A Study of the Variability Versus the Assumed Constancy of Manning's $n$

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A STUDY OF THE VARIABILITY VERSUS THE ASSUMED CONSTANCY OF MANNING’S n

by

Tyler G. Allen

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Civil and Environmental Engineering

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2014
ABSTRACT

A Study of the Variability Versus the Assumed Constancy of Manning’s $n$

by

Tyler G. Allen, Doctor of Philosophy

Utah State University, 2014

Major Professor: Blake P. Tullis
Department: Civil and Environmental Engineering

Quantifying hydraulic roughness coefficients is commonly required in order to calculate flow rate in open channel applications. An assumption typically coupled with the use of Manning’s equation is that a roughness coefficient ($n$) that is solely dependent upon a boundary roughness characteristic ($k$) may be applied. Even though Manning reported unique values of $n$ and $x'$ (the exponent of the hydraulic radius in Manning’s equation) for each of the different boundary roughness materials he tested, he chose $x' = 2/3$ as representative, assumed a constant $n$ value, and suggested that it was sufficiently accurate.

More recent studies have suggested that in addition to $k; R_h, S_o$, and $Fr$ can influence $n$. While research points to situations where $n$ may vary, it is always a temptation to simply apply the constant $n$ assumption especially in the case of more complicated channels such as composite channels where different roughness materials line different parts of a channel cross section.

This study evaluates the behavior of $n$ as a function of $Re, R_h, k, S_o$, and $Fr$ for four different boundary roughness materials ranging from smooth to relatively rough in a rectangular tilting flume. The results indicate that for the relatively rough materials $n$ is best described by its relationship with $R_h$ where it varies over a lower range of $R_h$ but approaches and at a point
maintains a constant value as $R_h$ increases. The constant value of $n$ is attributed to both the physically smooth boundary materials and a quasi-smooth flow condition in the rougher boundary materials. The study shows that an $x' = 2/3$ (the basis of Manning’s equation) correlated to the assumption of a constant $n$ value only applies to smooth boundary roughness materials and a quasi-smooth flow condition in the rougher boundary materials; otherwise, either $n$ or $x'$ must vary.

These findings are then applied to compare 16 published composite channel relationships. The results identify the importance of applying a varying $n$ where applicable due to the potential for error in assuming and applying a constant $n$. They also indicate that the more complicated predictive methods do not produce more accurate results than the simpler methods of which the most consistent is the Horton method.
PUBLIC ABSTRACT

A Study of the Variability Versus the Assumed Constancy of Manning’s $n$

Tyler G. Allen

Culverts have traditionally been designed to a minimal size to pass a specific design flood. The traditional culvert designs may result in a localized increase in velocity which can result in a blockage of animal or fish movement across a barrier effectively changing the ecosystem surrounding a number of affected species. While hydraulic loss coefficients are relatively well defined for such traditional culverts, the National Cooperative Highway Research Program (NCHRP) identified a need for further study of these coefficients for culverts more conducive to fish and animal passage.

A research team headed by Dr. Blake Tullis of Utah State University was contracted by the NCHRP to conduct physical, numerical, and computer modeling to conduct research to be used to refine the methods used to define hydraulic coefficients involved in the design of culverts more sensitive to the surrounding environment. This dissertation was conducted as a portion of that overall program and focuses on the hydraulic coefficient Manning’s ($n$) which is used to quantify the reaction of flow characteristics to the friction caused by the roughness of the surrounding channel. A project was conducted at the Utah Water Research Lab (UWRL) in order to better define Manning’s $n$ specifically for open channel applications which would be found in fish passage culverts as part of an overall $575,000 project.
ACKNOWLEDGMENTS

Thanks to the Transportation Research Board and the National Cooperative Highway Research Program who provided funding for this research by way of the NCHRP Report 734 authored by Dr. Blake P. Tullis. This dissertation comprises specifically the NCHRP Report 734, *Hydraulic Loss Coefficients for Culverts*, Chapters 6, 7, and 8, Pages 50-78.

I would like to give special thanks to Blake Tullis whose input, time, effort, and patience have been invaluable to my work on this project. The influence of his example has had an impact on my life and will continue to guide me throughout my career. Included in that special thanks are those of my committee whom I had the opportunity to work closely with at the Utah Water Research Lab for many years prior to and during my graduate studies; namely Mike Johnson and Steve Barfuss. I would also like to thank William Rahmeyer and Gary Merkley who also provided generous and important input to the project as members of my committee.

Special thanks are also given for my parents who have always granted me support in multiple ways in getting to this point. There will always be that guiding light as they continue to push me to reach my highest goals. I will never be able to repay them for their contribution to my life and getting me to this point.

Most of all I would like to give the highest form of appreciation to my wife for her support and patience in my endeavors to complete this work. Without her support and the support of my loving children I would not be where I am today.

