  \hspace{1cm} (3.1)

where $x_f$ is the position of the follower’s front bumper, $x_l$ is the position of the leader’s rear bumper, and $\dot{x}_l$ and $\dot{x}_f$ are the leader and follower’s velocity. The traditional state equation is used to describe a vehicle’s position as a function of initial velocity ($v_0$) and acceleration ($a$):

$$x(t) = v_0 t + \frac{1}{2} at^2$$ \hspace{1cm} (3.2)

For the emergency scenario, the leader begins braking at $t = 0$ and immediately sends a signal to the follower to indicate emergency braking. The signal is delayed by some communication delay $T$ which lumps computation time and other pure time delays, such as intravehicle responses. The follower receives the signal and immediately begins braking.
Both leader and follower continue decelerating until either they either collide or come to a complete stop. Vehicles are considered to brake at their maximum ability.

3.2.2 Preliminary Assumptions

It is necessary to define the relationship between the parameters, most importantly between $a$ and $t$. The proposed deterministic model considers $a$ as a binary value with respect to time: $a = 0$ before emergency braking, and $a = a_{min}$ (emergency braking rate, a negative value) as soon as emergency braking begins. This implies that the brake actuation constant ($\tau$) and the communication delay ($T$) are pure time delays. Total delay between when the leader brakes and the follower brakes ($T_{tot}$) can therefore be given by:

$$T_{tot} = T + \tau$$

(3.3)

This pure delay was used in AHS flow modeling [19,64,65]. This is not a perfectly accurate representation of real-world performance, as the binary nature of acceleration (instantaneous full braking) implies an infinite jerk value. However, it does allow for the construction of closed-form equations to describe vehicle kinematics that are not overly complex. For the stochastic model, a more realistic assumption is made, where $\tau$ is considered to represent a first-order response. Treating traditional (non-electric) vehicle braking as a first-order system with delay is proposed by Hedrick et al. [98]. The effects of treating $\tau$ in these two ways is addressed in Section 3.3.1.

As described in the previous chapter, $\Delta v_{safe}$ is the threshold used to determine the USZ. Since these collisions only occur under emergency scenarios, safety, not rider comfort, is considered. This model initially uses a $\Delta v_{safe}$ of $2.5 m/s$, though larger and smaller values are also considered. This value is similar to those reported in the literature [22,53,99].

Additionally, $v_0$ is considered to be constant for the two vehicles. This is not strictly the case, as the control model will likely result in small deviations of actual velocity from the desired value. However, it is expected that such differences will be negligible, and any initial difference could also be considered in the USZ model. The $S-\Delta v$ curves (an example
is provided in Figure 2.2) would undergo an upward or downward shift of the amount of the initial difference in speed. It is further assumed that the road conditions affect all vehicles approximately the same amount so all modeled variations come from vehicle capability.

### 3.2.3 Deterministic Collision Model

In this section, the development of the deterministic collision model is described. First, vehicle kinematic equations are provided. These are used to define the USZ and then give a quantification of the input variables necessary for safe platooning.

**Vehicle Kinematic Equations**

From Equations 3.2 and 3.3, vehicle positions and speeds during an emergency brake can now be given for two scenarios for the leader and three for the follower:

\[
x_l(t) = \begin{cases} 
S + v_0 t + \frac{1}{2} a_l t^2 & \text{The leader is braking} \\
S - \frac{v_0^2}{2a_l} & \text{The leader has fully stopped} \\
v_0 t & \text{The follower has not started braking} \\
\end{cases}
\]

\[
x_f(t) = \begin{cases} 
v_0 t + \frac{1}{2} a_f (t - T_{tot})^2 & \text{The follower is braking} \\
-\frac{v_0^2}{2a_f} & \text{The follower has fully stopped} \\
\end{cases}
\]

where \(S\) is the initial vehicle separation, \(v_0\) is the initial vehicle velocity, \(a_f\) is the follower’s braking rate, and \(a_l\) is the leader’s braking rate. Factors such as aerodynamic and tire drag are lumped into the braking rate terms.
The vehicles’ velocities can be obtained by taking the derivative of Equation 3.4:

\[
\dot{x}_l(t) = \begin{cases} 
    v_0 + a_l t & \text{The leader is braking} \\
    0 & \text{The leader has fully stopped} \\
    v_0 & \text{The follower has not started braking}
\end{cases}
\]

\[
\dot{x}_f(t) = \begin{cases} 
    v_0 + a_f(t - T_{tot}) & \text{The follower is braking} \\
    v_0 & \text{The follower has fully stopped}
\end{cases}
\]

Equations 3.1, 3.4, and 3.5 can now be combined to yield a set of equations giving \( \Delta v \) as a function of \( S \) for three sequential scenarios: The leader had not started braking (hereby known as Scenario 1), both vehicles are braking (hereby known as Scenario 2), and the lead vehicle has stopped while the follower is still braking (hereby known as Scenario 3). This does not consider the trivial case where the lead vehicle will have come to a stop before the follower begins to brake, as this would occur only when the vehicles are traveling at very low speeds or very great separations.

\[
\Delta v = \begin{cases} 
    \sqrt{-2S a_l} & \text{Scenario 1} \\
    \sqrt{2S(a_f - a_l) + a_f a_l T_{tot}^2} & \text{Scenario 2} \\
    \sqrt{v_0^2(1 - \frac{a_f}{a_l}) + 2a_f(S - v_0 T_{tot})} & \text{Scenario 3}
\end{cases}
\]

**Defining the USZ**

The USZ can be derived directly from the \( S - \Delta v \) curves given in Equation 3.6. This is accomplished by letting the USZ be defined by a single “worst case” scenario for leader and follower braking. For example, if the model is run where \( a_l = -10 \text{m/s}^2 \) and \( a_f = -8 \text{m/s}^2 \), this is the worst case that the system should encounter and be able to handle safely. This is the approach to measuring brake variance taken in Fenton [89] and Goto et al. [95], as opposed to Kanaris et al. [64] and James et al. [92], which assume braking rates to be given by bounded Gaussian distributions, resulting in a stochastic model.
The USZ can now be derived for the three scenarios of relative braking. First, let
the unsafe separation zone be defined as: USZ \propto [S_{iv,max}, S_{ip,min}]$, where $S_{ip,min}$ is the
minimum allowable safe interplatoon separation (lower bound of the USZ), and $S_{iv,max}$ is
the maximum allowable safe intraplatoon separation (upper bound of the USZ). Equation
3.6 is now rearranged and $\Delta v$ is replaced with $\Delta v_{safe}$, so that $S_1$, $S_2$, and $S_3$ represent
the acceptable separations as a function of $\Delta v_{safe}$, $a_l$, $a_f$, $T_{tot}$, and $v_0$ for each of the three
scenarios respectively.

\[ S_1 = \frac{\Delta v_{safe}^2}{-2a_l} \quad \text{Scenario 1} \quad (3.7a) \]
\[ S_2 = \frac{\Delta v_{safe}^2 - a_f a_l T_{tot}^2}{2(a_f - a_l)} \quad \text{Scenario 2} \quad (3.7b) \]
\[ S_3 = v_0 T_{tot} + \frac{\Delta v_{safe}^2 - v_0^2(1 - a_f/a_l)}{2a_f} \quad \text{Scenario 3} \quad (3.7c) \]

To determine which of $S_1$, $S_2$, and $S_3$ define each bound of the USZ, it is necessary
to determine which scenario is valid for a given situation. For example, one must identify
whether $\Delta v_{safe}$ is first reached before or after the follower has started braking, thus defining
$S_{iv,max}$ as being derived from Scenario 1 or 2, respectively.

To accomplish this, it is necessary to identify the intersections between the different
scenarios. Between Scenario 1 and 2 ($S_1 = S_2$), this corresponds to the point when the
follower first begins to brake, and between Scenario 2 and 3 ($S_2 = S_3$) to when the lead
vehicle has come to a complete stop. These intersection points can derived from Equations
3.6 and 3.7 and be given in terms of $S$ and $\Delta v_{safe}$, where Equation 3.8a represents the
intersection between Scenario 1 and 2, and Equation 3.8b the intersection between Scenario
2 and 3.

\[ \Delta v_{safe} = -a_l T_{tot} \quad (3.8a) \]
\[ S = \frac{a_f}{a_l} v_0 T_{tot} + \frac{1}{2} a_f T_{tot}^2 - \frac{1}{2} v_0^2 \frac{a_l - a_f}{a_l^2} \quad (3.8b) \]

$S_2$ is monotonically increasing where $a_l < a_f$, monotonically decreasing where $a_l > a_f$, and
flat where \( a_f = a_l \). Therefore, where \( a_l > a_f \), if \( \Delta v_{safe} \) is not reached in Scenario 1 (the follower has not reacted yet), then no USZ exists. By comparison, where \( a_l < a_f \), since \( S_2 \) is monotonically increasing, then \( \Delta v_{safe} \) may be reached in Scenario 2. Similarly, where \( a_l > a_f \), the upper bound of the USZ may occur in either Scenario 2 or 3 depending if the lead vehicle has come to a complete stop or not, but will always occur in Scenario 3 where \( a_l \leq a_f \). Lastly, if the obtained \( S_{iv,max} \geq S_{ip,min} \) then no USZ exists: for all separations \( \Delta v \) never exceeds \( \Delta v_{safe} \).

The USZ can now be analytically defined for the separate cases of relative braking by combining Equations 3.7 and 3.8:

A USZ does not exist if: \( S_{iv,max} \geq S_{ip,min} \)

Where: \( a_f \geq a_l \) (leader brakes stronger)

\[
S_{iv,max} = \begin{cases} 
S_1 & \text{if: } \Delta v_{safe} \geq -a_l T_{tot} \\
S_2 & \text{if: } \Delta v_{safe} \leq -a_l T_{tot}
\end{cases}
\]

\[
S_{ip,min} = S_3
\]

where: \( a_f \leq a_l \) (follower brakes stronger)

\[
S_{iv,max} = S_1
\]

\[
S_{ip,min} = \begin{cases} 
S_2 & \text{if: } S_2 \leq \frac{a_f}{a_l} v_0 T_{tot} + \frac{1}{2} a_f T_{tot}^2 - \frac{1}{2} v_0 (\frac{a_l-a_f}{a_l}) \\
S_3 & \text{if: } S_2 \geq \frac{a_f}{a_l} v_0 T_{tot} + \frac{1}{2} a_f T_{tot}^2 - \frac{1}{2} v_0 (\frac{a_l-a_f}{a_l})
\end{cases}
\]

where: \( a_f = a_l \) (both brake equally)

\[
S_{iv,max} = S_1
\]

\[
S_{ip,min} = S_3
\]

(3.9)
Defining Necessary Parameters for Safe Platooning

A primary concern for platooning is ensuring that the lower bound of the USZ is large enough that it can be maintained by the vehicle controllers at an acceptable level of reliability and comfort. This means that no undesirable extreme accelerations or jerks are needed to maintain the close separations. The minimum achievable intraplatooon separation at an acceptable level of comfort allowed by the vehicle controllers is notated as $S_{iv,ach}$. Since platooning is thereby considered safe where $S_{iv,ach} \geq S_{iv,max}$, and unsafe otherwise (since the maximum safe separation is too small to be maintained by the vehicle controllers), the necessary condition for safe platooning is given by:

$$S_{iv,ach} = S_{iv,max}$$  \hfill (3.10)

One of the benefits of a deterministic collision model with closed-form solutions is that it allows for a quantification of the parameters necessary for safe platooning. Equation 3.9 can be combined with Equation 3.10 to yield a description of the maximum delay and brake variance, and the relationship between them, necessary to ensure safe intraplatooon interactions:

If: $S_{iv,ach} \leq -\frac{\Delta v_{safe}^2}{2a_l}$

Then: $S_{iv}$ is always deemed safe

Otherwise:

$$a_{f,max} = \frac{\Delta v_{safe}^2 - 2S_{iv,ach}a_l}{2S_{iv,ach} + a_lT_{tot}^2}$$

If: $-a_lT_{tot} \geq \Delta v_{safe}$

$$T_{tot,max} = \frac{\Delta v_{safe}}{-a_l}$$  \hfill (3.11)

where $a_{f,max}$ is the maximum allowable $a_f$ and $T_{tot,max}$ is the maximum allowable $T_{tot}$. Equation 3.11 is valid for where the USZ exists as according to Equation 3.9. No USZ also
exists where $S_{iv,ach}$ is small enough so that an unsafe collision speed is not reached even if the follower does not react and maintains speed: the first term in Equation 3.11. Where $a_l = -10 \text{m/s}^2$ and $\Delta v_{safe} = 2.5 \text{m/s}^2$, this occurs where $S_{iv,ach} \leq 0.31 \text{m}$.

### 3.2.4 Matlab/Simulink Stochastic Model

A similar approach to the development of the deterministic model is used to obtain $S-\Delta v$ curves via coding in Matlab; this model was developed in conjunction with Spencer Jackson. The only differences between the above equations and the Matlab/Simulink model is that the latter: 1) Considers $\tau$ as a first-order brake actuation constant rather than as a pure time delay (the units are still in time), and 2) Uses distributions of maximum braking rates, rather than set values for the worst case scenario, to determine the USZ. This section first describes the construction of the Matlab/Simulink model and then its use.

