Predicting quaking aspen stand dynamics in Minnesota

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Predicting Quaking Aspen Stand Dynamics in Minnesota

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ABSTRACT. This paper presents equations for predicting future basal area, number of trees, and total cubic-foot volume of aspen stands in Minnesota. The modeling methodology uses a fully-stocked yield table for quaking aspen as a density standard. A relative density change equation based on observed growth from permanent plots provides the basis for predicting the future relative density and therefore the future basal area, number of trees, and volume. The equations are easy to apply and require only site index, age, and beginning basal area, number of trees, and volume. North. J. Appl. For. 10(1):20-27.

Quaking aspen is the most abundant commercial tree species in Minnesota (Jakes 1980). The aspen forest type covers approximately 34% of the commercial forestland in the state. This research focuses on predicting changes in volume, basal area, and number of trees of stands currently having a plurality of aspen. In this paper we describe the density standard models, the aspen permanent sample plot data available to us, and the relative density change models. We then test the different models, show how to use the models, and close with a discussion of the limitations of the aspen models developed.

Density Standard Models

Density standards for total cubic-foot stem volume, basal area, and number of trees, for site index (values >50) and age cells were taken from Brown and Gevorkiantz (1934). The equations given below represent the relations expressed in their quaking aspen Table 154 (temporary plots used in developing the table are from across the Lake States):

\[
V_{BG} = (-11908 + 416.5 \times s - 1.946 \times s^2) \left(1 - e^{-0.02556 \times age}\right)^{1.970} \tag{1}
\]

\[
BBG = (-177.4 + 8.801 \times s - 0.05289 \times s^2) \left(1 - e^{-0.02425 \times age}\right)^{0.8083} \tag{2}
\]

\[
TBG = 97.16 + 33773 \times e^{-0.02889 \times s - 0.04826 \times age} \tag{3}
\]

where
- \(s\) is site index (height in ft at base age 50),
- \(age\) is mean tree age,
- \(V_{BG}\) is total ft\(^3\) vol/ac in trees 1 in. dbh and larger,
- \(BBG\) is total basal area (ba)/ac in trees 1 in. dbh and larger, and
- \(TBG\) is number of trees/ac 1 in. dbh and larger.

Statistics summarizing the calibration of these equations using nonlinear regression are listed in Table 1.

Real Growth Series to Calibrate Relative Density Change Equations

Remeasured permanent growth plots are required to calibrate relative density change equations. Two North Central Forest Experiment Station research studies, NC-96 and NC-52, sup-
### Table 1. Calibration statistics for density standard equations fit to the quaking aspen yield table for trees 1 in. dbh and larger in Brown and Gevorkiantz (1934). Data consist of yields tabulated for 27 age-site cells with site index ranging from 50 to 80 ft.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Property</th>
<th>Measurement units</th>
<th>Adjusted $R^2$</th>
<th>$S_e$ a</th>
<th>$\bar{Res}$ b</th>
<th>$RE$ c (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>Total volume</td>
<td>$ft^3$ ac$^{-1}$</td>
<td>0.999</td>
<td>60.2</td>
<td>1.3</td>
<td>0.44</td>
</tr>
<tr>
<td>(2)</td>
<td>Total basal area</td>
<td>$ft^2$ ac$^{-1}$</td>
<td>0.996</td>
<td>1.91</td>
<td>0.01</td>
<td>0.004</td>
</tr>
<tr>
<td>(3)</td>
<td>Number of trees</td>
<td>ac$^{-1}$</td>
<td>0.996</td>
<td>48.5</td>
<td>-0.17</td>
<td>-0.70</td>
</tr>
</tbody>
</table>

a $S_e$ = Standard error of the estimate.

b $\bar{Res}$ = Mean residual $= \frac{\sum_{i=1}^{n} (obs - pred)}{n}$

c $RE$ = Mean relative error $= \frac{\sum_{i=1}^{n} \left( \frac{obs_i - pred_i}{obs_i} \right)}{n}$

Applied most of the long-term remeasurements. Both studies are on the Pike Bay Experimental Forest, Chippewa National Forest. Additional plot remeasurements came from Forest Inventory and Analysis permanent plots on the Chippewa National Forest, and from remeasurements of various studies located in north-central Minnesota. Figure 1 shows plot locations. Few plots are older than 50 yr (Table 2).