Tyler G. Allen
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NOTATION

\[\begin{align*}
A &= \text{sectional flow area of channel } [L^2] \\
\alpha &= \text{empirical coefficient to power law equation} \\
A_i &= \text{component flow area resulting from the partitioning of a composite-channel into subareas between the boundary roughness materials } [L^2] \\
b &= \text{empirical coefficient to power law equation} \\
C &= \text{boundary roughness coefficient} \\
C_c &= \text{boundary roughness coefficient (Chezy’s Equation)} \\
D &= \text{pipe diameter } [L] \\
D_r &= \text{representative particle diameter of the channel boundary where } r \text{ indicates the percentage of particles that are smaller than } D_r \ [L] \\
F &= \text{function of} \\
f &= \text{hydraulic roughness coefficient (Darcy-Weisbach Equation)} \\
Fr &= \text{Froude Number} \\
g &= \text{acceleration due to gravity } [L/t^2] \\
k &= \text{equivalent roughness height } [L] \\
K_n &= 1 \text{ for SI units and } K_n = 3.281^{(1-x')} (=1.486 \text{ when } x' = 2/3) \text{ for ES units} \\
L &= \text{length} \\
n &= \text{hydraulic roughness coefficient } [\text{Manning’s Equation } (x' = 2/3)] \\
n_{\text{average}} &= \text{average } n \text{ from experimental data} \\
n_c &= \text{equivalent } n \text{ value for Chezy and Darcy-Weisbach equations } (x' = 1/2) \\
n_e &= \text{composite Manning’s } n \\
n_{eq} &= \text{boundary roughness coefficient (dependent on } x' \text{ in Equation 4-6)} \\
n_i &= \text{component } n \text{ values of individual boundary roughness materials} \\
n_{opt} &= \text{equivalent } n \text{ value where } x' \text{ of Equation 4-6 is optimized}
\end{align*}\]
\[ P = \text{wetted perimeter [L]} \]
\[ PE = \text{predictive error [%]} \]
\[ P_i = \text{component wetted perimeter resulting from the partitioning of a composite-channel into subareas between the boundary roughness materials [L]} \]
\[ Q = \text{volumetric flow rate [L}^3/\text{t]} \]
\[ r^2 = \text{coefficient of determination} \]
\[ Re = \text{Reynolds number} \]
\[ R_h = \text{hydraulic radius [the flow area (A) divided by the wetted perimeter (P)] [L]} \]
\[ R_{hi} = \text{component hydraulic radius (A} / P_i \text{) [L]} \]
\[ \text{RMS} = \text{Root Mean Square [%]} \]
\[ \text{samples} = \text{number of data points sampled which contribute to the RMS} \]
\[ S_e = \text{energy grade line slope or friction slope} \]
\[ S_o = \text{channel slope} \]
\[ T = \text{width of the channel at the free water [L]} \]
\[ t = \text{time} \]
\[ V = \text{Mean channel velocity [L/t]} \]
\[ V^* = \text{Shear velocity} = (gR_hS_e)^{1/2} \text{[L/t]} \]
\[ x' = \text{exponent applied to } R_h \text{ in basic uniform-flow equation} \]
\[ y = \text{flow depth [L]} \]
\[ y' = \text{exponent to } S_e \text{ in basic uniform-flow equation} \]
\[ y_{\text{average}} = \text{average channel profile flow depth [L]} \]
\[ y_{\text{calculated}} = \text{flow depth calculated by a gradually varied flow computer program [L]} \]
\[ y_{\text{measured}} = \text{measured flow depth [L]} \]
\[ y_n = \text{normal depth [L]} \]
\[ v = \text{kinematic viscosity [L}^2/\text{t]} \]
CHAPTER 1
INTRODUCTION

GENERAL INFORMATION

Uniform flow head-discharge relationships for open channel applications correlate flow rate \( Q \) or mean channel velocity \( V \) to an energy gradient, taking into account the flow resistance associated with the channel cross sectional shape and boundary roughness. Most open channel head-discharge or uniform-flow equations are in the general form of Equation 1-1 (Chow 1959).

\[
V = CR_h^{x'} S_e^{y'}
\]  
\[\text{(1-1)}\]

In Equation 1-1, \( R_h \) is the hydraulic radius [the flow area \( A \) divided by the wetted perimeter \( P \)], \( S_e \) is the energy grade line slope or friction slope, \( C \) is a flow resistance or hydraulic roughness coefficient, and \( x' \) and \( y' \) are exponents. Under uniform flow conditions in open channel flow, \( S_e \) is equal to the channel slope \( S_o \).

The Chezy (Equation 1-2), Darcy-Weisbach (Equation 1-3), and Manning (Equation 1-4) equations represent three common open channel flow head-discharge relationships derived from Equation 1-1.

\[
V = C_c \sqrt{R_h S_e}
\]  
\[\text{(1-2)}\]

\[
V = \sqrt{\frac{8g R_h S_e}{f}}
\]  
\[\text{(1-3)}\]

\[
V = \frac{K_n}{n} R_h^{2/3} S_e^{1/2}
\]  
\[\text{(1-4)}\]

In Equations 1-2 through 1-4, \( g \) is the acceleration due to gravity; \( K_n = 1 \) for S.I. and \( K_n = 3.281^{(1-x')} \) for ES units; and \( C_c, f, \) and \( n \) are equation-specific hydraulic roughness coefficients.

Equations 1-2 and 1-3 are identical (i.e., same \( x' \) and \( y' \) values), with the exception of how the
hydraulic roughness for flow resistance is presented and a gravitational constant. Manning’s equation (Equation 1-4) is unique from the other two relationships in that \( x' = 2/3 \) instead of 1/2.

Manning (1889) based the change from \( x' = 1/2 \) to 2/3 (Equation 1-4) on empirical data with the intention that the roughness coefficient \( (n) \) would behave as a constant value dependent only on the roughness boundary of the channel. He applied this equation to over 100 data points and found the equation coupled with the constant \( n \) assumption was “sufficiently accurate” for his needs. Manning was conscious of the empirical nature of his equation (Equation 1-4) and cautioned that the use should only be applied to situations where it has been properly tested.

Chow (1959) indicated that Manning’s equation (Equation 1-4) is the most widely used uniform-flow equations due in part to its simple form. Streeter and Wylie (1979) and Yen (2002) agree that the simplicity and the popularity of Manning’s equation over Equations 1-2 and 1-3 is due to the fact that Manning’s \( n \) is often considered to be a constant value specific to a particular channel lining material type; whereas, \( C_c \) and \( f \) are generally considered to vary with stage and discharge. A committee organized by the American Society of Civil Engineers (ASCE) in 1957 called The Task Force on Friction Factors in Open Channels recommended that a nearly constant \( n \) value may be applied to open channels if the flow condition is found to be in the fully rough turbulent range defined by whether or not the hydraulic roughness coefficient is independent of Reynolds Number \( (Re) \), where \( Re = V4R/c/v \) (\( v \) represents the fluid kinematic viscosity), leaving it to be solely dependent on relative submergence. Relative submergence \( (R_c/k) \) is the ratio of the hydraulic roughness to the boundary material roughness height \( (k) \) (ASCE 1963). Chow (1959) described additional necessary conditions associated with a constant \( n \) as follows: the wetted perimeter of the channel must be lined with a material of uniform boundary roughness and the slope of the channel bottom must be uniform. However, consistent with Manning’s warnings on the empirical nature of Manning’s equation, the assumption of a constant \( n \) does not always apply. Limerinos (1970), Bray (1979), Jarrett (1984), Bathurst (2002), and Ugarte and Madrid
(1994) for example, all found specific situations for which Manning’s $n$ varies and determined the constant $n$ assumption was not “sufficiently accurate” for their needs. While these researchers agree that $n$ is variable, the conclusions are widespread in how best to apply the head-discharge relationships. Some have predicted varying $n$ values using various empirical equations to be applied to Manning’s equation (Equation 1-4) based on different combinations of $n = F(R_h, k, Fr, and S_e)$. $Fr$ is known as the Froude Number where $Fr = V/(2gA/T)^{1/2}$ ($T$ is the top width of the free water surface). Others, such as Jarrett (1984), have suggested changes to $x'$ and/or $y'$ of the fundamental equation (Equation 1-1) to predict uniform flow.