#### Model Construction

By considering the braking of each vehicle as a deceleration curve of the step response of a first order system with delay, this model approximates the performance of traditional hydraulic actuated calipers on disc or drum brakes used on today’s vehicles. The building of caliper pressure on the disc increases the braking force (and thus deceleration) in approximately a first order response [98]. In the model the time constant $\tau$ is separately assignable for both leader and follower (indicated by subscript $l$ and $f$, respectively). Factors such as aerodynamic and tire drag are lumped into the minimum acceleration term, $a_{min}$. A closed-form solution is solvable, but in order to account for full ranges of input values the piecewise equations become overly complicated. For example, if the leader actuation delay is larger than the communication delay the piecewise equations change order. Thus a Simulink model is used.

A block diagram of this model is given in Figure 3.1, which shows the main inputs and outputs considered. The platoon emergency braking scenario is simplified to only a leader and single follower. These could represent any vehicle in the system and its immediate follower, including the last vehicle in a platoon and the first vehicle of the following platoon.
Communication delays are only considered when there is delay in communication between the leader to the follower. If there are two platoons of 10 vehicles involved in the emergency brake scenario, 19 such interactions occur. It is apparent that in an automated highway with potentially thousands of vehicles, very low probabilities of dangerous impacts must be achieved.

The resulting $S-\Delta v$ curves are run through a binary function that specifies if a given separation is within the USZ. Parameters can be varied to visualize the effect upon the USZ. Plots showing the change in USZ with respect to parameter variation can identify sensitivity.

**Model Use**

Values of $a_{\text{min}}$ for both vehicles are selected from stochastic distributions. The safest system will have homogeneous vehicle performance, as variation causes the braking disparities, which expand the USZ. One potential way to accomplish this is through use of a single type of electric vehicle (EV), as proposed by concepts such as AET [1]; EVs promise more homogeneity than internal combustion engines. However, even with nominally homogeneous vehicles, there are variations in manufacturing, wear, and load. Therefore, $a_{\text{min}}$ is assumed to follow a bounded normal distribution. An upper bound is used under the assumption the braking capabilities of a vehicle could be evaluated at check in to the au-

![Block Diagram of the Matlab/Simulink Model](67)
tomated lane [100,101], and the vehicle would be rejected from the automated highway if insufficient. The lower bound represents the limits of the vehicle technology employed. Vehicles on the road are therefore assumed to lie someplace between these two bounds. Laws and regulations involving performance limits and inspection frequency could effect the standard deviation of the distribution as well as the bounds.

The model is run 1,000 times in order to consider rare instances within the distribution; each run used a random outcome from the distribution for minimum acceleration. The success function outputs are summed such that \( S \) is divided into 10cm bins, each containing the respective counts of an unsafe collision occurring. In order to account for model variation, this process is repeated 30 times such that a mean and variance could be calculated for each bin. The resulting curve is normalized to illustrate a probability for unsafe following distance correlated to vehicle parameter variations. In order to define the USZ, it is necessary to identify the maximum allowable probability for an unsafe collision to occur.

### 3.3 Comparing the Deterministic and Stochastic Collision Models

Chapter 4 provides the results of the deterministic and stochastic collision models described above. These results are compared and their similarities and differences are discussed. The Student’s t-test is then used to evaluate the two hypotheses described in Section 3.1.1: first that the two models are statistically identical, and second that the deterministic model is more conservative than and reasonably equivalent to the stochastic model. The goal of this section is to provide a more general overview of the relative strengths and weaknesses of the deterministic and stochastic collision models. It considers the effects of both treating \( \tau \) as a pure delay vs. a first order brake actuation constant and of treating brake variances as deterministic vs. stochastic values. It also shows how the deterministic model is the better choice to use as the foundation for capacity modeling.

#### 3.3.1 Effect of Treating \( \tau \) as a Pure Delay

To justify using \( \tau \) as a pure time delay in the deterministic model, the Matlab/Simulink model is used with deterministic inputs (no Monte Carlo simulations) to evaluate the effect
of varying $\tau$ when it is treated as a first-order brake actuation constant. First, let $v_0 = 30\text{m/s}$, $a_f = -8\text{m/s}^2$, and $a_l = -10\text{m/s}^2$, and as according to Equation 3.3, $T_{tot}$ is kept constant at 200ms. The input values are given in Table 3.1, and the results in Figure 3.2. As $\tau$ approaches zero, the system approaches a representation of $\tau$ as a pure delay.

As seen in Figure 3.2(b), at certain points considering $\tau$ as a first-order brake actuation constant can slightly increase $\Delta v$, though this has a maximum difference of only $\approx 0.2\text{m/s}$, and that is for the scenario when most of the delay is treated as $\tau$. Therefore, treating $\tau$ as a pure delay, as done in the deterministic model, yields approximate results to treating it as a first-order reaction and is a reasonable assumption.

### 3.3.2 Deterministic vs. Stochastic Brake Variance

This research claims that a deterministic model with a single brake variance (a worst case scenario for $a_f$ and $a_l$) can reasonably approximate the results obtained from a stochastic model using brake distributions. The two models effectively address the same question, “What likelihood of failure is acceptable?” The deterministic model states that any conditions worse than the “worst case” scenario are rare enough to represent an acceptable risk. By comparison, the stochastic model explicitly gives the probability of unsafe collisions, and uses an exogenous acceptable collision probability, for example a 1% likelihood of an unsafe collision during an emergency brake, to define the USZ. This provides greater specificity than the USZ from the deterministic model, which is given as a single range by Equation 3.9. However, the stochastic model requires a simulation to be run for each set of input

<table>
<thead>
<tr>
<th>$T$ (ms)</th>
<th>$\tau$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>190</td>
<td>10</td>
</tr>
<tr>
<td>170</td>
<td>30</td>
</tr>
<tr>
<td>135</td>
<td>65</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>65</td>
<td>135</td>
</tr>
</tbody>
</table>
Fig. 3.2: $S-\Delta v$ Curve Plots Showing the Effects of Varying $\tau$
parameters.

A deterministic model and a stochastic Matlab/Simulink model can interact in a similar way to traditional car-following equations and microscopic modeling software. Both methods have strengths, and while the latter are more flexible and specific, the former can provide the continuous relationships necessary to describe general trends. For example, this granularity allows for the deterministic model to define the USZ as a function of delay for all delay values. Additionally, the closed-form solutions can be expanded to yield flow relationships. Just as the Gazis-Herman-Rothery equations can be modified to give the Greenshields, Greenberg, and Underwood flow models, the collision model developed here can provide speed-density-flow relationships for platooning vehicles.

3.4 Determining the USZ with the Stochastic Model

Traditional hydraulic brake systems have inherent performance limits, and research is being conducted into drive-by-wire technology, electromechanical brakes, and regenerative braking, which all have the potential to extend these limits [102]. The nominal values, shown in Table 3.2, reflect the assumption that future technology will enable high vehicle braking and communication performance. The leader maintains high performance values while the follower performance is degraded. The Monte Carlo simulations are then performed using combinations of “short” or “long” delays $T$ and either a “strict” distribution of $a_{\text{min}}$ corresponding to high requirements on vehicle braking performance, or a “loose” distribution, corresponding to lower braking requirements. These 4 primary cases illustrate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0$ (m/s)</td>
<td>30</td>
</tr>
<tr>
<td>$a_{\text{min}}$ (m/s$^2$)</td>
<td>$-10$</td>
</tr>
<tr>
<td>$\tau$ (ms)</td>
<td>10</td>
</tr>
<tr>
<td>$T$ (ms)</td>
<td>20</td>
</tr>
</tbody>
</table>
the effects of these two main parameters. These case values are shown in Table 3.3.

The loose braking distribution corresponds to the braking distribution values used in AHS modeling, and the long delay to the worst delays in AHS modeling. In the following chapter, four figures are presented where one parameter is held constant while the other is varied. For example, for the loose distribution figure, all the $a_{\text{min}}$ are given by the loose distribution while delay is varied in 20ms intervals from short to long times. Thus two of the four primary cases are reflected in each figure. The largest and smallest curves are labeled with the respective value of the varied parameter.

3.5 Determining the USZ with the Simple Collision Model

This section uses the deterministic collision model (Equation 3.9) to define the USZ as a function of initial speed, brake variance, and communication delay. Unless otherwise stated, for all cases the USZ is given as a function of total delay ($T_{\text{tot}}$) where $a_l = -10^{m/s^2}$ and $\Delta v_{safety} = 2.5^{m/s}$ for different values of $a_f$ and $v_0$. The USZ is first given for where $a_f$ is near $a_l$ for $v_0 = 30^{m/s}$. It is then given for the different brake variances provided in Table 3.4 for $v_0 = 10$, 30, and $60^{m/s}$.

3.6 Enabling Safe Platooning

This section first describes the effects of the three amelioration protocols on the USZ and then defines the system parameters required for safe platooning.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Strict-Short</th>
<th>Strict-Long</th>
<th>Loose-Short</th>
<th>Loose-Long</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>ms</td>
<td>20</td>
<td>200</td>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>$\mu$</td>
<td>m/s$^2$</td>
<td>-9.5</td>
<td>-9.5</td>
<td>-7.75</td>
<td>-7.75</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>m/s$^2$</td>
<td>0.25</td>
<td>0.25</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Upper Bound</td>
<td>m/s$^2$</td>
<td>-10</td>
<td>-10</td>
<td>-10</td>
<td>-10</td>
</tr>
<tr>
<td>Lower Bound</td>
<td>m/s$^2$</td>
<td>-9</td>
<td>-9</td>
<td>-5.5</td>
<td>-5.5</td>
</tr>
</tbody>
</table>
Table 3.4: Different Brake Variances

<table>
<thead>
<tr>
<th>$a_l$ (m/s$^2$)</th>
<th>$a_f$ (m/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>-6</td>
</tr>
<tr>
<td>-10</td>
<td>-7</td>
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<tr>
<td>-10</td>
<td>-8</td>
</tr>
<tr>
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<td>-9</td>
</tr>
<tr>
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<td>-10</td>
</tr>
<tr>
<td>-10</td>
<td>-11</td>
</tr>
<tr>
<td>-10</td>
<td>-12</td>
</tr>
</tbody>
</table>

3.6.1 Amelioration Protocols

In cases where $S_{iv,ach} > S_{iv,max}$ as given by Equation 3.11, platooning is considered unsafe in an emergency braking scenario, and different amelioration protocols must be used to enable safe platooning. Increasing $\Delta v_{safe}$ implies either a greater risk acceptance or that the vehicles are designed to better withstand rear-end collisions, such as by using larger crumple zones. Brake derating effectively reduces brake variance, where the leader is instructed to brake weaker than it normally would in order to prevent collisions with the follower. Collaborative braking effectively reduces total delay by having the lead vehicle delay its braking.

To evaluate the effects of increasing $\Delta v_{safe}$, the USZ is calculated as a function of $\Delta v_{safe}$ using the deterministic model from 0 to 5 m/s for four different scenarios of $a_f$ and $T_{tot}$: $-6$ and $-8$ m/s$^2$ and 0.15 and 0.3s respectively. The stochastic model is also used to identify the importance of $\Delta v_{safe}$ and confirm the results of the deterministic model. Collision probabilities are calculated for the short delay case where $\Delta v_{safe} = 0$ m/s (no collisions are acceptable) and where $\Delta v_{safe} = 5$ m/s.

Increasing $\Delta v_{safe}$ is an exogenous amelioration protocol, while brake derating and collaborative braking are endogenous; the latter are applied by the system itself in order to increase $S_{iv,max}$ to $S_{iv,ach}$. As discussed briefly by Kanaris et al. [64], one potential downside to these endogenous protocols is that they could reduce capacity by increasing the necessary safe interplatoon separations. Chapter 4 addresses some of the qualitative concerns with
these strategies, including reasonableness and viability, but below the techniques to quantify their effects on $S_{ip,\text{min}}$ are presented.

Brake derating reduces the expected braking rate of the lead vehicle for intraplatoon interactions, but this lead vehicle is effectively the follower for the platoon-platoon interaction with the leading platoon. Therefore, the effect is reversed for interplatoon interactions. The leader brakes stronger than the follower, and the effective follower acceleration $a_{f,\text{actual}}$ used in calculating $S_{ip,\text{min}}$ can thus be defined as:

$$a_{f,\text{actual}} = (1 + \beta_d)a_f$$

$$\beta_d = \frac{(a_{\text{red}} - a_f)}{a_f}$$

(3.12)

where $a_{\text{red}}$ is the acceleration that must be reached in order to achieve safe intraplatoon interactions and $\beta_d$ is the unitless derating factor. When $\beta_d = 0$, no derating occurs. As with the AHS work, this research only considers derating for the two vehicle scenario, so it assumes some degree of coordination between all the following vehicles.