NC-96.—Data are from remeasurements of aspen thinning plots on the Pike Bay Experimental Forest. Quaking aspen site index ranged from 75 to 82 across the study area. The study was established in 1936 in a 13-yr-old stand. Treatments consisted of 6 different thinning intensities plus a control, each covering an area of 0.6 ac. There were no replications of thinning treatments. In the spring of 1946 a second thinning removed approximately 4.5 cords/ac of pulp and post material from each plot except the control plot. Stand summaries for the entire treated areas, based on a 10% sample of the area, were available for ages 13, 18, 23, 28, and 33. At age 33, four 1/7 ac permanent plots were established in each treatment and the control, each tree numbered, and its location mapped. Each of the four plots was summarized individually based on remeasurements at ages 33, 38, 43, 48, and 53. In summary, 9 measurements were available, extending from age 13 to 53, covering the period 1941–1981. The study area was harvested in the early 1980s.

NC-52.—The NC-52 remeasurements came from a thinning study established in a 10-yr-old aspen stand on the Pike Bay Experimental Forest in 1953. Half of the treatment area was thinned to about 750 trees/ac, 1/4 was left untreated, and 1/4 had hardwoods other than quaking aspen removed. Diameter at breast height was recorded for trees on forty 1/10-ac plots at stand ages 10, 19, 24, 29, 34, 39, and 46. Before measurement at age 34, individual tree identity was not recorded, so stand tables with 1 in. diameter classes formed the basis of the plot totals. Plot totals for the last three measurements were from diameters measured to the nearest 0.1 in. on numbered trees. The study is still active.

North-Central Minnesota Study Plots.—Additional data were obtained from a variety of studies established in north-central Minnesota (Table 3). The first study listed evaluated the effect of prescribed burning on a clearcut aspen stand. All other studies examined the effect of thinning. When available, we included control plots receiving no treatment as well as the thinned plots.

Chippewa NF plots.—Forest inventory plots located throughout the Chippewa National Forest were analyzed looking for plots with 50% or more quaking aspen (by basal area) which had no partial cutting during the measurement period 1970 to 1980. We found 13 plots that met the criteria. Each plot is a standard FIA design: 10 points, each point sampled with a 37.5 factor prism for trees 5 in. dbh and larger. Three 1/300-ac plots were sampled for trees smaller than 5 in. dbh. Per acre estimates of basal area and number of trees were obtained for each plot using standard FIA data analysis procedures.

Because data came from several sources, there was not a consistent method of calculating total cubic-foot volume per tree. To use a consistent method for each data source, we calculated total cubic-foot volume per acre for the plot using

---

![Figure 1. Location of long-term permanent growth plots in Northern Minnesota from which relative density change values were calculated (hatched area), and plots used to develop the stand volume Equation (4) (points).](image-url)
Table 2. Frequency of available permanent plot observations by site and age class for all data sources.\

<table>
<thead>
<tr>
<th>Site index class</th>
<th>46-55</th>
<th>56-65</th>
<th>66-75</th>
<th>76-85</th>
<th>86-95</th>
<th>96-105</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age class</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-19</td>
<td>1</td>
<td>31</td>
<td></td>
<td></td>
<td>71</td>
<td>9</td>
</tr>
<tr>
<td>20-29</td>
<td></td>
<td></td>
<td>28</td>
<td>71</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>4</td>
<td>17</td>
<td>62</td>
<td>71</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>40-49</td>
<td>4</td>
<td>1</td>
<td>87</td>
<td>71</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td>4</td>
<td>12</td>
<td>81</td>
<td>71</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>60-69</td>
<td>3</td>
<td>13</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>70-79</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80-89</td>
<td></td>
<td>27</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

a Boxed cells are combinations of site and age classes for which Brown and Gevorkiantz had temporary plot information in constructing their normal yield table. In parentheses are number of plots used by Brown and Gevorkiantz. Note the absence of permanent plot measurements in older stands on higher sites.