These equations are specific to a certain type of roughness, e.g., gravel (Limerinos 1970, Bray 1979) or channel type, e.g., steep mountain streams with relatively shallow flows compared to the height of the roughness elements which make up the boundary roughness (Bathurst 2002, Jarrett 1984, Ugarte and Madrid 1994); however, they may be considered broad in that they suggest a single equation is suitable for a range of size of gravel or any number of streams that are relatively different but are determined to be steep mountain streams as described by different parameters in each of the individual studies. Comparisons between the resulting Manning’s $n$ predicted values and the experimental data for these studies produced significant scatter and relatively low correlation values. Bathurst (2002) addressed this issue and found that by further segregating data according to a certain parameter (channel slope in his 2002 study) within a certain channel type (steep mountain streams) and using a different equation for each data set, a separate equation may be used with more accuracy to predict $n$ values, i.e., the correlation values increased.

The interest in the application of Manning’s $n$, when it might be appropriate to assume constant $n$ value versus applying varying $n$ values, and how that varying $n$ values might be determined, spurred this study because of a specific need to predict the effects of resistance in composite channels. Yen (2002) describes a composite channel as a channel where the wetted
perimeter is made up of more than one roughness material. Manning’s $n$ has been shown to vary in composite channels; however, the variable nature of $n$ has generally been attributed to the differing resistance effects of each of the multiple roughness materials in the channel. To compensate for this, relationships have been developed to predict an effective $n$ value ($n_e$), a weighted average of the $n$ values associated with each of the boundary roughness segments that make up the wetted perimeter.

The application of the constant $n$ assumption to the individual roughness materials within the $n_e$ relationships is not uncommon. Yen (2002) presented 16 different $n_e$ relationships with potential application to composite channels. Yen stated that due to the limited data available, the level of appropriateness of any of the relationships to engineering practice has yet to be determined. Of the research discovered for the literature review for this dissertation, only one took into consideration the effect of applying a varying $n$ versus the constant $n$ assumption to the individual roughness materials and, as stated by Yen (2002), the data were limited.

The purpose of this dissertation is to study the behavior of Manning’s $n$ in uniformly lined channels specifically addressing the underlying assumption of a constant $n$ (the principle reason for the development of Manning’s equation) and the seemingly conflicting data that shows $n$ to vary. Included in this study are data that provide support for both the assumption of a constant $n$ and use of a predictive equation for a varying $n$ within the same channel lined with uniform roughness depending on the parameters of the flow condition in the channel. This finding is evaluated with respect to the original derivation of Manning’s equation itself and the relationship between Manning’s equation and the use of varying $n$ predictive equations. The knowledge obtained on $n$ values in uniform channels is then applied to composite channel relationships in order to not only compare these relationships but to also determine how to best apply Manning’s $n$ to the individual roughness materials to optimize the results of these relationships.
RESEARCH OBJECTIVES

Chow (1959) indicates that quantifying Manning’s $n$ for use in the field is the most difficult part in applying Manning’s equation describing the process as anywhere from guesswork for beginners and sound engineering judgment for seasoned engineers. The main overlying objective of this dissertation is to help bridge the gap between the constant $n$ assumption and the research showing $n$ variability, thus aiding in the quantifying of Manning’s $n$. A list of sub-objectives intended to contribute to the main objective are listed below.

- Collect laboratory data for a relatively wide range of boundary roughness materials (smooth to rough and diverse in roughness type) in a uniformly lined channel quantify $n$ for each roughness material.
- Compare the experimental laboratory data to known and accepted roughness coefficient theory developed primarily in closed conduit pressurized flow (usually presented as Darcy-Weistbach $f$). This is accomplished by determining the behavior of both $n$ and $f$ (from laboratory data) with respect to $Re$, $R_b$, $k$, $S_o$, and $Fr$.
- Determine a relationship for satisfactorily quantifying $n$ for each material where appropriate.
- Determine the conditions under which $n$ behaves as a constant versus a variable in a uniformly roughened channel as described by Chow (1959).
- Compare the experimental data from this study to those used by Manning in the development of Equation 1-4 to better understand the appropriate application of the constant $n$ assumption versus a variable $n$.
- Explore other research that reported variable $n$ behavior and compare their findings to the current study.
• Collect laboratory data for composite channel situations using combinations of the same roughness materials to quantify Manning’s $n$ in composite roughness channels.

• Determine how to best apply the $n$ values of each of the individual roughness materials to a composite channel configuration (e.g. should $n$ be quantified differently for a roughness lining the wall of a channel versus a roughness lining the base of the channel) through the use of the composite channel relationships proposed by Yen (2002).

• Compare the results from applying the constant $n$ assumption and a varying $n$ value to the composite roughness channel relationships and the laboratory data.

• Using the laboratory data, determine which of the 16 composite channel relationships is the most suitable for use, if any.

ORGANIZATION

This dissertation has been written based on the multi-paper format, with each paper representing a separate chapter (Chapters 3 to 5) specifically intended for publication in peer-reviewed journals. Chapter 2 addresses the experimental method used in determining Manning’s $n$ for each of the channel configurations evaluated in the study. Gradually varied flow computations have been previously used at the Utah Water Research Laboratory to determine $n$ values; however, these computations were based on the assumption that $n$ is a relatively constant value. A major component of this dissertation addresses the fact that $n$ is variable. Chapter 2 discusses how the GVF profile computations were used to appropriately determine $n$ with the consideration that $n$ may vary. This provides a basis for the experimental setup used to collect the data which were applied in each of the subsequent Chapters 3, 4, and 5. In Chapter 3, the behavior of $f$ and $n$ with respect to $Re$, $R_h$, $k$, $S_o$, and $Fr$ is addressed with a focus on a variable $n$ versus a constant $n$. The examination of these relationships is important regarding the consideration of when Manning’s $n$ should be applied as a constant or should be applied as a
variable. The information in Chapter 3 sets up the discussion in Chapter 4 which examines the behavior of \( x' \) [determined to be \( \frac{2}{3} \) by Manning (1889)] in Equation 1-1 with respect to constant \( n \) assumptions for the boundary roughness materials examined in this study and the roughness materials analyzed by Manning (1889). A better understanding of the behavior of \( x' \) coupled with the behavior of \( n \) (or the equivalent of an \( n \) value when \( x' \) is anything other than \( \frac{2}{3} \)) leads to a better understanding of Manning’s Equation in general and its relationship with \( n \). The intention of Chapter 4 is to better equip the reader to make decisions regarding the use of the variable \( n \) equations when available versus using constant \( n \) values. Chapter 5 applies the results of Chapters 3 and 4 to a practical situation (composite channels) to show the potential impact of the decision to use a variable \( n \) equation (including which equation to apply) versus assuming a constant \( n \) while at the same time fulfilling the need as described by Yen (2002) to compare composite channel \( n_r \) relationships. Chapter 5 evaluates the application of the \( n \) values for each of the individual roughness materials studied to the composite channel relationships and compares these relationships based on their performance. The final chapter (Chapter 6) provides a brief summary of Chapter 2 and presents a brief summary and conclusions of each of the other chapters.
BACKGROUND