Intraplatoon collaboration could result in longer effective interplatoon delays for two reasons. First, the collaboration itself requires the lead vehicle to delay its braking, and the effective intraplatoon delay ($T_{\text{tot,\text{actual}}}$, the delay used in calculating $S_{ip,\text{min}}$) must be increased by this amount. Second, perfect collaboration is unrealistic, and the difference between expected and actual intraplatoon communication delays must then be integrated into the intraplatoon delays. $T_{\text{tot,\text{actual}}}$ can thus be given as:

$$T_{\text{tot,\text{actual}}} = 2T_{\text{tot}} - T_{\text{red}} + \alpha_c Z (T_{\text{tot}} - T_{\text{red}})$$

(3.13)

where $T_{\text{red}}$ is the delay achieved through collaboration and $\alpha_c$ is a constant representing the effects of imprecise coordination. Where no collaboration is used, $T_{\text{tot}} = T_{\text{red}}$, so $T_{\text{tot,\text{actual}}} = T_{\text{tot}}$. The $2T_{\text{tot}} - T_{\text{red}}$ term corresponds to the additional delay caused by coordination itself, and the $\alpha_c Z (T_{\text{tot}} - T_{\text{red}})$ term corresponds to the additional delay caused by imprecise coordination. This research makes the assumption that achieving effective
coordination results in an 10% penalty for interplatoon delay: \( \alpha_c = 0.1 \). For example, if one wants to decrease the intraplatoon delay by 100ms (\( T_{tot} - T_{red} = 100\text{ms} \)), they must increase the interplatoon delay by 10ms for each vehicle in the platoon.

3.6.2 Necessary Total Delay and Brake Variance for Safe Platooning

Equation 3.11 is used to define the required parameters for safe platooning under different \( S_{iv,ach} \) using the values in Table 3.5 and three different \( \Delta v_{safe} \) values of 2.5, 3.5, and 4.5m/s. The results show the total delay and variance necessary to ensure safe intraplatoon interactions, describing the relationship between these two parameters and which is more important in different scenarios.

3.7 Capacity Modeling

This section describes the calculation of the flow of automated platooning systems using the deterministic collision model and identifies the maximum capacity as a function of speed and platoon size for scenarios with different input parameters: brake variance, communication delay, and needed amelioration techniques.

3.7.1 Assumptions for Capacity Analyses

For all capacity analyses, \( \Delta v_{safe} = 2.5\text{m/s} \), \( L = 5\text{m} \) and \( a_l = -10\text{m/s}^2 \). Since “worst case” scenarios are considered, follower braking is never more severe than the leader’s (\( a_l \geq a_f \)). Furthermore, as shown by NAHSC [19] and Kanaris et al. [64], intraplatoon delays can be assumed to be smaller than interplatoon delays. This work makes the assumption that

| Table 3.5: Defining Conditions for Safe Platooning |
|-----------------|-----------------|
| Condition       | Values          |
| \( a_l \)       | \(-10\text{m/s}\)² |
| \( a_f \)       | \([-0.16]\text{m/s}\)² |
| \( T_{tot} \)   | \([0, 0.6]\text{s}\) |
| \( S_{iv,ach} \) | 0.5, 1, 1.5, 2, 2.5, 3m |
interplatoon delays will be three times greater, as done by Carbaugh et al. [65], though more research into AET communications is needed to obtain a more reliable value. $S_{iv,ach}$ is initially assumed to be 1m; this is better than the 2m value used in AHS [19,64,65], and is chosen to represent improvements in automated vehicle control during the ensuing years. Lastly, it is assumed that the minimum safe interplatoon separations must be maintained to avoid any collision: $S_{ip,min}(v_{safe} = 0)$.

3.7.2 Resulting Capacity Equation

Equation 2.15 is reproduced here.

$$ q = 3600v * \frac{Z}{S_{ip} + (Z - 1) * S_{iv} + Z * L} $$

It can now be combined with the above assumptions and Equations 3.9, 3.12, and 3.13 to yield a capacity equation in terms of follower braking rate ($a_f$), leader braking rate ($a_l$), initial velocity before braking ($v_0$), delay ($T_{tot}$), level of collaborative braking ($T_{red}$), and level of brake derating ($\beta_d$):

$$ q = \frac{3600v_0Z}{3v_0[2T_{tot} - T_{red} + 0.1Z(T_{tot} - T_{red})] - \frac{v_0^2}{2a_f(1-\beta_d)}(1 - a_l/a_f) + (Z - 1) + 5Z} $$ (3.14)

3.7.3 Capacity as a Function of Speed and Platoon Size

Four different cases for brake variance and total delay are chosen to represent different types of systems. Case 1 is just inside the “safe” boundary, so that no amelioration is needed. For Case 2, either collaborative braking or brake derating could be used to make platooning safe. For Case 3, only brake derating could make platooning safe, as collaborative braking would be insufficient. Case 4 is the reverse, where only collaborative braking could be used.

Capacities are given as a function of speed for $Z = \{1, 3, 5...21\}$ first assuming that no amelioration is used. For Case 2-4 when platooning is initially unsafe ($S_{iv,max} < S_{iv,ach}$), Equation 3.11 is used to determine the necessary level of amelioration. Brake derating and collaborative braking are considered separately in order to compare the effects of each.
Where collaboration is used, $T_{red}$ is calculated from Equation 3.11 and $\beta_d$ is set at zero, and where brake derating is used, $\beta_d$ is calculated from Equation 3.11 and $T_{red}$ is set equal to $T_{tot}$.

The implications and quantitative effects on capacity of brake derating and collaborative braking are then addressed and two stricter forms of them are introduced: cumulative brake derating and exponential collaborative braking. These results show what capacities are achievable in an AET system, the speed-flow relationships of an automated platooning system, and the effects of varying platoon size. The conditions under which optimal platoon size exists vs. when flow asymptotically approaches a maximum as platoon size increases to infinity are analytically shown and examples of optimal platoon sizes are given.

3.8 Anticipated Contributions

Chapter 4 uses the methods presented here to obtain results describing: 1) The parameters that allow for safe platooning, including the factors that affect the size of the USZ, and 2) The capacity of AET systems under a free-flow environment limited only by safety concerns. These results are compared with both AHS values and traditional roadways. This shows how the general speed-density-flow relationships of traditional traffic flow theory will be maintained, but at different output values.

This work performed in this thesis indicates the viability of very high roadway capacities for AET, in excess of 10,000\text{veh/hr}, achievable due to the benefits of reduced communication delay and brake variance promised by AET. It also evaluates how in certain situations, platooning allows for higher and higher capacities without having to reduce speed; maximum flow occurs at a higher speed than the design speed of the roadway. Additionally, this work considers both stochastic and deterministic collision models, showing how the former is a useful starting point for describing vehicle behavior during emergency braking at the nanoscopic level, and the latter is useful in describing general flow patterns and quantifying the requirements for safe platooning. Where platooning is not initially safe, it considers different amelioration protocols and calculates their ability to make platooning safe at the expense of reducing capacity.
CHAPTER 4
RESULTS AND ANALYSIS

This chapter first provides Unsafe Separation Zone (USZ) plots that are the outputs of the deterministic and stochastic collision models, and uses the results to perform a statistical analysis comparing the two models. It then examines the necessary parameters for safe platooning. Lastly, it provides the results of the capacity model. These show the effects of different brake variances, delays, initial speeds, platoon sizes, and amelioration protocols on automated vehicle flow. The possibility of an optimal platoon size to maximize capacity and variations on the amelioration protocols to better represent real-world performance are also evaluated.

4.1 Unsafe Separation Zone

This section provides the USZ curves derived from both the deterministic and stochastic collision models.

4.1.1 Deterministic Model

Equation 3.9 is used to define the USZ for a variety of different scenarios, and is reproduced below for ease of reference, as is Equation 3.7, which gives the equations for unsafe separation for each of the three scenarios: the leader is braking and the follower has not started braking ($S_1$), both vehicles are braking ($S_2$) the leader has stopped and the follower is still braking ($S_3$).
A USZ does not exist if: $S_{iv,max} \geq S_{ip,min}$

Where: $a_f \geq a_l$ (leader brakes stronger)

$$S_{iv,max} = \begin{cases} S_1 & \text{if: } \Delta v_{safe} \geq -a_l T_{tot} \\ S_2 & \text{if: } \Delta v_{safe} \leq -a_l T_{tot} \end{cases}$$

$$S_{ip,min} = S_3$$

where: $a_f \leq a_l$ (follower brakes stronger)

$$S_{iv,max} = S_1$$

$$S_{ip,min} = \begin{cases} S_2 & \text{if: } S_2 \leq \frac{a_f}{a_l} v_0 T_{tot} + \frac{1}{2} a_f T_{tot}^2 - \frac{1}{2} v_0 (\frac{a_l-a_f}{a_l}) \\ S_3 & \text{if: } S_2 \geq \frac{a_f}{a_l} v_0 T_{tot} + \frac{1}{2} a_f T_{tot}^2 - \frac{1}{2} v_0 (\frac{a_l-a_f}{a_l}) \end{cases}$$

where: $a_f = a_l$ (both brake equally)

$$S_{iv,max} = S_1$$

$$S_{ip,min} = S_3$$

$$S_1 = \frac{\Delta v_{safe}^2}{-2a_l}$$

Scenario 1

$$S_2 = \frac{\Delta v_{safe}^2 - a_fa_l T_{tot}^2}{2(a_f-a_l)}$$

Scenario 2

$$S_3 = v_0 T_{tot} + \frac{\Delta v_{safe}^2 - v_0^2 (1 - a_l/a_l)}{2a_f}$$

Scenario 3

For all Figures 4.1-4.4 the USZ is given as a function of total delay ($T_{tot}$) where $a_l = -10 m/s^2$ and $\Delta v_{safe} = 2.5 m/s$ for different values of $a_f$ and $v_0$. Figure 4.1 provides the USZ when $a_f$ is near $a_l$ for $v_0 = 30 m/s$. Figures 4.2, 4.3, and 4.4 provide the USZ for $a_f$ values
from $-6$ to $-12\,m/s^2$ for $v_0 = 10, 30,$ and $60\,m/s$ respectively. While the USZ for $v_0 = 10\,m/s$ is presented as a single chart, the difference between $S_{iv,max}$ and $S_{ip,min}$ becomes too great for $v_0 = 30$ and $60\,m/s$, so two different subcharts are presented in Figures 4.3 and 4.4. In all figures, the top line, which has a positive slope, represents the upper bound of the USZ ($S_{ip,min}$) and the bottom line, which has a negative slope, represents the lower bound ($S_{iv,max}$).

For all Figures 4.1-4.4, one can observe that there are points where USZ curves become non-differentiable. These indicate the conditions where the USZ switches between being defined by different scenarios as given in Equation 3.9: $S_1$, $S_2$, and $S_3$. The existence of these non-differentiable points is thus governed by Equation 3.9, and the intersections between scenarios that they represent vary based on the relationship between leader and follower braking rates.

As an illustration, take two instances, both from Figure 4.1, one where the follower brakes weaker than the leader, and the second where the follower brakes stronger. For the first, two non-differentiable points occur for $a_f = -9.25\,m/s^2$, one at approximately a total delay of 0.05s and a separation of 4m and another at a total delay of 0.25s and a separation of 0.31m. The first represents the intersection between Scenarios 1 and 3. This indicates the conditions where $S_{ip,min} = S_{iv,max}$, so that for smaller delay values, no USZ exists at any separation. The second non-differentiable point indicates the point where $S_{iv,max}$ switches between being governed by Scenarios 1 and 2. For the second instance, where $a_f = -10.25\,m/s^2$, two non-differentiable points also exist. The non-differentiable point at approximately a total delay of 0.3s and a separation of 8m represents the intersection between Scenarios 2 and 3, given by the $S_{ip,min}$ term in Equation 3.9 for situations where the follower brakes stronger than the leader. In such cases where $a_f \leq a_l$, no USZ exists until $T_{tot} = 0.25s$. The second non-differentiable point also occurs at a total delay of 0.25s and a separation of 0.31m, and as in the above instance represents the intersection between Scenarios 1 and 2. However, in such cases where $a_f \leq a_l$, this is the intersection of the lines for $S_{iv,max}$ and $S_{ip,min}$. In all such cases, no USZ exists until $T_{tot} = 0.25s$, and for greater
Fig. 4.1: USZ for $a_f$ from $-9.25$ to $-10.75 \text{m/s}^2$ ($v_0 = 30 \text{m/s}$)

Fig. 4.2: USZ Where $v_0 = 10 \text{m/s}$
Fig. 4.3: USZ Where \( v_0 = 30 \text{m/s} \)
(a) View of Safe Intraplatoon Separations

(b) View of Safe Interplatoon Separations

Fig. 4.4: USZ Where $v_0 = 60\text{m/s}$
delays $S_{iv,max}$ remains constant at 0.31m, but $S_{ip,min}$ increases as a function of delay.

The above two instances both have two non-differentiable points, but one can observe that all cases have either one or two such points. For example, in Figure 4.2 for $a_f = -6 m/s^2$, the only non-differentiable point is at 0.25s and a separation of 0.31m. Cases that have only one non-differentiable point occur when the is always a USZ for any delay value. A theoretical second non-differentiable point does exist in such cases, but at a negative delay value. Where the USZ does not exist, it can be understood as representing conditions where small delays and tight brake variances ensure that regardless of initial separation, the vehicles will never collide above the exogenous unsafe collision speed ($\Delta v_{safe}$).