Equation (4) (Schlaegel 1971). Figure 1 shows the geographical distribution of plots used by Schlaegel to develop Equation (4).

\[ V = 0.41898 \times B \times h \] (4)

where

- \( B = \) total stand basal area in ft²/ac
- \( h = \) average total height in feet of dominants and codominants

Average height of dominants and codominants \( h \) was computed using age and site index in the Lundgren and Dolid (1970) equation:

\[ h = s \times (1.46 - 1.4337e^{-0.02274 \times \text{age}}) \] (5)

Relative Density Change Equation Calibration

Leary and Smith (1990) and Leary (1991) detail a method to calibrate relative density change equations. Briefly, the method requires calculations of relative density for each stand property (volume, basal area, and number of trees) on each permanent plot. Relative density is the ratio of the plot’s stand property to the standard [in our case calculated with Equations (1), (2), or (3) for the same age and site index]. The difference between relative density at the end of the measurement interval and relative density at the beginning of the measurement interval gives an estimate of relative density change over the period. Due to the differing measurement intervals, relative density changes were standardized to the most common interval of 5 yr in length (71% of the intervals) by linear interpolation (23% of the intervals) or extrapolation (6% of the intervals).

Several relative density change equations were tested, starting from experiences in Leary (1991) and Leary and Smith (1990). Models selected produced predictor variables with greater statistical significance than the alternatives and did not show serious departures from the assumptions for linear regression. The calibration data consisted of 387 observed density changes. One number of trees relative density change was clearly deviant and was not used in the calibration of number of trees equation. The equations are:

\[ \frac{\Delta RD_v}{\Delta t} = a_1 \times s + a_2 \times RD_v \] (6)

\[ \frac{\Delta RD_{ba}}{\Delta t} = b_1 \times RD_{ba} + b_2 \times RD_{ba}^2 + b_3 \times s + b_4 \times \text{age} \] (7)

\[ \frac{\Delta RD_t}{\Delta t} = c_1 \times s + c_2 \times RD_t \] (8)

where

- \( a_1, a_2, b_1, b_2, b_3, b_4, c_1, c_2 \) are numerical contents
- \( s \) is site index,
- \( \text{age} \) is mean tree age,
- \( RD_t \) is volume relative density (observed volume divided by

Table 3. Selected information for aspen data from north-central Minnesota studies. The first column lists a reference that further describes the study.

<table>
<thead>
<tr>
<th>Author</th>
<th>County</th>
<th>Measurement intervals</th>
<th>Site index</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perala 1974</td>
<td>Cass</td>
<td>7</td>
<td>70</td>
<td>2-10</td>
</tr>
<tr>
<td>Hubbard 1972</td>
<td>Koochiching</td>
<td>6</td>
<td>90</td>
<td>7-24</td>
</tr>
<tr>
<td>Noreen 1968</td>
<td>Koochiching</td>
<td>6</td>
<td>80</td>
<td>4-20</td>
</tr>
<tr>
<td>Perala &amp; Laidly 1989</td>
<td>St. Louis &amp;</td>
<td>24</td>
<td>83-103</td>
<td>5-21</td>
</tr>
<tr>
<td></td>
<td>Koochiching</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perala 1978 (H-70)</td>
<td>Cass</td>
<td>54</td>
<td>73</td>
<td>30-62</td>
</tr>
<tr>
<td>Perala 1978 (M-371)</td>
<td>Cass</td>
<td>8</td>
<td>85</td>
<td>37-47</td>
</tr>
<tr>
<td>Perala 1978 (NC-52)</td>
<td>Cass</td>
<td>4</td>
<td>80</td>
<td>10-34</td>
</tr>
<tr>
<td>Perala 1978 (NC-93)</td>
<td>Cass</td>
<td>4</td>
<td>60-75</td>
<td>16-21</td>
</tr>
<tr>
<td>Unpub.</td>
<td>Koochiching</td>
<td>19</td>
<td>80</td>
<td>15-39</td>
</tr>
</tbody>
</table>

22 NJAF 10(1) 1993
Table 4. Calibration results for relative density change equations. Units for volume and basal area are cubic feet and square feet.