Listed in the objectives to this study is the collection of laboratory data from which Manning’s $n$ can be quantified for both uniformly lined channels and composite channels. Manning’s $n$ can be directly calculated via Equation 1-4 when uniform flow exists in the channel when the flow depth ($y$) and $Q = VA$ are known. The laboratory flume used for this study is relatively short at 48-ft in length, and previous work completed in this flume showed that uniform depth is not achieved for all test conditions assuming the flow is allowed a free over-fall at the tail end of the flume. In laboratory practice, a tailgate is often used to help establish uniform flow by increasing the downstream flow depth, truncating part of the gradually-varied flow (GVF) profile, and facilitating the establishment uniform flow depth in the channel. According to Yen (2003), however, establishing a uniform flow depth via a gate-controlled backwater curve does not guarantee the establishment of uniform flow in the channel. In addition to a constant depth, uniform flow also requires uniformity in the velocity distribution, pressure, and turbulence characteristics. Yen (2003) states that even though a constant depth may be forced in a short channel with the use of a tailgate, the flow conditions associated with the channel inlet and tailgate may affect the other flow characteristics, resulting in a flow condition that is not truly uniform. It was apparent that collecting data to demonstrate the existence of uniform flow in the laboratory flume or in the field could prove to be very time consuming and costly.

Instead of a tailgate-controlled backwater curve approach, this study adopted the practice a free-overfall downstream boundary condition. For flow conditions where uniform depth was not established naturally, Manning’s $n$ values were determined using a computational GVF profiling technique. Traditional GVF profiling techniques, however, typically apply the constant
n value assumption, where n is independent of both stage and discharge. As addressed by the results of this study, this assumption may not always apply depending on the type of roughness material and the flow conditions in the channel. Not only is Manning’s n typically determined based on a constant n assumption, but other important parameters such as Re, R_h, V, and Fr are also determined based on the n-dependent, calculated normal depth (y_n). This chapter describes the GVF profile experimental method and how this method was used and adapted to account for for variable n conditions.

EXPERIMENTAL METHOD

For each steady state flow condition, a measured GVF profile was determined by measuring flow depths (y_{measured}) at 33 different locations along the length of the flume (due to entrance effects, on average, only the downstream 22 measured points were used in the GVF profile calculations). By using Equation 2-1, which calculates the change in flow depth (dy) relative to a specified change in distance along the length of a channel (dx), a computed GVF profile may be predicted.

\[
\frac{dy}{dx} = \frac{S_o - S_e}{1 - Fr^2}
\]  

(2-1)

In Equation 2-1, S_o is the slope of the channel and Fr is the Froude Number as defined by Equation 2-2.

\[
Fr = \frac{V}{\sqrt{g \frac{A}{T}}}
\]  

(2-2)

In Equation 2-2, T is the cross sectional water surface top-width in a channel. The energy grade line slope (S_e) is calculated using Equation 1-4 through which Manning’s n is introduced into the process. A spreadsheet was developed which allows the user to adjust Manning’s n until the
computed water surface profile best matches the measured profile. To determine the “best fit” of the data, the Coefficient of Determination ($r^2$), Equation (2-3), was maximized.

$$r^2 = 1 - \frac{\sum (y_{measured} - y_{calculated})^2}{\sum (y_{measured} - y_{average})^2}$$  \hspace{1cm} (2-3)$$

In Equation (2-3), $y_{calculated}$ is the flow depth calculated by the GVF computer program and $y_{average}$ is the average value of $y_{measured}$.

Figure 2-1 shows the resulting Manning’s $n$ data for a channel uniformly lined with the block roughness used in this study (see Chapter 3 for a full description of the block roughness). In Figure 2-1 two sets of data are featured: Manning’s $n$ determined directly by Equation 1-4 from experiments where the flow conditions were such that uniform flow occurred naturally in the flume (represented by the solid squares) and Manning’s $n$ determined from experiments where flow depths were not uniform using the GVF profile method (represented by the hollow squares). The uniform flow data clearly indicates that Manning’s $n$ varies with stage. Figure 2-1(A) shows the data based on calculations where $y_n$ was calculated using the constant $n$ assumption. The data in Figure 2-1(A) are scattered and the $n$ data calculated using the GVF method are not consistent with the $n$ data calculated based on naturally occurring uniform flow. However, there are a few points that are relatively close. These $n$ values were determined from data where the GVF profile either reached or came very close to reaching $y_n$ within the length of the flume.

Three assumptions were made in order to justify the use of the GVF profile method for calculating Manning’s $n$ values and to correctly correlate those $n$ values with the parameters that are also dependent on depth ($Re$, $Rh$, $V$, and $Fr$). These assumptions included:

- Because the GVF profiles used in determining Manning’s $n$ were relatively short (maximum of 28-ft), the assumption of a constant $n$ value may be applied through that
Figure 2-1: Comparison of Manning’s $n$ values determined using the uniform flow depth and the GVF technique (non-uniform flow conditions) vs. $y$ (data from Block lined channel).
(A)  

![Graph A](image)

- n-normal depth
- n-GVF (y-calculated assuming constant n)

(B)  

![Graph B](image)

- n-normal depth
- n-GVF (y-average)
length and the resulting $n$ value may be assumed to be an average $n$ value over the range of depths making up the water surface profile.

- This average $n$ value corresponds to the average measured depth over the length of GVF profile.
- The basis for the justification of this method is the experimental data associated with test conditions where uniform flow occurred naturally in the flume.

Figure 2-1(B) shows the results of the experiments based on these 3 assumptions and the data show that Manning’s $n$ values calculated by the GVF profile method are relatively consistent with the data from the naturally occurring uniform flow data. Based on these results, the GVF profile method was used for each flow condition tested where uniform flow did not occur naturally and the calculated Manning’s $n$ values were paired with the $y_{average}$ values for each condition.
ABSTRACT

Quantifying hydraulic roughness coefficients is commonly required in order to calculate flow rate in open channel and closed conduit applications. Much of the theory of resistance on open channel flow is derived from studies on pressurized circular pipe, which features the Darcy-Weisbach roughness coefficient, $f$, which is dependent upon $Re$, $R_h$, and/or $k$. Relative to full-pipe flow, however, the behavior of open channel flow resistance is more complicated because of the presence of a free surface and because the flow area does not remain constant.