Overall, both tighter brake variances and smaller delays serve to decrease the size of the USZ, indicating that improvements in these characteristics should allow for safer, higher capacity platooning than was initially possible. Additionally, as seen by comparing Figures 4.2, 4.3, and 4.4, $S_{ip,min}$ and the relative differences caused by different variances and delays increases as a function of speed, indicating that systems with tight variances and small delays would be especially important for high speed operation. However, since as given by Equation 3.7, $S_1$ and $S_2$ are not dependent on initial speed, $S_{iv,max}$ remains relatively constant as a function of speed. The only situation where $v_0$ affects $S_{iv,max}$ is where it becomes small enough for no USZ to exist: for example, at $T_{tot} = .05s$ where $a_f = -8 m/s$ in Figure 4.2.

The relatively small values for $S_{iv,max}$ indicate that this is a primary limiting factor for enabling safe platooning. If vehicles cannot maintain these tight separations, then platooning can becomes unsafe during emergency braking scenarios. The remainder of this chapter deals extensively with this issue, including identifying ways to increase this value. The above results indicate that for most cases tight brake variances appear to be more important than small communication delays in providing moderately large $S_{iv,max}$ values. For example, where $a_f = -6 m/s$, even when there is no delay, $S_{iv,max}$ is still only 0.78m.

4.1.2 Stochastic Model

Monte Carlo simulations are then performed with the Matlab/Simulink model using
combinations of short or long delays and strict or loose distributions as described in Table 3.3. Figures 4.5-4.8 provide the results for each of these four primary cases. As first described in Chapter 3, to better show the effects of varying different parameters on collision probability, each figure shows a range of values, including two of the four primary cases. For example, Figure 4.6 shows the USZ using a loose braking distribution for a range of different delay values, from 20ms (the Loose-Short case) to 200ms (the Loose-Long case).

As with the results of the deterministic model, for some input parameter values the USZ does not exist. This is most apparent in Figure 4.5, where no USZ is observed until the total delay is 120ms or greater. Also as with the deterministic model, these results indicate the importance of brake variance compared with total delay in ensuring safe platooning under the given assumption. For example, even with long delays, a strict braking distribution allows for safe platooning when intraplatoon separations are less than 2m, assuming a maximum acceptable unsafe collision probability is $10^{-3}$. It can be observed in Figure 4.7 that even with short delays the braking distribution can degrade to only a mean of $-9.35^{m/s^2}$ before USZ occurs, whereas with a strict distribution the delay can be 5 times the short delay value before USZ appears. Even in the Strict-Long case the USZ is less than 6m wide at the $10^{-4}$ level of acceptable unsafe collision likelihood.

As discussed in Chapter 3, this unsafe collision likelihood is a characteristic that does not appear in the deterministic model results and is a benefit of using the stochastic approach. The USZ for each case (a given delay and brake variance) is given as a function of unsafe collision likelihood; the larger the acceptable likelihood, the smaller the USZ range (larger $S_{iv,max}$ and smaller $S_{ip,min}$. This a reasonable result, as one would expect that accepting a higher percentage of emergency braking maneuvers will result in unsafe collisions would allow for more generous system parameters. Such an acceptance represents an explicit tradeoff between safety and system performance.

4.2 Comparing the Stochastic and Deterministic Results

The USZ curves derived from the stochastic and deterministic models, as presented in the previous 2 subsections, show similar patterns. For example, both indicate that a
Fig. 4.5: Unsafe Collision Probability Curves for Strict Distribution [67]

Fig. 4.6: Unsafe Collision Probability Curves for Loose Distribution [67]
Fig. 4.7: Unsafe Collision Probability Curves for Short Delay [67]

Fig. 4.8: Unsafe Collision Probability Curves for Long Delay [67].
strict braking distribution is more important than a short delay in ensuring a smaller USZ, especially with respect to maintaining a large $S_{iv,max}$. However, it is necessary to prove that they are at least approximately equivalent in order to effectively evaluate the hypotheses posed in this paper. Since the capacity model uses the deterministic model as inputs, the capacity outputs can only be considered valid if the simpler deterministic model can be shown to be a reasonable approximation of the more complex and accurate stochastic model. To accomplish this, a Student’s t-test is used to compare the two models for three of the four primary cases given in Table 3.3. Strict-Short is not considered; for this case no USZ exists, so no statistical analysis is possible.

To compare the two models, two different hypotheses are test. The first ($H_1$) is that the deterministic model is equivalent to the stochastic model: $S_{stochastic} = S_{deterministic}$. The null hypothesis here is that the two are not equivalent: $S_{stochastic} \neq S_{deterministic}$. The second hypothesis ($H_2$) is more complex: the deterministic model yields results that are more conservative than and reasonably close (within 10%) to those of stochastic model. Mathematically this can be given as: $0.9 S_{iv,max,stochastic} \leq S_{iv,max,deterministic} \leq S_{iv,max,stochastic}$ for intraplatoon separations and $S_{ip,min,stochastic} \leq S_{ip,min,deterministic} \leq 1.1 S_{ip,min,stochastic}$ for interplatoon separations. The null hypothesis here is that the safe separations obtained from the deterministic model ($S_{iv,max,deterministic}$ and $S_{ip,min,deterministic}$) fall outside of these ranges. Conceptually, if $H_2$ can be accepted, this implies that the deterministic model gives a larger USZ (smaller $S_{iv,max}$ and larger $S_{ip,min}$) than the stochastic model, so that by using the deterministic model outputs as inputs for the capacity model, one will obtain smaller (more conservative) results for AET capacity. The 10% level is chosen to show that while the deterministic model may be more conservative, it is still yields similar results to the stochastic model.

$H_1$ is evaluated using a two-tailed t-test. $H_2$ is evaluated using two one-tailed t-tests. The first of these two tests is for $S_{deterministic} = S_{stochastic}$ (a one-tailed version of the test for $H_1$). The second is for where $S_{deterministic}$ is set at the 10% bound: $0.9 S_{iv,max,stochastic} = S_{iv,max,deterministic}$ and $S_{ip,min,deterministic} \leq 1.1 S_{ip,min,stochastic}$ for
intraplatoon and interplatoon separations respectively. \( H_2 \) is rejected if it fails either of these two tests.

Tables 4.1-4.3 show the results of this analysis for each of the three primary cases under consideration. For each of the three primary cases, three levels of probability, as derived from the deterministic model, are tested: the extreme level, the \( 10^{-4} \) level, and the \( 10^{-2} \) level. The extreme level gives the worst case scenario encountered by the stochastic model; the smallest observed \( S_{iv,max} \) and the largest observed \( S_{ip,min} \). The \( 10^{-4} \) and \( 10^{-2} \) levels give the USZ respectively where a 0.01% and 1% collision probability at or above an unsafe collision speed are the allowable risk levels. \( H_1 \) and \( H_2 \) are either accepted or rejected each for scenario. For each of the stochastic model cases, the standard deviation (\( \sigma \)) is also given, as is the bias from the deterministic results. Lastly, as the stochastic model uses 30 trials, \( df = 29 \) \((n - 1)\). A t-table is then used to obtain \( t_{0.995} \) (confidence at the 99.5% level): \( t_{0.995} = 2.756 \) for a one-tailed test and \( t_{0.995} = 3.038 \) for a two-tailed test.

First, these results show that for almost all cases, \( H_1 \) cannot be accepted; the deterministic model does not yield precisely the same results as the stochastic model at statistically significant levels. The only exceptions occur for the Loose-Short case. These results are not unexpected; as discussed in Chapter 3, the two models are fundamentally different, so it is unlikely that they will return identical results.

However, it is not necessary for the two models to give identical results to be considered acceptably equivalent for the purposes of this research. As shown in the results for \( H_2 \), it can be concluded that the deterministic model gives results that are more conservative than and within 10% of the stochastic model results when the extreme scenario is used. The \( 10^{-4} \) scenario generally meets these parameters, with the exception of \( S_{ip,min} \) in the Loose-Long case. However, \( H_2 \) is always rejected here for the \( 10^{-2} \) scenario.

Importantly, the results of the deterministic model are always equal to or more conservative than those of the stochastic model. The deterministic model is more conservative when the bias is negative for intraplatoon separation and positive for interplatoon separation. The one instance where this does not hold is a -0.1% bias for \( S_{ip,min} \) in the extreme
### Table 4.1: Comparing the Stochastic and Deterministic Results for the Loose-Long Case

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$S_{iv,max}$ (m)</th>
<th>$\sigma$ (m)</th>
<th>Bias (%)</th>
<th>$H_1$</th>
<th>$H_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>0.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extreme</td>
<td>0.5</td>
<td>0.0224</td>
<td>-10</td>
<td>Rejected</td>
<td>Accepted</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>0.6</td>
<td>0.0227</td>
<td>-25</td>
<td>Rejected</td>
<td>Rejected</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>0.9</td>
<td>0.0243</td>
<td>-50</td>
<td>Rejected</td>
<td>Rejected</td>
</tr>
</tbody>
</table>

Interplatoon Separation

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$S_{ip,min}$ (m)</th>
<th>$\sigma$ (m)</th>
<th>Bias (%)</th>
<th>$H_1$</th>
<th>$H_2$</th>
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</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>42.25</td>
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<td></td>
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<tr>
<td>Extreme</td>
<td>42.0</td>
<td>0.205</td>
<td>0.6</td>
<td>Rejected</td>
<td>Accepted</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>40.4</td>
<td>0.194</td>
<td>4.5</td>
<td>Rejected</td>
<td>Accepted</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>24.7</td>
<td>0.171</td>
<td>71.1</td>
<td>Rejected</td>
<td>Rejected</td>
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</table>

### Table 4.2: Comparing the Stochastic and Deterministic Results for the Loose-Short Case

<table>
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<tr>
<th>Scenario</th>
<th>$S_{iv,max}$ (m)</th>
<th>$\sigma$ (m)</th>
<th>Bias (%)</th>
<th>$H_1$</th>
<th>$H_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>0.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extreme</td>
<td>0.7</td>
<td>0.0255</td>
<td>-1.1</td>
<td>Accepted</td>
<td>Accepted</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>0.7</td>
<td>0.0255</td>
<td>-1.1</td>
<td>Accepted</td>
<td>Accepted</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>1.0</td>
<td>0.0305</td>
<td>-30.8</td>
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<td>Rejected</td>
</tr>
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</table>

Interplatoon Separation

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$S_{ip,min}$ (m)</th>
<th>$\sigma$ (m)</th>
<th>Bias (%)</th>
<th>$H_1$</th>
<th>$H_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>36.85</td>
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<tr>
<td>Extreme</td>
<td>36.9</td>
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<td>Accepted</td>
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<tr>
<td>$10^{-4}$</td>
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<td>0.182</td>
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<td>Accepted</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>24.5</td>
<td>0.149</td>
<td>50.3</td>
<td>Rejected</td>
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</table>
### Table 4.3: Comparing the Stochastic and Deterministic Results for the Strict-Long Case

**Intraplatoon Separation**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$S_{iv,\text{max}}$ (m)</th>
<th>$\sigma$ (m)</th>
<th>Bias (%)</th>
<th>$H_1$</th>
<th>$H_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>2.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extreme</td>
<td>2.6</td>
<td>0.0540</td>
<td>−5.8</td>
<td>Rejected</td>
<td>Accepted</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>2.6</td>
<td>0.0540</td>
<td>−5.8</td>
<td>Rejected</td>
<td>Accepted</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>3.1</td>
<td>0.659</td>
<td>−21.0</td>
<td>Rejected</td>
<td>Rejected</td>
</tr>
</tbody>
</table>

**Interplatoon Separation**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$S_{ip,\text{min}}$ (m)</th>
<th>$\sigma$ (m)</th>
<th>Bias (%)</th>
<th>$H_1$</th>
<th>$H_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>8.04</td>
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<td></td>
</tr>
<tr>
<td>Extreme</td>
<td>8.0</td>
<td>0.0894</td>
<td>0.5</td>
<td>Rejected</td>
<td>Accepted</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>8.0</td>
<td>0.0894</td>
<td>0.5</td>
<td>Rejected</td>
<td>Accepted</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>7.6</td>
<td>0.0164</td>
<td>5.8</td>
<td>Rejected</td>
<td>Accepted</td>
</tr>
</tbody>
</table>

scenario for the Loose-Short case. However, this difference is small enough that the t-test shows that the two are statistically equivalent ($H_1$ is accepted). Therefore, the only reason why $H_2$ is rejected here is that the deterministic model gives too conservative results, greater than a 10% difference. In terms of percent bias, it is impossible to distinguish where the effect is stronger for interplatoon or intraplatoon separations. However, in absolute values, the effect is far larger for intraplatoon separations. For example in the Loose-Short case, going from the extreme to $10^{-2}$ yields a difference of only 0.3m in $S_{iv,\text{max}}$ but of 22.4m in $S_{ip,\text{min}}$.