<table>
<thead>
<tr>
<th>Overall regression</th>
<th>Vol./ac</th>
<th>Basal area/ac</th>
<th>No. of trees/ac</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth intervals</td>
<td>387</td>
<td>387</td>
<td>386</td>
</tr>
<tr>
<td>F-ratio</td>
<td>181</td>
<td>121</td>
<td>44.5</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.482</td>
<td>0.554</td>
<td>0.184</td>
</tr>
<tr>
<td>S_e</td>
<td>0.0975</td>
<td>0.0882</td>
<td>-0.0009</td>
</tr>
<tr>
<td>Res</td>
<td>-0.0013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean sq. residual</td>
<td>0.0094</td>
<td>0.0077</td>
<td>0.0109</td>
</tr>
<tr>
<td>Mean abs. residual</td>
<td>0.0697</td>
<td>0.0587</td>
<td>0.0680</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Numerical constants</th>
<th>a₁</th>
<th>a₂</th>
<th>b₁</th>
<th>b₂</th>
<th>b₃</th>
<th>b₄</th>
<th>c₁</th>
<th>c₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.00354</td>
<td>-0.2912</td>
<td>5.953</td>
<td>-7.622</td>
<td>0.00153</td>
<td>-0.00191</td>
<td>0.001275</td>
<td>-0.1453</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.0002</td>
<td>0.0196</td>
<td>1.056</td>
<td>0.819</td>
<td>0.00004</td>
<td>0.0005</td>
<td>0.0001</td>
<td>0.0153</td>
</tr>
<tr>
<td>t-ratio</td>
<td>17.9</td>
<td>14.9</td>
<td>5.64</td>
<td>-9.30</td>
<td>4.18</td>
<td>-3.39</td>
<td>9.09</td>
<td>-9.18</td>
</tr>
</tbody>
</table>

volume from the standard for the same age and site index), \( RD_a \) is basal area relative density (observed basal area divided by basal area from the standard for the same age and site index), and

\( RD_i \) is number of trees relative density (observed trees per acre divided by trees per acre from the standard for the same age and site index).

Table 4 summarizes how well the models fit the observations. The relatively poor fit for number of trees demonstrates the large variability in mortality found on small plots.

**Model Tests**

Both the density standard equation and relative density change equation are needed to predict stand dynamics. We tested the combined effect of the model for cubic volume, basal area, and number of trees in three ways:

1. comparing accuracy of predicted and observed values for periodic relative density change and final stand property (volume, basal area, number of trees).
2. calculating, i.e., deducing, selected individual future stand properties and checking the resulting numbers against published values and our collective experience, and
3. deducing selected stand property interrelations, and comparing with theory.

Accuracy tests were made using methods outlined in Rauscher (1986). Tests were done on the calibrated combinations of relative density change and density standard for each property (volume, basal area, number of trees). One hundred thirty-four observations had been randomly drawn from the database of permanent growth plot observations for accuracy testing. One hundred nineteen were 10 yr of age or older and are used in tests described below. These observations were not used to calibrate the models.

Future individual stand properties of special interest were: mean annual property change by 5 yr increments, the maximum mean annual property change value, and the age at which mean annual property change is a maximum. Thus, our tests related primarily to the first column of the modified Bakuzis matrix (Leary 1988).

Tougher tests are possible by examining stand property interdependence as shown in the interior cells of the modified Bakuzis matrix. Two relations are examined: whether the models violate the -3/2 power law of self-thinning, and whether the models violate assumptions regarding the A and B lines in the stocking guide framework.

Our equations estimate 5 yr change. In tests described below we used the equations as they would be applied in practice. For example, if a plot remeasurement interval was 9 yr, the equations were iterated twice, but only 4/5 of the last increment used.

**Accuracy Tests**

To test models for volume, basal area, and number of trees we:

1. divided the initial and final stand property for the 119 remeasurement intervals by appropriate values from the density standard to obtain initial and final observed relative density,
2. predicted relative density change for the measurement interval,
3. added predicted relative density change to initial relative density to obtain final predicted relative density, and
4. multiplied final predicted relative density by the standard to estimate final predicted stand property.

In most cases, the measurement interval was 5 yr. Results are given in Table 5. The models for volume, basal area, and number of trees produce unbiased estimates of relative density change and standing crop (because each 95% confidence interval about the bias contains zero). As is usual, percentage errors are larger for growth (relative density change) than yield (standing crop).

**Deduction Tests**

Our deduction test for volume was simply to select sets of initial conditions, make projections for a number of 5 yr periods, and calculate mean annual volume increment at each period until MAI reached a maximum and started to decrease. We are particularly interested in the age at which MAI peaked, the value of MAI at that age, and how close the projections came to the 80,80,80 (80 cords/ac on site index 80 at age 80) condition reported in Brown and Gevorkiantz (1934). Table 6 presents the results from the test deductions. Numbers in parentheses under maximum MAI and age 80 columns are cord-equivalents, at 79 ft³/cord.