A primary objective behind the development of Manning’s equation was to create a simple open channel flow equation with a roughness coefficient ($n$) that was solely dependent upon the boundary roughness characteristic (e.g., roughness height, $k$). Currently, hydraulic engineering handbooks publish singular representative $n$ values (or a small range to account for variations in material surface finish) per boundary material type (e.g., concrete, cast iron, clay, etc.). More recent studies, however, have suggested that $R_h$, $k$, $S_o$, and $Fr$ can influence $n$.

The behavior of $f$ and $n$ as a function of $Re$, $R_h$, $k$, $S_o$, and $Fr$ for open channel flow was evaluated for four different boundary roughness materials, ranging from smooth to relatively rough, by conducting stage-discharge tests in a rectangular tilting flume. The test results showed that when plotting $f$ or $n$ vs. $Re$, a family of curves resulted, with each curve corresponding to a specific channel slope ($S_o$). For a given $S_o$, both $f$ and $n$ decrease with increasing $Re$. The $S_o$-specific family of $f$ curves converges to a bounding curve, unique to each boundary roughness material tested, with increasing $Re$, which represents a quasi-smooth flow boundary condition.

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For the $n$ data, the quasi-smooth flow condition caused the $n$ values to converge to a constant-$n$ value at larger $Re$ values. A quasi-smooth flow boundary condition describes a condition where a boundary layer develops adjacent to the channel boundary that consists of a layer of flow eddies. The boundary layer thickness exceeds the material roughness height, reducing the influence of the boundary roughness elements of flow resistance.

$f$ and $n$ also decrease with increasing $R_h$, with $n$ eventually approaching a constant value.

The constant $n$ assumption ($n$ is independent of $Re$ and $R_h$) is most appropriate for smoother boundary materials or rough boundary materials where a quasi-smooth flow boundary condition exists. Where a quasi-smooth condition does not exist, the constant $n$ assumption is less appropriate for rougher boundary roughness materials.

INTRODUCTION

Quantifying hydraulic roughness coefficients is commonly required for discharge calculations for both closed conduit and open channel flow applications. Common open channel discharge equations include the Darcy-Weisbach equation, Equation 3-1, and Manning’s equation, Equation 3-2, which include the friction factor ($f$) and Manning’s $n$, respectively, as hydraulic roughness coefficients.

$$V = \sqrt{\frac{8g}{f} R_h S_e}$$  \hspace{1cm} (3-1)

$$V = \frac{K_n}{n} R_h^{2/3} S_e^{1/2}$$  \hspace{1cm} (3-2)

In Equations 3-1 and 3-2, $V$ is the mean velocity, $g$ is acceleration due to gravity, $K_n$ is 1.0 (SI) units and 1.486 (ES), $R_h$ is the hydraulic radius [the cross sectional area ($A$) divided by the wetted perimeter ($P$), $R_h = D/4$ for a pipe of diameter $D$], and $S_e$ is the slope of the energy grade line or
the friction slope. Under uniform flow conditions in open channel flow, $S_e$ is equal to the channel slope ($S_o$).

According to Streeter and Wylie (1979), Manning’s equation has been continually and with great popularity applied to open channel calculations because $n$ is thought to be an absolute roughness coefficient, i.e., dependent upon surface roughness only. Representative Manning’s $n$ values for common channel lining materials are typically presented in hydraulic handbooks, many of which refer to Chow (1959), as singular values or as a high, average, and low value to account for surface variations. In contrast to the constant $n$ assumption, Streeter and Wylie (1979) go on to state that $n$ actually depends upon the size and shape of the channel cross section in some unknown manner. This dependency and others have been described by researches with equations where $n = F(R_h, k, Fr, and S_e)$ (Limerinos 1970, Jarrett 1984, Bathurst 2002, Bray 1979, Griffiths 1981, Ugarte and Madrid 1994).

A committee set up by ASCE in 1957, being assigned the task to evaluate friction factors in open channels, suggested that $n$ may be applied as a constant value so long as the flow condition could be described as turbulent and fully rough. They defined a fully rough turbulent flow as one where $f$ and $n$ are independent of Reynolds Number ($Re$), where $Re=V4R_h/v$ ($v$ represents the fluid kinematic viscosity), leaving $f$ and $n$ to be solely dependent on relative submergence, quantified as $R_h/k$, where $k$ is a representative value for the boundary material roughness height. The equations proposed by Bathurst (2002), Griffiths (1981), Bray (1979), and Limerinos (1970) all show $f$ as a function of relative submergence and independent of $Re$, yet the result of these equations is a variable Manning’s $n$ suggesting that the assumption of a constant $n$ value even in fully rough turbulent flow, as defined by the committee, is not always valid. This leads to possible confusion as to when a constant $n$ is appropriate and when a variable $n$ might need to be applied.
The current study uses both Equations 3-1 and 3-2, per suggestion of the ASCE committee, to evaluate the seemingly contradictory results of the assumption of a constant \( n \) versus a variable \( n \) comparing this behavior to what has been learned through past research involving both open channel and full pipe flow. The behavior of the friction factors \( f \) and \( n \) are evaluated with respect to \( Re, R_h, k, S_o, \) and \( Fr \). The analysis was based on open channel flow testing conducted in a rectangular tilting flume featuring boundary roughness materials ranging from smooth to relatively rough.

**BACKGROUND**

*Darcy-Weisbach f*

The Darcy-Weisbach equation, Equation 3-1, dates back to the mid 1800’s (Rouse and Ince, 1957). Nikuradse (1933) performed tests on turbulent flow in artificially roughened pipes (pipe walls roughened with uniformly-sized sand grains) flowing full to investigate the behavior of \( f \). Nikuradse made two important conclusions. At low \( Re \) for pipe with relatively small sand grains (high \( R_d/k \) values), the values of \( f \) were similar to smooth-pipe values \([f = F(Re)\) only and the flow condition is known as smooth turbulence or smooth-wall pipe flow]. At relatively low \( R_d/k \) values and high \( Re \) values, \( f \) is solely a function of \( R_d/k \), and the flow condition is known as fully rough turbulence. A transitional turbulence \( Re \) range also exists where \( f \) is a function of both \( Re \) and \( R_d/k \). Colebrook (1939), using commercial pipe data, developed an empirical equation that describes the dependencies of \( f \) on \( R_d/k \) and \( Re \). From Colebrook’s equation, the Moody Diagram was developed and has become a common source for assigning a value to \( f \) for smooth-wall, full-pipe flow under turbulent conditions.