Ultimately, it can be concluded that at very low acceptable unsafe collision probabilities (i.e. less that 0.01%) the stochastic and deterministic model are reasonable equivalent, with the deterministic model giving more conservative results. It is therefore reasonable to use the deterministic model to help define the parameters for safe platooning in the following section and later the capacity for automated platooning vehicles. However, this must be taken with the caveat that at higher acceptable unsafe collision probabilities (i.e. 1%), the deterministic model may be too conservative, giving a USZ that is too broad and as a result lowering the estimated capacity. Identifying the acceptable collision probability is beyond
the purview of this research, but as these results show, it is an important consideration that should be addressed in future studies.

4.3 Parameters for Safe Platooning

This section first shows the effects of varying $\Delta v_{safe}$ on the USZ and gives the required brake variance and delay values to ensure safe intraplatoon interactions for different and $\Delta v_{safe}$ values and achievable intraplatoon separations ($S_{iv,ach}$); as described in the previous chapter, this value represents the smallest intraplatoon separation at an acceptable level of comfort than can be achieved by the vehicle controllers.

4.3.1 Varying $\Delta v_{safe}$

Figure 4.9 gives the USZ as a function $\Delta v_{safe}$ where $v_0 = 30\text{ m/s}$ as obtained from the deterministic collision model. This shows that while increasing $\Delta v_{safe}$ has a relatively minor effect on $S_{ip,min}$, it has large effects on $S_{iv,max}$. As in Figures 4.1-4.4 where $a_f \geq a_l$, the inflection (non-differentiable) points for each $S_{iv,max}$ curve occurs at the point where it becomes governed by $S_1$ rather than $S_2$. These inflection points are a function of $T_{tot}$, occurring at $-T_{tot}/a_l$, so at $1.5\text{ m/s}$ for $T_{tot} = 0.15s$ and at $3\text{ m/s}$ for $T_{tot} = 0.3s$. Therefore, especially for low $T_{tot}$ values, increasing $\Delta v_{safe}$ can substantially increase $S_{iv,max}$ even when there is a large brake variance.

The results of the stochastic model evaluating the effects of changing $\Delta v_{safe}$ are given in Figures 4.10 and 4.11, which show the probability curves where no collisions at all are acceptable ($\Delta v_{safe} = 0\text{ m/s}$) and where more dangerous collisions are tolerated ($\Delta v_{safe} = 5\text{ m/s}$) respectively. These confirm the outputs of the deterministic model, showing the potential of increasing $\Delta v_{safe}$ to increase $S_{iv,max}$ and allow for safe platooning even with large brake variances. Additionally, as in Figure 4.9(b), Figure 4.10 suggests that a platooning system could not operate under the requirement that no impact can occur in emergency scenarios unless vehicles are perfectly matched due to the near unity probability of collision at short distances even in the Strict-Short case. It can be observed that under very specific conditions the loose distribution is actually better than the strict, most apparent in the
Fig. 4.9: USZ as a Function of $\Delta v_{\text{safe}}$

(a) $S_{ip,\text{min}}$

(b) $S_{iv,\text{max}}$
near-zero separations in Figure 4.10. This is because the higher probability of disparity in $a_{\min}$ increases the number of instances where the follower has much better braking than the leader, whereas if they are better matched, delays will surely cause an incident.

4.3.2 Necessary Brake Variance and Delay Values

Figure 4.12 presents the necessary $a_l$ and $T_{tot}$ values for platooning obtained from Equation 3.11, where $a_l = -10\text{m/s}^2$ for 5 different $S_{iv,ach}$ values from 0.5 to 3m and for three different $\Delta v_{safe}$ values of 2.5, 3.5, and 4.5$m/\text{s}$. Equation 3.11 is reproduced below for ease of reference.

If: $S_{iv,ach} \leq -\frac{\Delta v_{safe}^2}{2a_l}$

Then: $S_{iv}$ is always deemed safe

Otherwise:

If: $-a_lT_{tot} \leq \Delta v_{safe}$

$$a_{f,\text{max}} = \frac{\Delta v_{safe}^2 - 2S_{iv,ach}a_l}{2S_{iv,ach} + a_lT_{tot}^2}$$

If: $-a_lT_{tot} \geq \Delta v_{safe}$

$$T_{tot,\text{max}} = \frac{\Delta v_{safe}}{-a_l}$$

Where $\Delta v_{safe} = 3.5m/\text{s}$ and $4.5m/\text{s}$, not all $S_{iv,ach}$ curves appear, since for these parameters no unsafe collisions occur for small separations even if the follower has not reacted before the collision. As stated previously, where $a_l = -10\text{m/s}^2$ and $\Delta v_{safe} = 2.5m/\text{s}^2$, this occurs where $S_{iv,ach} \leq 0.31m$. For the larger $\Delta v_{safe}$ values of 3.5 and 4.5$m/\text{s}$, this occurs at 0.61 and 1.01m respectively; these values are obtained from Equation 3.11, where no USZ exists when $S_{iv,ach} \leq -\frac{\Delta v_{safe}^2}{2a_l}$. Therefore, the 0.5m $S_{iv,ach}$ curve does appear in where $\Delta v_{safe} = 3.5m/\text{s}$ and the 0.5m and 1m curves do not appear where $\Delta v_{safe} = 4.5m/\text{s}$.

As given in Equation 3.11, where $T_{tot} > -\frac{\Delta v_{safe}}{a_l}$, $S_{iv,\text{max}}$ is always $-\frac{\Delta v_{safe}^2}{2a_l}$; this is given by the “Limit” line in Figure 4.12, and the term itself is derived from the Scenario 1 term.
Fig. 4.10: Any Collision ($\Delta v_{safe} = 0$ m/s) Probability Curves for Short Delay [67]

Fig. 4.11: Dangerous Collision ($\Delta v_{safe} = 5$ m/s) Probability Curves for Short Delay [67]
Fig. 4.12: Necessary $T_{tot}$ and $a_f$ for Safe Platooning
in Equation 3.7. All $S_{iv,ach}$ curves are given by $\frac{\Delta v_{safe}^2 - 2S_{iv,ach}a_l}{2S_{iv,ach} + a_lT_{tot}}$: this term is derived from the Scenario 2 term in Equation 3.7. For each $\Delta v_{safe}$ value, all lines intersect at a single point, $-10m/s^2$ and 0.25s for $\Delta v_{safe} = 2.5m/s$. As it represents the intersection between Scenarios 1 and 2, it is equivalent to the non-differentiable point that occurs at 0.25s in Figures 4.1-4.4. Therefore, as with those results, for larger delays (greater than the Limit value), safe separations only occur when $a_f < a_l$. Here, the $S_{iv,ach}$ curves actually define the upper bound of the USZ. For example, in Figure 4.12(a), where $a_f = -14m/s^2$ and $S_{iv,ach} = 0.5m$, the USZ only occurs when $T_{tot} > 0.27s$; whenever $T_{tot}$ is lower, platooning is safe at any separation.

Since Equation 3.11 is not dependent on initial velocity ($v_0$), Figure 4.12 is valid for all conditions where the USZ exists, as according to Equation 3.9. Where $v_0$ becomes very small, the USZ can disappear, since $S_2 \geq S_3$. However, where $S_{iv,ach} = 1m$, the first time this occurs is at $v_0 = 7m/s$ and $t = 0.04s$, and such a scenario is considered unrealistic for the purposes of modeling automated platooning flow at highway speeds.

### 4.4 Capacity as a Function of Platoon Size and Speed

This section provides the total lane capacity as a function of speed for platoon sizes ranging from one to twenty-one vehicles for four cases of brake variance and total delay. The specific values for each of the cases, as well as the amelioration protocols that can be used are provided here in Table 4.4. It is assumed that $\Delta v_{safe} = 2.5m/s$, $S_{iv,ach} = 1m$, and $a_l = -10m/s^2$. Figure 4.12(a) is used to choose the cases according to the guidelines described in Section 3.7.3.

<table>
<thead>
<tr>
<th>Case</th>
<th>$a_l$ ($m/s^2$)</th>
<th>$T_{tot}$ (s)</th>
<th>Amelioration Protocol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-8$</td>
<td>0.168</td>
<td>No Amelioration Needed</td>
</tr>
<tr>
<td>2</td>
<td>$-8$</td>
<td>0.200</td>
<td>Collaborative Braking and Brake Derating</td>
</tr>
<tr>
<td>3</td>
<td>$-6$</td>
<td>0.168</td>
<td>Collaborative Braking Only</td>
</tr>
<tr>
<td>4</td>
<td>$-7$</td>
<td>0.400</td>
<td>Brake Derating Only</td>
</tr>
</tbody>
</table>

Table 4.4: Brake Variance and Total Delay for the Four Cases
Figures 4.13-4.16 show capacity as a function of speed for the four cases if no amelioration were used, though for Cases 2-4 this would result in unsafe platooning where $S_{iv,ach} = 1\text{m}$. Section 4.6 provides the flow relationships for Cases 2-4 when amelioration is needed.

4.5 Flow Analysis

A number of inferences can be drawn from Figures 4.13-4.16. These are presented below and addressed in order.

- Automated vehicle flow patterns roughly mirror those of traditional vehicles, especially for the congested portion of the curve, but at different magnitudes.
- However, there is no free-flow speed.
- Autonomous vehicles, with no platooning or coordination, do not promise large capacity improvements over current roadway conditions.
- Additionally, platooning allows for flow increases during uncongested travel without requiring decreases in speed.
- Each additional vehicle in a platoon brings decreasing marginal returns for vehicle flow; flow is asymptotically increasing as a function of platoon size.
- The optimal speed ($v_o$) where capacity occurs increases as a function of platoon size.
- The tight brake variance and small communication delay in AET can offer a safer system with higher capacities than those promised by AHS.

4.5.1 Similarity to Traditional Flow Patterns

First, recall that the congested portion of the curve refers to the conditions where an increase in vehicle speed results in an increase in capacity. As seen in Figures 4.13-4.16, this portion of the curve is roughly parabolic, mimicking the output of the Greenshields model. Also like traditional flow models, as speed increases from zero, flow first increases
Fig. 4.13: Case 1 Platooning Capacity: No Amelioration

Fig. 4.14: Case 2 Platooning Capacity: No Amelioration
Fig. 4.15: Case 3 Platooning Capacity: No Amelioration

Fig. 4.16: Case 4 Platooning Capacity: No Amelioration
to the capacity level and then decreases. However, the uncongested portion of the curve is *not* parabolic as in Greenshields. Instead, flow asymptotically approaches zero as speed increases.

This is a result of the lack of free-flow speed. The capacity model developed here, based on the collision model, does not consider a roadway’s design speed. Safety is only considered based on safe-following distances. Therefore, this can be seen as the equivalent of using the AIMSUM traffic simulation software with only the Gipps’ model for impeded traffic, and ignoring the potential for unimpeded flow. In real-world scenarios, each roadway would have a maximum safe traveling/design speed, though this value could be higher for automated platooning lanes than for traditional roadways [19]. Therefore, once this design-speed restraint is considered, flow increases in a roughly vertical line (independent of speed) until flow starts to become impeded and car following beings. While not the focus of this research, this consideration allows for the general pattern automated platooning flow to more closely represent real-world traffic flow [36] and traditional flow modeling [27].

The differences in flow patterns created by the inclusion of platooning are addressed in the following subsection. For the single autonomous vehicle scenario \((Z = 1)\), the results of this capacity model agree with those of AHS research, showing how autonomous, non-platooning vehicles do not promise massive capacity improvements [19,64,65]. Even in the high-performing Case 1, capacity is only \(3,423\text{veh/hr}\) at \(v_o = 22\text{m/s}\). Since it is assumed in the collision model that interplatoon delay is three times that of intraplatoon delay, the delay between the autonomous vehicles in this case is set at \(0.168 \times 3 = 0.504\text{s}\). If we use a 0.168s delay between these autonomous vehicles, capacity increases to \(5,029\text{veh/hr}\), also at \(v_o = 22\text{m/s}\). However, as stated in Chapter 3, this capacity model provides capacity in an ideal scenario of a free-flowing, straight lane, with no platoon merging nor splitting, lane changing, or lane exiting/entering. Previous research indicated that these could reduce capacity from 5% to 70% [94], which at the higher end would bring this value down to current highway levels.
Case 4 more closely represents current manually-driven vehicles, with a delay (reaction time) of 1.2s. This is a commonly used value for unexpected maneuvers, such as an emergency brake [103]. However, even with a relatively tight brake variance of $-10$ vs. $-7\,\text{m/s}^2$, capacity is only $1,878\,\text{veh/hr}$ at $v_o = 17\,\text{m/s}$. A more realistic $a_f$ of $5\,\text{m/s}^2$ would reduce capacity to $1,568\,\text{veh/hr}$ at $v_o = 11\,\text{m/s}$. Deviations from the free-flowing, isolated, straight lane case would further degrade these values. However, it was expected that this capacity model would return lower than observed values, since it demands a higher level of safety than traditional roadways, where it has been estimated that drivers maintain only 60% of the necessary minimum safe separations [19].