The qualitative directions shown in Table 6 seem correct; age at maximum MAI is later if starting relative density is small and
Table 5. Test of models showing accuracy measures of observed and predicted relative density change and final stand properties for
volume, basal area, and number of trees (dbh ≥ 1 in.). The confidence interval is for a two-sided probability of 0.05.

<table>
<thead>
<tr>
<th></th>
<th>Relative density change error</th>
<th>Final standing crop error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute</td>
<td>Percent</td>
</tr>
<tr>
<td>Total volume</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bias (obs. - pred.)</td>
<td>0.0117</td>
<td>47.2</td>
</tr>
<tr>
<td>confidence interval ±</td>
<td>0.0167</td>
<td>56.2</td>
</tr>
<tr>
<td>Total basal area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bias (obs. - pred.)</td>
<td>0.0094</td>
<td>46.0</td>
</tr>
<tr>
<td>confidence interval ±</td>
<td>0.0146</td>
<td>406.0</td>
</tr>
<tr>
<td>Number of trees</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bias (obs. - pred.)</td>
<td>0.0104</td>
<td>53.7</td>
</tr>
<tr>
<td>confidence interval ±</td>
<td>0.0173</td>
<td>928.0</td>
</tr>
</tbody>
</table>

maximum MAI is higher on better sites. Qualitative behavior
differs from expected in that the age of maximum MAI is greater
on good than poor sites. Ages at which mean annual increment
are maximum are reasonable. The model comes close to meeting
the (80,80,80) condition (Table 6).

The deduction test for basal area dynamics was similar to that
for volume. We calculated cubic volume mean annual incre-
ment[by calculating basal area and using Equations (4) and (5)],
and determined the age at which it is a maximum and MAI at
maximum (Table 7). Ages of maximum mean annual increment
are very similar to those obtained from projected volume for
stands fully stocked at age 10. When initial density is lower, the
basal area equation predicts an earlier peak than shown in Table
6. The decline in basal area at later ages is consistent with
Zehngraaff (1947), who indicates basal area decline after age 60.

An important criterion for number of trees per acre deduction
test is the Sukatchev effect (Harper 1977): trees are lost to
mortality more quickly on good sites than on poor sites, if both
start with the same number of trees. The model does not violate
the effect at age 40 or 60, within the range of our calibration data,
but does by age 80 (Table 8).

Stand Property Interrelations

The -3/2 Power Law of Self-Thinning.—The -3/2 power law
of self-thinning asserts that tree frequency and volume per tree
are closely linked in evenaged monocultures as follows:

\[
\text{volume/ tree} = k_n \text{(number of trees)}^{3/2}
\]

where \( k_n \) is approximately -3/2.

Because quaking aspen often grows in evenaged monocul-
tures, one might expect that it behaves according to Equation (9).

Indeed, Perala and Cieszewski (in review) showed quaking
aspen stands do self-thin according to the -3/2 rule. In our test we
used reasonable initial conditions, applied the relative density
change equations for volume, Equation (6), and number of trees
per acre, Equation (9), to estimate change in each property, and
checked if the resulting trajectory gives evidence of self-thinning.
Because actual stands obey the rule, our models should
produce it. If the combined use of the models [Equations (6) and
(8)] do not produce self-thinning, then one or the other is
incorrect, even if each performed reasonably well when tested
separately. Results are shown in Figure 2. Clearly, the equation
combinations show our models predict a period of self-thinning.

Trajectories in the Stocking Guide Framework.—A second
test of stand property interrelations is to use the number of trees
per acre and basal area prediction equations to examine stand
trajectories in

\[
< \text{basal area} - \text{number of trees} >
\]

space, the framework in which evenaged stand stocking guides
is expressed. Figure 3 shows trajectories for NC-52 plots used in
validation testing. Upper limits of basal area range from 130 to
159 ft²/ac. Perala’s aspen stocking guide (Perala 1986) shows
that upper management level peaks at about 170 ft²/ac and that
maximum density peaks at about 220 ft²/ac. Figure 3 shows the
basal area increase, as the number of trees decreases, found for
other stocking guides (Gingrich 1967, Benzie 1977, Roach
1977). We conclude that the model produces reasonable shapes
for the trajectories in the stocking guide framework, but the
maximum basal areas attained may be low. Although not shown
in Figure 3, regeneration of only, say, 400 trees/ac at age 10, will

Table 6. Summary of mean annual volume increment deduction test. All initial conditions were applied at age 10. Site index designates
height at age 50, and rd designates volume relative density.