Chow (1959) compiled data from various open channel flow tests performed in rough channels with turbulent flow and made the following observations. Some of the data show that, at relatively high \( Re \), \( f \) becomes independent of \( Re \) and is solely dependent on \( R_d \) and \( k \). He also
observed for some data that $f$ decreased with increasing $Re$, with the minimum $f$ values bounded by an equation in the form of Equation 3-3, where $f$ is a function of $Re$ and the coefficients $a$ and $b$ are boundary roughness ($k$) specific.

$$\frac{1}{\sqrt{f}} = a \log \left( \frac{Re \sqrt{f}}{b} \right)$$

(3-3)

In Equation 3-3, $a$ and $b$ are empirical coefficients specific for a given channel shape and boundary roughness. Prandtl developed an equation (commonly referred to as the Prandtl-von Kármán equation) in the form of Equation 3-3, which reasonably describes $f$ data for smooth-walled pipe, with $a$ and $b$ equal to 2 and 2.51, respectively (Crowe et al. 2001). The open channel flow stage-discharge data presented by Chow (1959) suggest that $a$ and $b$ will vary with boundary roughness type, i.e., $f$ values increase with increasing boundary roughness or increasing $k$ values. Chow (1959) also suggests that when the behavior of $f$ for a given boundary roughness material can be described by Equation 3-3 with a constant set of empirical coefficients ($a$ and $b$), a *quasi-smooth* flow condition exists. The idea of a *quasi-smooth* boundary flow condition was introduced by Morris (1955) and describes a flow state where the areas between the roughness elements are filled with stable eddies, creating a pseudo wall flow boundary similar to a smooth wall (see Figure 3-1). The results from this study confirm that Equation 3-3 is a relative limiting boundary to $f$ and also show that this limiting boundary has relevance to the assumption of a constant $n$.

*Manning’s n*

Equation 3-4 relates the Manning’s $n$ roughness coefficient and $f$.

$$\frac{V}{V^*} = \sqrt{\frac{8}{f}} = \frac{K_{*} R^{1/6}}{n \sqrt{g}}$$

(3-4)
In Equation 3-4, $V^*$ is the shear velocity [$V^* = (gR_hS_e)^{1/2}$]. Manning (1889) developed Equation 3-2 with the expressed intent of providing a simplified open channel flow equation where, contrary to existing equations, the empirical coefficients (including the roughness coefficient) would remain constant for a given channel boundary type, independent of $Q$ and $R_h$ variations. Manning applied Equation 3-2 with river-reach-specific constant $n$ values to more than 100 data points taken from various rivers and concluded it was “sufficiently accurate.”

Chow (1959) states that, if the bed and banks of a channel are equal in roughness and the slope is uniform, then $n$ is usually assumed constant for all flow depths ($y$). Chow (1959) presents Manning’s $n$ data (constant values) and photographs for a number of different channel types as a reference for designers. More recent studies, however, have shown that $n$ is not necessarily a constant even under the conditions described by Chow (1959). A number of relationships have been developed, based on the results of these studies in order to predict the behavior of $n$. For example, Limerinos (1970), Bray (1979), Griffiths (1981), and Bathurst (2002) have presented relationships suggesting that $n$ is a function of $R_h/k$. Jarrett (1984) suggested that $n$ is dependent upon $S_e$ and $R_h$. Ugarte and Madrid (1994) proposed relationships for $n$ involving $R_h$, $k$, $S_e$, and the Froude Number ($Fr$). These relationships were developed based on studies where Manning’s equation was applied to a specific type of channel. The Limerinos,
Bray, and Griffith relationships were developed for rivers with gravel beds; the Bathurst, Ugarte and Madrid, and Jarrett relationships were specific to “mountain streams” characterized as steep with relatively small $R_h/k$ values. Yen (2002) maintains, however, that for a given boundary roughness, $n$ should be relatively constant, independent of $Re$, and $R_h$, provided that the equivalent $f$ value per Equation 3-4 is in the fully turbulent range [i.e., $f = F(R_h$ and $k$)].

**Froude Number Effects**

Open channel flow state is commonly characterized by the value of the Froude number (per Equation 3-5), which represents the ratio of inertial to gravitational forces. In Equation 3-5, $T$ is the channel top width. When $Fr < 1$, gravitational forces are dominant, flow velocities are low, and the flow condition is referred to as subcritical. When $Fr > 1$, the inertial forces are dominant, the velocity is high, and the flow condition is referred to as supercritical.

$$Fr = \frac{V}{\sqrt{gA/T}}$$  (3-5)

Chow (1959) states that when $Fr < 3$, the influence of $Fr$ on open channel roughness coefficients is negligible. Chow concedes, however, that as more data becomes available, the influence of $Fr$ on open channel roughness coefficients may need to be reconsidered. Ugarte and Madrid (1994) concluded that $n$ has $Fr$ dependencies; however, it is important to note that their study was generally limited to relatively small $R_h/k$ values. Bathurst et al. (1981) also found that $Fr$ was a factor in quantifying $n$; however, instead of using the traditional $Fr$ definition, $A/T$ in Equation 3-5 was replaced with $R_h$.

**OBJECTIVES**

The objectives of this study are to investigate the relationships of the open channel roughness coefficients $f$ and $n$ with $Re$, $R_h$, $k$, $S_0$, and $Fr$ in a controlled laboratory setting in an
effort to better understand the appropriateness of the constant $n$ value assumption for a given boundary roughness. Comparisons are made for four different roughness materials ranging from smooth (acrylic sheeting) to relatively rough (block and trapezoidal corrugated roughness elements).

**EXPERIMENTAL METHOD**

The behavior of Manning’s $n$ for four different boundary roughness materials was investigated by conducting flow tests in a 4-ft wide by 3-ft deep by 48-ft long adjustable slope rectangular laboratory flume. The four channel boundary materials tested include smooth acrylic sheeting (see Figure 3-2); a low-profile, commercially available expanded metal lath adhered to the acrylic walls and floor of the flume (see Figure 3-3); regularly spaced wooden blocks (see Figure 3-4 and 3-5); and trapezoidal corrugations oriented normal to the flow direction (see Figures 3-6 and 3-7). The wooden blocks, measuring 4-inches wide (normal to flow direction) by 3.5 inches long by 1.5 inches tall, with the top edges rounded (1-inch radius round-over), featured a painted exterior and were assembled in a closely spaced, uniform pattern. The wooden trapezoidal corrugation elements were 1.5 inches tall, had a 1.5-inch wide top width and a 4.5-inch wide base, and were spaced 1.5 inches apart. The blocks and trapezoidal corrugation elements were attached to sheets of painted marine grade plywood, which were attached to the flume floor and walls.