### 4.5.2 Effects of Platooning

Platooning fundamentally changes the flow behavior by adding another variable to speed, density and flow, shifting it from a 2-dimensional to a 3-dimensional relationship. While the difficulties inherent with merging and splitting are not the focus of this thesis and are being addressed in other research, if it is assumed that average platoon size can vary as a function of demand, then flow can be increased without a decrease in speed. For example, assume that there is an AET roadway with a design speed of $30\,\text{m/s}$ with the parameters given in Case 1. Vehicles could act independently in unimpeded conditions until demand reaches $3,336\,\text{veh/hr}$. Then, instead of decreasing speed to accommodate increased demand, vehicles could begin platooning and maintain speed. Average platoon size could be two vehicles until demand is $5,629\,\text{veh/hr}$, three vehicles until demand is $7,302\,\text{veh/hr}$, etc.

As seen with these values and in Figures 4.13-4.14, the marginal improvements in flow decrease as platoon size increases. Figure 4.17 directly shows the flow marginals as a function of platoon sizes for different speeds. This decreasing rate of return is due to the decreasing contribution of interplatoon separations ($S_{ip}$) to vehicle density and thus flow. Conceptually, as $Z \to \infty$, the system starts to approximate one of constant vehicle separations based on the intraplatoon separation ($S_{iv}$). Here, Equation 2.15 simplifies to
the traditional flow relationship to yield the maximum flow when $Z \to \infty$:

$$q = 3600 \frac{v}{L + S_{iv,ach}}$$

Therefore, while increasing platoon sizes can yield flow improvements, very large platoons, such as greater than 20 vehicles, may not be necessary. Where $v_0 = 30\text{m/s}$, the maximum theoretical capacity is $18,000\text{veh/hr}$ according to Equation 4.2. Table 4.5 provides the platoon size where 75% of this value can be reached for Cases 1-4 if no amelioration is used. This shows that in general, stricter parameters (a higher-performing system) allow for high capacity levels to be reached at lower platoon sizes. Where no amelioration is needed, as in Case 1, relatively small platoon sizes can be sufficient.

Also as seen in Figures 4.13-4.16, $v_0$ increases proportionately as a function of $Z$; as platoon size grows, the speed at which maximum capacity occurs increases as well. As with the decreasing marginal returns on flow as a function of platoon size, this is a result of the decreasing importance of $S_{ip}$ on determining flow for larger platoon sizes. Since $S_{iv}$ is fixed at $S_{iv,ach}$, the only vehicle separations that increase as a function of speed are interplatoon.
Table 4.5: Platoon Size Required for 75% of the Theoretical Maximum Capacity: No Amelioration

<table>
<thead>
<tr>
<th>Case</th>
<th>Platoon Size (Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>28</td>
</tr>
</tbody>
</table>

For the theoretical case of infinite platoon size, there is no \( v_o \); flow increases linearly as a function of speed as according to Equation 4.2.

### 4.5.3 Situations That Do Not Require Amelioration

As discussed in Section 4.3.2, relatively small delays and tight brake variances are required to enable safe platooning, even where \( S_{iv,ach} \) is set at the relatively low value of 1m. Even tighter conditions are required if \( S_{iv,ach} = 2m \), the value assumed by AHS research for intraplatoon separations in capacity calculations [19, 64, 65]. However, the combination of a single vehicle type, check-in procedures, modern wireless communications, and improved vehicle responses, both intravehicle delays and braking responses, promised by the AET’s system of electric, collaborative platooning vehicles will provide superior delays and brake variances. For the conditions, such as in Case 1, where no amelioration is needed, very high capacity values are achievable. For Case 1, \( v_o \) occurs at values greater than 65\( \text{m/s} \) for both 10 and 20 vehicle platoons. If we assume that capacity is instead limited by the roadway’s design speed, set by at 30\( \text{m/s} \), the capacities are 12,504\( \text{veh/hr} \) for \( Z = 10 \) and 14,757\( \text{veh/hr} \) for \( Z = 20 \). For comparison, the maximum theoretical capacity for \( v_0 = 30\text{m/s} \) is 18,000\( \text{veh/hr} \) as given by Equation 4.2. The lowest possible capacity for cases where amelioration is not needed occurs where \( T_{tot} = 0.14\text{s} \) and \( a_f = -7.62\text{m/s}^2 \): at \( v_0 = 30\text{m/s} \), 12,265\( \text{veh/hr} \) and 14,730\( \text{veh/hr} \) for 10 and 20 vehicle platoons respectively. Therefore, we can conclude that very high capacities, over 10,000\( \text{veh/hr} \) are attainable if the input parameters can ensure safe platooning according to the collision model. Capacity in situations where platooning
is initially unsafe is addressed in the following section.

4.6 Capacity When Platooning Is Initially Unsafe

This section addresses the effects of amelioration on capacity. It first gives flow as a function of speed for different platoon sizes using different amelioration protocols on Cases 2-4. It then provides an analysis of the flow characteristics of platooning vehicles when the amelioration protocols are used.

4.6.1 Cases with Amelioration

Figures 4.18 and 4.19 gives the flow for Case 2 when brake derating and collaborative braking are used, respectively. Figure 4.20 gives the flow for Case 3 with brake derating, and Figure 4.21 gives the flow for Case 4 with collaborative braking.

4.6.2 Flow Analysis with Amelioration

As seen in Figures 4.18-4.21, both brake derating and collaborative braking reduce flow. This effect is relatively minor where mild amelioration is needed, as in Case 2, and large where stronger amelioration is needed, as in Cases 3 and 4. Table 4.6 shows the flow ($q$) at $Z = 10$ for $v_0 = 30$ and $60 \text{ m/s}$ and the relative reduction in flow ($q_{red}$) from the equivalent case with no amelioration.

These results show a reasonable agreement with the AHS models. Case 3 represents a typical input for these models with a brake variance on the smaller end, and its flow

<table>
<thead>
<tr>
<th>Case</th>
<th>Amelioration Protocol</th>
<th>$v_0 = 30 \text{ m/s}$</th>
<th>$q_{red}$ (%)</th>
<th>$v_0 = 60 \text{ m/s}$</th>
<th>$q_{red}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Brake Derating</td>
<td>11,980</td>
<td>96</td>
<td>14,160</td>
<td>92</td>
</tr>
<tr>
<td>2</td>
<td>Collaborative Braking</td>
<td>11,367</td>
<td>94</td>
<td>11,980</td>
<td>78</td>
</tr>
<tr>
<td>3</td>
<td>Brake Derating</td>
<td>8,991</td>
<td>87</td>
<td>7,999</td>
<td>78</td>
</tr>
<tr>
<td>4</td>
<td>Collaborative Braking</td>
<td>6,119</td>
<td>65</td>
<td>7,381</td>
<td>71</td>
</tr>
</tbody>
</table>
Fig. 4.18: Case 2 Platooning Capacity: Brake Derating

Fig. 4.19: Case 2 Platooning Capacity: Collaborative Braking
Fig. 4.20: Case 3 Platooning Capacity: Brake Derating

Fig. 4.21: Case 4 Platooning Capacity: Collaborative Braking
of 8,991 veh/hr is correspondingly on the higher end of AHS predictions. By comparison, a larger brake variance of \( a_f = -5 \text{ m/s}^2 \) input into this capacity model would reduce capacity to 5,756 veh/hr. Additionally, these figures indicate that while both brake derating and collaborative braking can substantially degrade flow, by comparing the flows for the two amelioration protocols in Case 2 and the relative drop in flow in Cases 3 and 4, it appears that brake derating is the superior option, at least where \( v_0 = 30 \text{ m/s} \). This confirms the findings of the collision model, which claims that lower brake variances are more important than smaller delays.

However, collaborative braking is comparatively more attractive at \( v_0 = 60 \text{ m/s} \). This is because collaborative braking’s effect is independent of vehicle speed. Interplatoon separation must be increased by the appropriate amount to accommodate the collaboration as given by Equation 3.13. By comparison, brake derating demands that for interplatoon interactions \( a_f \) increases as according to Equation 3.12. Since this increased brake variance applies to the entire time both vehicles are braking (Scenario 2), brake derating results in larger increases in interplatoon separations as speed increases.

Additionally, brake derating and collaborative braking have different effects on system flow patterns with respect to platoon size. Table 4.7 helps to show this, providing both the potential maximum capacity \( (q_{\text{max, pot}}, \text{ the maximum capacity that can be achieved when amelioration is used}) \) and the platoon size necessary to reach 75% of \( q_{\text{max, pot}} \) where \( v_0 = 30 \text{ m/s} \).

If collaborative braking is used, flow asymptotically approaches a value less than the

<table>
<thead>
<tr>
<th>Case</th>
<th>Amelioration Protocol</th>
<th>( q_{\text{max, pot}} ) (veh/hr)</th>
<th>( Z )</th>
<th>( Z_{\text{no amelioration}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Brake Derating</td>
<td>18,000</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>Collaborative Braking</td>
<td>17,175</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>Brake Derating</td>
<td>18,000</td>
<td>30</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>Collaborative Braking</td>
<td>11,920</td>
<td>28</td>
<td>28</td>
</tr>
</tbody>
</table>
theoretical maximum. When brake derating is used, flow approaches the theoretical maximum, but larger platoon sizes are needed to reach equivalent flow levels. For example, in Case 3 platoons of 30 vehicles are needed to reach 75% of the theoretical maximum, while only 21 vehicle platoons are required if no amelioration were used. However, with both amelioration protocols, no optimal platoon size exists. Though the marginal improvements in flow can change, they are always positive, and a larger platoon size will always result in a larger flow.

4.7 Identifying an Optimal Platoon Size

That flows increase asymptotically as a function of platoon size can be shown analytically from Equation 3.14, which is reproduced here:

\[ q = \frac{3600v_0Z}{3v_0[2T_{tot} - T_{red} + 0.1Z(T_{tot} - T_{red})]} - \frac{v_0^2}{2a_f(1-\beta_d)}(1 - ax/a_i) + (Z - 1) + 5Z \]

This equation can be simplified to be given as a function of platoon size and the derivative taken as follows:

\[ q = \frac{aZ}{bZ + c} \]

\[ \frac{\partial q}{\partial Z} = \frac{ac}{(bZ + c)^2} \] (4.3)

Capacity occurs where \( q' = 0 \), so an optimal platoon size would exist where \( \frac{\partial q}{\partial Z} = 0 \). With the above formulation, this never occurs, so there is no optimal platoon size; assuming that \( ac \) is positive, \( \frac{\partial q}{\partial Z} \) starts positive and asymptotically approaches zero as \( Z \) increases. This corresponds to what is observed in the above flow-speed-platoon size curves and in the representation of marginal improvements in flow given in Figure 4.17. However, if the formulation in Equation 4.3 does not hold true, then identifying optimal platoon sizes may become possible. The following two sections address situations where optimal platoon sizes could exist by developing different equations for the influence of brake derating and collaborative braking. These equations represent stricter conditions and limiting factors.
that could be encountered in real-world scenarios.

4.7.1 Optimal Platoon Size for Cumulative Brake Derating

As discussed in the previous chapter, brake derating as initially formulated only includes a two vehicle scenario; all subsequent vehicles after the follower are assumed to be able to match the braking rate of the follower precisely. This is the assumption made by the AHS work for setting a “brake amplification factor,” but it is not necessarily valid. To ensure safety throughout the platoon, it might be necessary to provide derating for each vehicle interaction. For example, assume a 10% derating was required to provide safety in a three vehicle platoon, where vehicle 1 is the leader. To avoid colliding with vehicle 3, vehicle 2’s braking would have to be derated by 10%, so that it is 90% of its maximum. The lead vehicle would then need to be derated an additional 10%, bringing its total braking to 81% of its maximum. Therefore, Equation 3.12 becomes:

$$a_{f,actual} = (1 + \beta_d)Za_f$$

(4.4)

If this equation for cumulative brake derating is used, then the platooning capacity for Cases 2 and 3 is given by Figures 4.22 and 4.23 respectively where $v_0 = 30\text{m/s}$. For Case 2, capacity is $8,059\text{veh/hr}$ and occurs at $Z = 9$. For Case 3, capacity is $2,411\text{veh/hr}$ and occurs at $Z = 3$. Therefore, an optimal platoon size does exist where brake derating is assumed to have a cumulative effect. Furthermore, this drastically reduces system capacity, even in situations such as Case 2 when only relatively mild amelioration is needed; actual follower braking is $-8\text{m/s}^2$, but no amelioration would be needed where $T_{tot} = 0.2s$ if $a_f$ were slightly stronger at $-8.59\text{m/s}^2$. Therefore, if brake derating were to behave in such a manner in real-world scenarios, it may not be able to provide sufficient capacity to justify an AET system.
Fig. 4.22: Case 2 Platooning Capacity: Cumulative Brake Derating

Fig. 4.23: Case 3 Platooning Capacity: Cumulative Brake Derating
4.7.2 Optimal Platoon Size for Exponential Collaborative Braking

Here, it is assumed that the additional delay due to collaboration scales with the square of $Z$ instead of linearly, so that Equation 3.13 becomes:

$$T_{\text{tot,actual}} = 2T_{\text{tot}} - T_{\text{red}} + \alpha_c Z^2 (T_{\text{tot}} - T_{\text{red}})$$  (4.5)

While provisional, such a formulation could account for increasing delays due to more difficult wireless communications between platoons as platoon size increases. For example, since this requires more intraplatoon communications, if interplatoon communications were given a lower priority, then they would experience a higher frequency of packet loss. Though the above formulation only considers the cases where collaborative braking is used, the same effect could also occur without it.