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th>Age at MAI</th>
<th>Maximum MAI</th>
<th>Volume in ft³/ac at age (cd/ac)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>ft³/ac (cds/ac)</td>
<td>40</td>
</tr>
<tr>
<td>Site index 60</td>
<td>40</td>
<td>48.3 (0.61)</td>
<td>1926</td>
</tr>
<tr>
<td>rd 1.0</td>
<td>50</td>
<td>46.0 (0.58)</td>
<td>1798</td>
</tr>
<tr>
<td>rd 0.6</td>
<td>57</td>
<td>44.7 (0.56)</td>
<td>1670</td>
</tr>
<tr>
<td>Site index 80</td>
<td>47</td>
<td>91.6 (1.16)</td>
<td>3629</td>
</tr>
<tr>
<td>rd 1.0</td>
<td>55</td>
<td>89.3 (1.13)</td>
<td>3440</td>
</tr>
<tr>
<td>rd 0.2</td>
<td>57</td>
<td>87.6 (1.11)</td>
<td>3251</td>
</tr>
</tbody>
</table>
Table 7. Summary of mean annual volume increment predicted by the relative density change equation for basal area. All initial conditions were applied at age 10. Site index designates height at age 50, and rd designates basal area relative density.

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th>Age at MAI maximum</th>
<th>Maximum MAI ft³/ac (cds/ac)</th>
<th>Basal area in ft²/ac at age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site index 60</td>
<td>40</td>
<td>55.8 (0.71)</td>
<td>100</td>
</tr>
<tr>
<td>rd 1.0</td>
<td>40</td>
<td>54.5 (0.69)</td>
<td>98</td>
</tr>
<tr>
<td>rd 0.6</td>
<td>40</td>
<td>47.9 (0.61)</td>
<td>84</td>
</tr>
<tr>
<td>Site index 80</td>
<td>50</td>
<td>96.4 (1.22)</td>
<td>130</td>
</tr>
<tr>
<td>rd 1.0</td>
<td>45</td>
<td>95.2 (1.21)</td>
<td>135</td>
</tr>
<tr>
<td>rd 0.6</td>
<td>45</td>
<td>88.8 (1.12)</td>
<td>114</td>
</tr>
</tbody>
</table>

give an increase in number of trees to about 450 from age 10 to 15, but then a monotonic decrease in number of trees.

Implementation

For each stand property, the equations for the density standard and relative density change operate as a linked pair—a different density standard would require a different equation for relative density change. To apply our equations requires temporary inventory plot measurements of age, site index, and any one, or all three, stand properties (total cubic-foot volume, basal area, and number of trees) for trees 1 in. dbh and larger. The current relative density is the ratio of the value of the current stand property to the value of the density standard [Equations (1), (2), or (3)] with the same age and site index. Calculate relative density change [Equations (6), (7), or (8)] and add it to the current relative density to predict a new relative density. The product of the new relative density and the density standard [Equations (1), (2), or (3) using the original age + 5] is the predicted stand property. The procedure is repeated for additional 5-yr increments. When the interval is less than 5 yr, proportionately reduce the calculated relative density change.

For example, Table 9 shows, first in spreadsheet notation and then with numeric values, the prediction of number of trees at age 35 starting from a 28-yr-old, site index 78 aspen stand with 1000 trees/ac. The predicted number of trees is 697 trees/ac.

Discussion and Conclusions

Quaking aspen occurs in mixed stands with a number of other species. While most of the permanent growth plots used in calibration and validation had over 90% quaking aspen by basal area, the range extended to as little as 50%. We plotted residuals from fitting relative density change equations for each stand property against percentage quaking aspen for remeasurement intervals and found no trend. We also plotted residuals in the validation data set against percentage quaking aspen and found no trend. This strongly suggests that the models developed are robust relative to species composition, and may be used in stands having, say, 60% (by basal area) or more quaking aspen.