Assigning a $k$ value to various types of roughness materials is not an exact process. For gravel-lined channels, the mean grain size diameter is often used. In this study, all roughness materials, save the acrylic sheeting, have more than one geometric dimension that influences the hydraulic roughness (e.g., the block height, width, length, and spacing). Chow (1959) explains that while $k$ represents a measure of a boundary’s roughness, it is an empirical parameter that
Figure 3-2. Acrylic boundary roughness material

Figure 3-3. Metal Lath boundary roughness material
Figure 3-4. Block boundary roughness material

Figure 3-5. Schematic of block boundary roughness material (dimensions shown in inches)
Figure 3-6. Trapezoidal corrugation boundary roughness material

Figure 3-7. Schematic of trapezoidal corrugation boundary roughness material (dimensions shown in inches)
doesn’t necessarily correspond to a specific geometric dimension of the roughness element that can be measured using a linear scale and that \( k \) is influenced by many factors such as roughness element shape, orientation, and distribution. In this study, \( k \) was assumed equal to the physical height of the roughness elements, for lack of a more appropriate alternative. The acrylic sheeting \( k \) value was selected to be consistent with published values (\( k = 0.00006 \) inches).

Water was supplied to the flume from a reservoir located adjacent to the laboratory and was metered using calibrated orifice flow meters. Flow depths were measured using a precision point gage, readable to 0.008-in, attached to a movable carriage located above the flume. At each measurement location, the channel invert was measured and recorded. Subsequent flow depths were determined by subtracting the difference between the measure water surface and invert elevations.

Manning’s \( n \) can be directly calculated via Equation 3-2 when uniform flow exists in the channel and \( y \) and \( Q \) are known. Due to the limited length of the laboratory flume, uniform flow depth could not be achieved for all test conditions. In laboratory practice, a tailgate is often used to help establish uniform depth in a flume by increasing the downstream flow depth and truncating part of the gradually varied flow (GVF) profile. According to Yen (2003), this method does not guarantee the presence of a uniform flow condition. In addition to a constant flow depth; the velocity distribution, pressure, and turbulence characteristics must also be uniform for uniform flow to exist. Yen (2003) states that even though a constant depth may be forced in a short channel with the use of a tailgate, the flow conditions associated with the channel inlet and tailgate may affect the characteristics of the flow, resulting in a flow condition, which is not “uniform.”

In the current study, all tests featured a free-over fall downstream boundary condition. For flow conditions that did not achieve normal depth naturally, Manning’s \( n \) values were determined using a computational GVF profiling technique. For each steady state flow condition,
the GVF profile was determined by measuring flow depths \(y_{measured}\) at 33 different locations distributed along the length of the flume. The Manning’s \(n\) coefficient was determined for each flow condition by adjusting the Manning’s \(n\) value in a GVF computer program until the computed water surface profile best matched the measured profile. To determine the “best fit” of the data, a coefficient of determination \((r^2)\), Equation 3-6, was maximized.

\[
r^2 = 1 - \frac{\sum (y_{measured} - y_{calculated})^2}{\sum (y_{measured} - y_{average})^2}
\]  

(3-6)

In Equation 3-6, \(y_{calculated}\) is the flow depth calculated by the GVF computer program and \(y_{average}\) is the average of \(y_{measured}\).

The data collection proceeded as follows. For each slope and discharge, the water surface was measured in relation to the flume floor at 2-ft intervals over the upstream half of the flume and at 1-ft intervals over the downstream half. Due to the nature of the block and trapezoidal corrugation roughness materials, no single channel invert datum was present. Consequently, a representative datum was determined by calculating the total volume of the roughness elements (blocks or trapezoidal corrugations) divided by the total flume floor area and adding the resulting height to the elevation of the plywood floor upon which the roughness elements were installed. This artificial boundary was used as the channel invert reference for the block and corrugated roughness tests.

Using this GVF method, a separate Manning’s \(n\) value was determined for each flow condition. Early in the data collection process, however, it became apparent that, for the relatively rough boundary materials (blocks and trapezoidal corrugations), Manning’s \(n\) exhibited variability with flow depth for a common flow rate. Figure 3-8, for example, shows Manning’s \(n\) data for a number of flow conditions in the block-lined channel. With steeper channel slopes, where uniform flow conditions were more prevalent, \(n\) values were determined using the measured normal depth \((y_n)\), \(Q\), and Equation 3-2. For milder sloping channels, where uniform
flow profiles were less common, $n$ values were determined using the GVF profile method. A comparison of the block-lined Manning’s $n$ values determined using both techniques is presented in Figure 3-8, which plots $n$ vs. the average channel profile flow depth ($y_{\text{average}}$). The uniform flow depth data in Figure 3-8 show that for the block roughness, $n$ varies ($0.087 \geq n \geq 0.048$) with changes in uniform flow depth ($0.13 \geq y \geq 0.9$). Analysis of various truncated sections of a single GVF profile, using the GVF $n$ method, also produced different predictive values for $n$, suggesting that $n$ is also variable with depth throughout a GVF profile. Based on the variable nature of $n$ with flow depth in GVF profiles, the predicted normal depths associated with the variable $n$ values should also vary. Consequently, based on the good correlation between the uniform flow and GVF $n$ data presented in Figure 3-8, $y_{\text{average}}$ was selected as the representative flow depth parameter for calculating $R_h$, $Re$, $V$, etc. when using the GVF method.
Manning’s $n$ data for the acrylic boundary were collected at three different slopes (i.e., 0.0002, 0.0003, and 0.0022) with the number of flow conditions at each slope ranging from 6 to 17. The metal lath boundary was tested at four different slopes (i.e., 0.0066, 0.0118, 0.0179, and 0.022) with 4 to 29 flow conditions tested at each slope. The block and trapezoidal corrugation boundaries were each tested at five slopes (i.e., 0.0004, 0.0018, 0.0095, 0.0237, and 0.05) with 7 different flow conditions per slope. The channel discharges ranged from 0.24 to 23 cfs.

DISCUSSION AND ANALYSIS

$f$ Relationships

Figure 3-9 plots the Darcy-Weisbach $f$ versus $Re$ data for each of the roughness materials in a uniformly lined channel. The data from the acrylic-lined channel generally follow the Prandtl-von Kármán smooth-wall pipe flow curve. Though not necessarily discernible in Figure 3-9 due to the scale of the y-axis, the acrylic experimental $f$ values exceed the Prandtl-von Kármán curve values at higher $Re$ values. At a given $Re$ value, $f$ increases with increasing boundary roughness (i.e., $f$ of the blocks is greater than the metal lath, which is greater than the acrylic) as expected.