It is assumed that where Equation 4.5 is used, $\alpha_c$ is 0.01. This represents minor delays due to collaboration in small platoons and large delays in large platoons. Figures 4.24 and 4.25 give platooning capacity where this exponential collaborative braking is used for Cases 2 and 4 respectively. As with the above section $v_0 = 30\,\text{veh/hr}$.

For Case 2, capacity is $8,934\,\text{veh/hr}$ and occurs at $Z = 11$. For Case 4, capacity is $2,806\,\text{veh/hr}$ and occurs at $Z = 5$. Therefore, as with cumulative brake derating, where exponential collaborative braking is used, an optimal platoon size does exist. These results indicate that with the given assumptions, exponential collaborative braking is slightly better than cumulative brake derating: flows of $8,934\,\text{veh/hr}$ vs. $8,059\,\text{veh/hr}$ respectively for Case 2. However, as with cumulative brake derating, only when mild amelioration is needed are large flows possible. The larger amelioration demanded by Case 3 can bring capacity down towards the level of traditional roadways.

4.8 Summary of Analytical Findings

This section addresses the findings of this research, providing the main takeaway points for both the collision and capacity models.
Fig. 4.24: Case 2 Platooning Capacity: Exponential Collaborative Braking

Fig. 4.25: Case 4 Platooning Capacity: Exponential Collaborative Braking
4.8.1 Collision Model

One major conclusion from Section 4.5 is that the most important factor in ensuring safe platooning is a tight braking variance, followed by communication delay. Even when it is assumed that the control model can maintain a steady-state intraplatoon separation of 1m, as opposed to the 2m used in AHS modeling, there cannot be a brake variance higher than $a_f = -6.88\text{m/s}^2$ and $a_l = -10\text{m/s}^2$ even if there is no delay; otherwise platooning will not be safe in an emergency braking scenario. Additionally, the collision model results show that while AHS was able to demonstrate platooning in a controlled environment, the systems would not have been safe using that emergency braking standard without amelioration protocols such as brake derating and collaborative braking. This does not indicate a flaw in their research, but rather the technological limitations of the 1990s. However, the advent of wireless communications combined with the single type of electrical vehicle proposed by AET should allow for the small delays and tight braking variances necessary for safe platooning without amelioration.

The results also show the potential benefits of increasing $\Delta v_{safe}$, which could be achieved by designing the vehicles specifically to better handle rear-end collisions. Even relatively small increases in this value, such as from 2.5 to 3.5\text{m/s}, could greatly expand the parameter sets enabling safe platooning. For example, in a system with no delay, a leader acceleration of $-10\text{m/s}^2$, and a steady-state intraplatoon separation of 1m, this increase would allow for the far greater brake variance, with allowable follower acceleration going from $-6.88\text{m/s}^2$ to $-3.875\text{m/s}^2$. Similarly, as shown in the stochastic model results, increasing the acceptable unsafe collision likelihood can also shrink the USZ and allow for “safer” performance under a wider variety of input parameters. However, while steps can be taken to increase $\Delta v_{safe}$ while maintaining the same level of safety (i.e. vehicle design), increasing the acceptable unsafe collision likelihood explicitly sacrifices system safety for system performance.

Lastly, the comparison of the stochastic and deterministic collision models using student’s t-tests shows that that two are not statistically identical. However, at low acceptable
unsafe collision likelihood (i.e. less than $10^{-4}$), the deterministic model is more conservative than and reasonably equivalent to the stochastic model. However, at higher acceptable unsafe collision likelihoods, while the deterministic model is always more conservative than the stochastic model, the USZ values are not necessarily very close (i.e. greater than 10% different). Therefore, while the deterministic model can be reasonably used as conservative input values for the capacity model, if decision makers determine that for any eventual automated platooning system higher unsafe collisions likelihoods are acceptable, then USZ should be modified in accordance with this value.

4.8.2 Capacity Model

Two general scenarios are addressed by the capacity model: platooning flow with and without amelioration. Capacity varies greatly depending on the system parameters, including the parameters described in each of the four cases used for analysis here and also on vehicle speed, platoon size, and type of amelioration used. For example, at speeds of $30\text{m/s}$, capacity can range from over $12,504\text{veh/hr}$ for high performing systems (Case 1) with 10 vehicle platoons to $1,137\text{veh/hr}$ for low performing systems (Case 4) with a single autonomous vehicle. However, even such systems can achieve high capacities is platooning is allowed. For example, if the above scenario for Case 4 had 10 vehicle platoons, capacity would be $6,119\text{veh/hr}$ if the amelioration protocol of collaborative braking were used.

Therefore, identifying the scenarios that will enable unsafe platooning is of the utmost importance. However, of the four cases used for analysis, only Case 1 does not require amelioration where steady-state intraplatoon separation is 1m as according to the collision model. These results show that for this case specifically and for all other cases where amelioration is not required, flows of over $12,000\text{veh/hr}$ are possible for platoons of 10 vehicles where $v_0 = 30\text{m/s}$. Higher flows are possible at higher speeds and platoon sizes. Therefore, the technological advances promised by AET should allow for higher capacities than were modeled by AHS.

Where amelioration is required, two protocols are evaluated: collaborative braking and brake derating. For most of the scenarios analyzed, brake derating is the superior option
when both can potentially be used, though at higher speeds and platoon sizes collaborative braking becomes more attractive. When these protocols are implemented as initially modeled, the flow results reasonably correspond with those of AHS when using similar input parameters for vehicle performance. Mild amelioration results in small reductions in capacity, and more substantial amelioration gives larger drops.

The general flow-speed-platoon size relationships are the same for both the amelioration and no amelioration scenarios. Namely, there is no optimal platoon size; for any given speed, flow asymptotically approaches a maximum as \( Z \to \infty \). However, collaborative braking can decrease this maximum, and brake derating requires more vehicles in a platoon to reach the same flow level. Additionally, optimal speed \( v_o \), the speed that gives capacity for a given platoon size) increases as a function of platoon size. Therefore, while the general flow-speed-density relationships for platooning vehicles is similar to that of traditional vehicles, as platoons get longer, \( v_o \) often occurs at higher speeds than would be safe even in free-flowing situations, such as 70 m/s. As a result, instead of decreasing speed to increase flow as done on traditional roadways during uncongested travel, a platooning system could accommodate increased demand while maintaining speed by increasing average platoon size.

The practicality of the amelioration protocols are also addressed, and stricter conditions are proposed for both: cumulative brake derating and exponential collaborative braking. These revised protocols drastically degrade vehicle flow, especially for Cases 3 and 4, where substantial amelioration is required. These results show the potential downsides of using the amelioration protocols, further indicating the benefits of the tight brake variances and small delays promised by AET. For example, in Case 4 with exponential collaborative braking, maximum capacity for any platoon size or speed is only 2,806 veh/hr. This instance, as well as the other results and the mathematical analyses presented in Section 4.7 show that the addition of exponents to the capacity equation caused by these new amelioration protocols results in the emergence of an optimal platoon size. With the given assumptions, this optimal level is relatively small, less than 11 vehicle platoons for all cases.
CHAPTER 5
CONCLUSIONS AND FUTURE RESEARCH

Automated platooning has the potential to offer superior performance to the current system of manually-driven vehicles, and as technology improves and research continues, it comes closer to real-world implementation. Therefore, it is increasingly important to identify how automated platooning can function in a safe manner and model what capacities it can achieve. The work presented in this thesis seeks to answer these questions by developing interlinked collision and capacity models. These models are used to identify the necessary parameters for safe platooning under an emergency braking standard and describe automated platooning flow and the different parameters that affect it. They can be applied to a wide range of platooning systems beyond that proposed by AET, including automated platooning within mixed traffic scenarios. The obtained results show the ability for automated platooning to provide high-capacity roadway travel that could solve many existing transportation problems: roadway congestion, the last-mile problem for transit, high emissions, and safety risks. However, the results also show the difficulty of maintaining safe intraplatoon interactions, and the importance of tight brake variances and small communication delays as promised by AET and enabled by improvements in electric vehicle technology and wireless communications.

This chapter summarizes the work performed in this thesis. It first reviews the research efforts, including the literature review and the development of the collision and capacity models. It then discusses the obtained results, showing the necessary parameters for AET to achieve safe platooning with capacities of over 10,000 veh/hr. The impacts of these findings are then addressed and future work is recommended.

5.1 Summary of Work

This section addresses the three main areas of work: literature review, collision model development, and capacity model development. This includes a summary of the statistical analysis used to compare the two different collision models developed here (stochastic and
deterministic), and how the deterministic model outputs are used as inputs for the capacity model. The Section 5.2 summarizes the obtained results of these assorted models, which are presented in full in Chapter 4.

5.1.1 Literature Review

First, previous research pertaining to safety and capacity in automated vehicle platoons is considered, including both traditional traffic models and automation-specific studies, with a special emphasis on the Automated Highway Systems (AHS) project. Both microscopic (car-following) and macroscopic (traffic flow) models are reviewed, as well the capacity models that form a subset of the macroscopic models. The relationship between microscopic and macroscopic models is also discussed, as is the difference between the older analytic car-following models with closed-form solutions, such as the GHR family of equations, and the more complicated stochastic traffic simulation softwares, such as Paramics. Since the approach taken by this work is to use a microscopic collision model as the foundation for developing the capacity model, existing safety models, such as Gipps’ equation, are a focus. However, such models do not perfectly describe real-world conditions, especially because of their implicit assumption that a vehicle is always following a leader. Therefore, more accurate estimations require the inclusion of different regimes, such as the two in AIMSUM: Gipps’ for impeded travel and a free-driving equation for unimpeded travel.

The AHS research takes a similar approach to the one advocated in this thesis, developing collision models and testing them under an emergency braking standard. The Unsafe Separation Zone (USZ) concept, which went unnamed in AHS research, shows that very small vehicle separations can be as safe as very large ones. In the case of an emergency brake, vehicles close to each other may collide, but at an acceptably low speed. Vehicles with these very small separations form platoons, and the separations can be maintained due to the improvements allowed by automation: vehicle collaboration, more rapid vehicle-vehicle communications, and smaller intervehicle response delays.

However, there are some areas in the AHS research that merit further work, which this thesis seeks to perform. First, most of the developed models are not replicable. For
the capacity models, inputs and outputs are provided, but not the models themselves. The results of the collision models are not presented in detail. They are either directly used to build collision models or the expected intraplatoon collision speed is presented as a deterministic output despite the stochastic inputs. Second, AHS research assumes a mix of traditional vehicles with internal combustion engines, with a correspondingly wide range of maximum braking rates. Third, while the amelioration techniques of brake derating and collaborative braking are considered, only select values are used. Fourth, AHS studies do not consider the potential amelioration effects of varying the maximum acceptable collision speed ($\Delta v_{\text{safe}}$).

5.1.2 Collision Model Development

Two separate but related collision models are developed to quantify the USZ of automated platooning vehicles and the effects on safety of three main parameters: brake variance, communication delay, and maximum acceptable collision speed. The USZ is defined as the intermediate separations for which an emergency brake results in a collision speed greater than the maximum acceptable level. Collisions are therefore deemed acceptable when the vehicle separation fall outside this range: less than or equal to maximum safe intraplatoon separation ($S_{iv,\text{max}}$) and greater than or equal to minimum safe interplatoon separation ($S_{ip,\text{min}}$). Very small separations, less than or equal to $S_{iv,\text{max}}$, can be safe since while a collision does occur as a result of an emergency brake, it is at a low enough speed to avoid the risk of serious injury or death.

The two developed models are: a simpler deterministic model with closed-form solutions and a more complicated stochastic model coded in Matlab/Simulink and incorporating a Monte Carlo analysis. The stochastic model is more flexible and has the potential to better simulate real-world conditions. Specifically, it models braking as a first-order response (mimicking the behavior of real-world caliber braking), instead of the binary approach of instantaneous full braking used in the deterministic model (and also previously used in AHS research). Additionally, while the deterministic model uses a worst-case scenario for braking (weakest expected leader braking and strongest expected follower braking), the Monte
Carlo simulation used by the stochastic model allows for the unsafe collision probability to be calculated for a given braking distribution of the vehicles on the roadway. This in turn allows for the USZ to be defined as a function of acceptable unsafe collision likelihood. For example, if a 1% likelihood of unsafe collisions is deemed acceptable, than there will be a smaller USZ (narrower range of unsafe vehicle separations) than if a 0.01% likelihood were the maximum acceptable level.

However, by providing closed-form equations, the deterministic model allows for a consideration of a wide range of input parameters without requiring a simulation run for each case. This allows for a more in-depth analysis of the USZ and how it could possibly be decreased to allow for easier platoon, especially by allowing for the explicit mathematical definition of the necessary parameters for safe platooning. Moreover, a deterministic collision model allows for the development of a deterministic capacity model to describe the flow of automated vehicle platoons in far greater detail than was possible in the AHS research, which did not use such deterministic inputs for safe vehicle separations.