Based on our analysis of total volume as a function of stand age, the models appear to be overestimating stand volume somewhat (more than 80 cords/ac at age 80 and site index 80 in Table 6). Some sources of possible overprediction are:

1. We ignored explicit inclusion of defect in volume computations,
2. Our permanent plot remeasurements came from an area of Minnesota having good growing conditions, and
3. Study areas were subjectively selected and then given special treatment during the study period.

Each possibility is addressed briefly.

1. Our predictions assume all trees are solid wood. Cull and defect were ignored in the volumes predicted. Hahn (1984) estimates that Lake States quaking aspen trees classified by Forest Inventory and Analysis as growing stock have about

**Figure 2.** Check of predicted number of trees/ac and volume per tree stand property interdependence. Initial conditions were 10 yr, 600 trees/ac, and 250 ft³. The regression was fit to observations after omitting predictions for the first 5 time steps. First plotted point corresponds to age 10, with later points having 5-yr age increments.

![Figure 2](image_url)
combinations of number of trees per acre and basal area at age 10 from NC-52 plots not used in calibrating relative density change equations. Basal area is projected using Equation (7) and number of trees/ac with Equation (8). Tick marks on the trajectories signal 5 yr elapsed time.

5% cull. Cull percentage in other tree classes (rough, rotten) are much larger. W.B. Smith (personal communication) estimates about 90% of trees in aspen stands are growing stock. To estimate cubic volume of sound wood, our predictions must be decreased by about 15%.

There is an inadequate geographical distribution of the permanent growth plots used in calibrating the relative density change equation. The omission is not purposeful; all known sources of permanent growth plots having tree measurements to 1.0 in. dbh were used in our calibration. A result is that a “complete” range of the soils and climate of northern Minnesota is not included. Most plots were located in the portion of north central Minnesota, known as a generally good, although not the best, aspen growing area in Minnesota.

The method of selecting the physical location of studies 96 and 52 is unknown to the authors, and probably subjective. In similar cases, subjective location of scientific studies results in above average site qualities being selected. The use of subjective criteria in locating research study plots, as well as subsequent plot treatment, leads to what Bruce (1977) calls the “research plot effect” bias in growth studies. In at least one study we used, plots received special treatment during the study and should be included in the research plot effect; i.e., historical records for NC-52 indicate it was aerially sprayed, along with recreational and administrative sites, during a severe outbreak of forest tent caterpillar in 1952 (letter to the NC-52 study files by M.L. Heinselman 1952). The amount of the enhanced growth, sprayed over unsprayed, is not known although significant slowing of quaking aspen height growth is evident for the early 1950s in unpublished stem analysis information collected in the Pike Bay area.

When possible sources of overestimation are added—they would seem to be conformable for addition—estimates given by our projection models may approach 25% too high (say 15% for cull and 10% for “research plot effect”). Users should check our models against their own permanent growth plot remeasurement information to calculate their own reduction factors or to calibrate their own relative density change equations. When no permanent growth plots are available, we suggest a 15–25% reduction to give a conservative yield figure.

**Literature Cited**


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**Table 9.** An example showing how to predict number of trees at age 35 for an aspen stand currently 28 yr old, with 1000 trees/ac and site index 78. Equation form and parameters are described in the paper. The first part of the table is in spreadsheet format and the last part shows the results of the calculations.

<table>
<thead>
<tr>
<th>Age</th>
<th>Density standard</th>
<th>Starting density</th>
<th>Starting relative density</th>
<th>5-yr relative density change</th>
<th>Projected relative density</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28</td>
<td>Eq. (3)</td>
<td>Observed</td>
<td>C1/B1</td>
<td>Eq. (8)</td>
</tr>
<tr>
<td>2</td>
<td>33</td>
<td>Eq. (3)</td>
<td>F1*B2</td>
<td>C2/B2</td>
<td>Eq. (8)</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>Eq. (3)</td>
<td>F2*B3</td>
<td>0.9846</td>
<td>-0.0436</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>1016</td>
<td>1000</td>
<td>0.9410</td>
<td>0.9410</td>
</tr>
<tr>
<td></td>
<td>33</td>
<td>818.7</td>
<td>770</td>
<td>-0.0373</td>
<td>0.9261</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>752.3</td>
<td>697</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