5.1.3 Comparing the Two Collision Models

As automated platooning is an emerging technology, there is no real-word data against which to compare and calibrate the models. However, it is necessary to show that the simpler deterministic model is a reasonable substitute for the stochastic model in order to be acceptably used as an input for the capacity model. To accomplish this, a statistical analysis utilizing the Student’s t-test is used to compare the two models. The analysis shows that while the two models do not provide statistically identical USZ results, the deterministic model yields USZ values that are more conservative than and, in most cases, reasonably close to (within 10%) those of the stochastic model. The deterministic model is always shown here to be more conservative than or equal to the stochastic model. However, the two can be more than 10% different. This occurs at lower levels of acceptable unsafe collision likelihood. For example, for the Loose-Short case a 0.01% acceptable likelihood yields USZ values (both $S_{iv, max}$ and $S_{ip, min}$) within the 10% threshold for the two models, but this threshold is exceeded at a 1% acceptable likelihood, by 30.8 and 50.3% for $S_{iv, max}$
and $S_{ip,\text{min}}$, respectively. Therefore, for these higher acceptable likelihood levels, the deterministic model may be considered too conservative when compared with the stochastic model.

### 5.1.4 Capacity Model Development

The USZ derived from the deterministic collision model is then used to develop the capacity model. This is the same model that was used to estimate capacity and flow for AHS research [19, 91], which is the traditional flow relationship equating flow, density, and speed that has been modified to consider platooning vehicles. The collision model helps provide the two different distance headway inputs (interplatoon and intraplatoon separation), and the other exogenous independent variables are vehicle speed, platoon size, and vehicle length. Therefore, the capacity model is used to estimate the effect of five main parameters on automated platooning vehicle flow: the three parameters discussed above for the collision model (brake variance, communication delay, and maximum acceptable collision speed) as well as flow-specific parameters of vehicle speed and platoon size.

Importantly, this capacity model estimates the flow of automated platoons under ideal conditions: an isolated lane with no merging nor splitting, traveling on a straight road with no vertical curves nor inclement weather. Flow is provided at different platoons sizes and speeds for four primary cases of different brake variances and delays. For conditions where platooning is initially unsafe, the amelioration protocols of brake derating and collaborative braking are used, and their potential to reduce flow is quantified.

As discussed above the lack of real-world data makes it difficult to verify the models using a statistical analysis. However, it is possible to show the model’s reasonableness by comparing its results those that were obtained by AHS work. The capacity model meets this reasonableness test, as the results presented here corroborate with AHS results using similar input values: i.e. approximately 6,000-9,000\text{veh/hr} for the ”better-case” scenarios presented in the AHS work as compared with 8,991 and 6,119\text{veh/hr} for Cases 3 and 4, respectively, as estimated in this research. Additionally, compared with the AHS models, the model developed here has the benefit of being easy to replicable and manipulate.
5.2 Research Impacts

This section first provides an overview of the major findings of this research and then focuses on the impacts of the results obtained from the collision and capacity models.

5.2.1 Overview

The three most important conclusions from the research presented in this thesis are:

- Ensuring safe platooning during emergency braking scenarios requires small communication delays and, most importantly, tight brake variances.

- If platooning can be made safe without amelioration, then very high capacities are possible at reasonable speeds and platoon sizes: for example 12,504 veh/hr at a speed of 30 m/s and a platoon size of 10 vehicles.

- Though the general flow outlines for automated platooning remain consistent with traditional roadways and models, the consideration of different platoon sizes results in functional differences, such as an increased optimal speed.

Overall, this work seeks to show the effects of varying different parameters (vehicle speed, platoon size, brake variance, communication delay, and maximum acceptable collision speed) on both the vehicle separations that are unsafe for platooning vehicles (the USZ) and on the capacity of those platooning vehicles. AET is a technology for the future of transportation, promising travel that is faster, safer, cleaner, and less congested. As a future technology, there is no single, unique system that one can point to. Changing the above AET parameters can result in huge changes in AET safety and capacity. For example, capacity can range from its theoretical maximum of 18,000 veh/hr for an infinitive platoon size at a speed of 30 m/s, to 12,504 veh/hr for 10 vehicle platoons under the high performing system given by Case 1 (small communication delays and tight brake variances) to 1,137 veh/hr for a single autonomous (non-platooning) vehicle under Case 4, which has higher delays and greater brake variance than Case 1.
As alluded to here, platooning itself is shown to be the most important factor in ensuring high capacity systems. Even in lower performing systems like Case 4, capacities in excess of 10,000 veh/hr are possible at large enough platoon sizes. However, there are explicit tradeoffs between safety and capacity. Case 4 can achieve these capacities only if platooning can be considered safe, which as this research shows may prove to be quite difficult. The amelioration protocols of brake derating and collaborative braking can help, but as described in Chapter 4, both of these could be difficult to practically implement in real-world scenarios, and their stricter variants (cumulative brake derating and exponential collaborative braking) can severely reduce capacity. For example the capacities under Case 4 with 10 vehicle platoons and a speed of 30 m/s range from 9,414 veh/hr if no amelioration were used to 6,119 veh/hr with collaborative braking to 2,388 veh/hr with exponential collaborative braking.

The above two amelioration protocols are endogenous approaches, altering the AET system to improve safety within a given set of confines. However, an exogenous approach is also possible by evaluating and altering the definition of safe itself. No transportation system can ever be 100% safe, with zero collisions, injuries, or fatalities. Unexpected incidents can and will occur, and any system designer for any infrastructure project, from roads to bridges to oil rigs, must decide on reasonable tradeoffs between safety, cost, and system performance. For AET, two such areas that this research addresses are maximum acceptable collision speed and maximum acceptable unsafe collision likelihood; the latter value can only be obtained from the stochastic collision model. The results presented in this research show that variations in either of these parameters can have a large effects on the USZ, and by extension platooning capacity. For example, when leader and follower braking and \(-10 \text{ m/s}^2\) and \(-8 \text{ m/s}^2\) respectively, communication delay is 0.2s, and vehicle speed is 30 m/s, changing maximum acceptable collision speed from 2.5 to 3.5 m/s increases the maximum allowable intraplatoon separation from 0.76m to 2.26m

### 5.2.2 Collision Model Impacts

Automated platooning was first proposed in 1979, but relatively few models have been
developed to define the USZ’s lower bound \((S_{iv,\text{max}})\). This research defines the necessary parameters for safe platooning, and the results show that fairly tight tolerances are required, especially for tight brake variances. Small communication delays also play a major role, but the collision models show that these tend to be less important. For example, assume that no amelioration is used and that the control model can maintain a steady-state intraplatoon vehicle separation \((S_{iv,\text{ach}})\) of 1m and a \(\Delta \text{v}_{\text{safe}}\) of \(2.5\text{m/s}\). In this scenario, delay can never be larger than 0.25s, which is a reasonably large delay for wireless communications and electric vehicles. However, even if there is zero delay, brake variance can never be greater than \(-6.875\text{m/s}^2\) for follower maximum braking and \(-10\text{m/s}^2\) for leader maximum braking.

Previous AHS models require a substantial brake amplification factor of 30% to enable tighter effective brake variance and overcome their initial assumptions of a mixed vehicle fleet powered by internal combustion engines [64]. However, as explained in detail Chapter 4, there are practical difficulties with this approach, especially the potential to drastically reduce vehicle flow. As proposed by AET, electric vehicles should have more predictable responses with shorter delays, further aided by the advent of wireless communications that were not available during AHS. Still, these results indicate the importance of maintaining a single vehicle type in platoons and requiring that strict maintenance protocols be maintained, such as a check-in procedure to the platooning lanes, in order to ensure a tight braking variance. This is true for dedicated automated platooning systems such as AET, but also platoons in mixed traffic as proposed by Europe’s SARTRE and KOVOI projects.

Where platooning is initially unsafe, brake derating and collaborative braking enable safe intraplatoon interactions when the brake variance and total delay are initially unsafe, but this comes at the expense of increased interplatoon separations. Therefore, when these amelioration protocols are used, flow degrades, though the effect is relatively small when only minor amelioration is needed. In general, brake derating is a superior option to collaborative braking, allowing for higher capacities, though at higher speeds and platoon sizes, collaborative braking becomes more attractive. This is consistent with the results of the collision model, which showed brake variance to generally be the most important factor.
The collision model also evaluates the effects of a third amelioration protocol, increasing $\Delta v_{safe}$. This does not have a major effect on $S_{ip,min}$, so will not substantially change capacity in the no amelioration cases. However, it is a highly attractive option when amelioration is required, as small changes in its value can substantially increase $S_{iv,max}$.

### 5.2.3 Capacity Model Impacts

Automated vehicle platooning as proposed by AET allows for very high vehicle flows, especially where no amelioration is needed. This has the potential to fundamentally change roadway transportation in America: reducing congestion, increasing safety, and improving emissions. Automated platooning could combine these benefits, which are usually associated with mass transit, with the flexibility of the private automobile, helping to solve the last-mile problem. Vehicles could travel on high-capacity AET lanes that mimic the performance of public transit, and then check-out of the system and drive the final mile or two to their destination on traditional roads under manual control. This would provide a viable high-capacity transportation option to medium-density areas, such as Los Angeles, that cannot support large levels of traditional transit but also lack the road infrastructure to support the resulting high number of private vehicle trips.

While platooning promises higher vehicle flows than are possible on current roadways, the general speed-density-flow relationships remain consistent with traditional roadways and models. As speed increases, flow also increases to a capacity $q_{max}$ at an optimal speed $v_o$, then begins to decline as speed increases further. However, the consideration of platooning results in functional differences. First, at larger platoon sizes the $v_o$ may become higher than the design speed of the roadway, such as $70m/s$, so that for practical purposes flow can be seen as monotonically increasing as a function of speed. Here, capacity occurs at the highest safe speed for the roadway. Additionally, in a dynamic system, it is possible to increase density and thus vehicle flow in response to increased demand without decreasing speeds by instead increasing average platoon size.

As platoon size increases for a given speed, flow asymptotically increases to theoretical maximum. Therefore, no optimal platoon size exists to maximize capacity; the highest
allowable platoon size will allow for the highest capacity. This is true for the modeled results except where cumulative brake derating or exponential collaborative braking is used, in which case optimal platoon sizes can exist. This research analytically proves this and provides illustrative examples. These stricter amelioration protocols can also substantially reduce capacity, approaching the levels of traditional roadways.

5.3 Future Work

Most important for future work is to test and expand upon the assumptions made in developing the collision and capacity models. For example, the real-world implementation of brake derating and collaborative rating must be better shown in order to allow for more accurate models. Initially, as with AHS, brake derating is treated as occurring only a single time within a platoon for a single leader-follower pair. By comparison, with cumulative brake derating every vehicle in a platoon must derate its braking with respect to the vehicle in front of it. This is arguably a more realistic assumption, and given the large difference in flows using the two different types of derating, it is important to identify precisely how brake derating would work in a real-world scenario. Exponential collaborative braking produces a similarly large decrease in flow by implying that the effective interplatoon delay increases exponentially, rather than linearly, as a function of platoon size. This could represent increased delay due to packet interference as more intraplatoon communications are sent as platoon size increases, crowding out interplatoon communications.

More work is also needed to identify what $S_{iv,ach}$ values are reasonably achievable. This work assumes 1m for the capacity modeling, and AHS uses 2m, but neither value is shown in real-world testing. For this and the amelioration protocols, more research on the platooning control models, which describe vehicle-to-vehicle and vehicle-to-infrastructure communications and responses during steady-state flow, can help solidify the assumptions made here. In particular, while this research assumes a constant $S_{iv,ach}$, it is necessary to show if it varies as a function of speed, which in turn could be a limiting factor on maximum safe platooning speed. The removal of human drivers allows for the potential for higher speeds than on current roadways, which could provide decreased travel times and
higher capacities, and the limiting safety factors for these higher speeds must be identified. To further improve safety, since this work shows the large potential benefits of increasing $\Delta v_{\text{safe}}$, more research into vehicle design and collision dynamics would help give the highest value that is reasonably achievable.

Additionally, as this research considers the ideal case of a single automated lane with no merging/exiting nor vertical or horizontal curves, more work is needed to see what factors could degrade flow from its optimal level. The potential effects of merging and exiting are especially important; past AHS work showed that this could reduce capacity by up to 70% [94]. More work is also needed to identify both the acceptable safety levels and the type and frequency of emergencies for which automated platooning systems must account. The research presented here only considers a single emergency braking scenario, but a wide variety of potentially dangerous situations exist, such as temporary loss of communications, unexpected short-term braking, and tire blowouts. Considering a wide range of conditions could result in a multi-regime collision model, and would help quantify more precisely the necessary parameters for safe platooning. Lastly, live demonstrations, both full-size and table top, would be very useful in justifying assumptions and calibrating and modifying models. These demonstrations can illustrate how communication systems, vehicle dynamics, and emergency protocols respond to real-world stimuli.
REFERENCES