At first glance, there appears to be considerable scatter in the data for the two larger roughness materials (block and trapezoidal roughness materials) in Figure 3-9; however, a closer look reveals families of curves segregated by $S_o$. The data show that, for a prismatic channel where $S_o$ is held constant, $f$ decreases as $Re$ increases. For a constant $Re$, $f$ increases with increasing $S_o$. As $Re$ increases, the roughness-element-specific, slope-dependent family of curves converges to a single curve. There is no singular $Re$ value, however, at which the individual curves converge. The $Re$ value at which a slope-specific curve converges to the bounding curve for an individual roughness material increases with increasing $S_o$. 
Figure 3-9. $f$ vs. $Re$ data for acrylic, metal lath, trapezoidal corrugation (A), and block (B) roughness materials.
The bounding curve to which the acrylic, metal lath and trapezoidal corrugation data converge is consistent with Equation 3-3, which, as described by Chow (1959), becomes a limiting boundary to the decreasing effect of the boundary roughness on the total resistance to the flow. Figure 3-9 shows that the block slope-specific data curves do not fully converge to a single curve within the range of Re tested; however, the trend lines appear to be converging toward a single bounding curve with increasing Re. The convergence of the metal lath and the trapezoidal corrugation roughness data to a single bounding f vs. Re curve indicates that the conditions in the channel have reached a quasi-smooth boundary flow conditioning consistent, in theory, with the illustration in Figure 3-1.

$n$ Relationships

If $n$ were constant (as is often assumed) and solely dependent on $k$, four horizontal lines, one for each roughness material tested, should result when plotting $n$ versus Re. The results in Figure 3-10 show relatively constant $n$ values for the smooth acrylic data and over most of the Re data range for the metal lath. There is a small range of relatively small Re values over which $n$ for the metal lath varies. $n$ varies significantly for the two rougher materials (block and corrugation roughness) over the range of Re tested. The data for these roughness materials show trends similar to the $f$ data presented in Figure 3-9: there is a family of curves segregated by $S_o$, $n$ decreases with increasing Re, $n$ increases with an increasing slope (at a constant Re value), and the $S_o$-specific curves converge as Re increases. An inspection of the data in Figures 3-9 and 3-10 reveals a subtle but important difference between the behavior of $n$ and $f$ with Re. In Figure 3-10, the slope-dependent Manning’s $n$ data curves converge to a constant (minimum) value as Re increases, indicating that $n$ is solely dependent upon $k$ at higher Re values. In contrast, the $S_o$-specific $f$ curves in Figure 3-9 converge to a bounding curve in the form of Equation 3-3 as Re increases. Though the slopes of the bounding curves become relatively small at higher Re values,
Figure 3-10. $n$ vs. $Re$ data for acrylic, metal lath, trapezoidal corrugation (A), and block (B) roughness materials
the bounding curves do not reach a zero slope, indicating that $f$ remains a function of $Re$ and $k$ over the range of $Re$ numbers tested.

The data in Figure 3-10 also suggest that the appropriateness of a constant-$n$ value assumption increases as the relative smoothness of the channel boundary increases. The $n$ values for the acrylic and metal lath channels are constant over the majority of the $Re$ range tested. As the relative roughness increases (e.g., the blocks and trapezoidal corrugations), the range of $Re$ over which $n$ is constant diminishes. Based on the data presented in Figure 3-10, the constant-$n$ assumption, commonly used when applying Manning’s equation (Equation 3-2), is appropriate for smooth-wall channel lining materials (e.g., smooth acrylic sheeting) or for “rougher” boundary materials when a quasi-smooth boundary condition is present (e.g., metal lath and trapezoidal corrugation roughness material $n$ vs. $Re$ data becomes constant). Under these conditions, $n$ is a function of $k$ and is no longer dependent on $Re$, $S_o$, or $R_h/k$.

The behavior of the block roughness $n$ data in Figure 3-10 is similar to that of the $f$ data in that the $n$ data do not fully converge to a constant value (a bounding curve for the $f$ data) due to the limited range of experimental $Re$ values. It is assumed, however, that, similar to the trapezoidal corrugations, the block data will converge to a constant $n$ value at higher $Re$ values.

Figure 3-11 presents $n$ vs. $R_h/k$ for the block and trapezoidal corrugations. The block data show a strong dependence on $R_h/k$ ($n$ decreases with increasing $R_h/k$) and are relatively independent of $S_o$, as the data essentially collapse to a single curve. The fact that the $n$ vs. $R_h/k$ data are essentially independent of $S_o$ means that, for the rectangular flume used in this study, $n$ was solely a function of flow depth. $n$ will be the same for two different channel slopes, provided that flow depths are the same, independent of the differences in $Q$, $V$, and $Re$ for the two slope conditions. As a result, when correlating $n$ vs. $R_h/k$, $n$ is essentially independent of $V$ and $Re$. The trapezoidal corrugation data in Figure 3-11 also show a strong dependence on $R_h/k$; but a slight data segregation (family of curves) associated with $S_o$ exists (more than with the block data).
Figure 3-11. $n$ vs. $R_d/k$ for Block and Trapezoidal Corrugation roughness materials

The reason for the variation in the behavior of $n$ vs. $R_d/k$ between the block and trapezoidal corrugation materials isn’t clear, but it may be related to the nature of the flow paths near the boundaries. With the blocks, flow passes over and around the individual roughness elements. With the trapezoidal corrugations, the flow only passes over the roughness elements, making the velocity profile near the boundary primarily two-dimensional rather than three-dimensional like the blocks. The disparity between the $S_\phi$-specific $n$ vs. $R_d/k$ curves in Figure 3-11, however, is significantly reduced relative to the $n$ vs. $Re$ data in Figure 3-10.

It is also interesting to note that, despite the fact that the block and trapezoidal corrugation roughness elements are the same height, the $n$ vs. $R_d/k$ data trend differently in Figure 3-11. At smaller flow depths (e.g., $R_d/k = 1.0$), the flow resistance of the blocks is larger (larger $n$) than the trapezoidal corrugations. For the trapezoidal corrugations, $n$ decrease more rapidly with increasing $R_d/k$ than the blocks, and the point at which $n$ becomes constant occurs at a lower $R_d/k$.
value. This suggests that equating $k$ to the height of the roughness element does not adequately characterize the influence of the roughness elements on flow resistance. Though perhaps not a general conclusion, it is interesting to note that the trapezoidal corrugations and the block, which were approximately the same height, both approach approximately the same constant $n$ value at high $R_h/k$ values ($n \approx 0.033$). More research is recommended to investigate the characteristic differences between the flow resistance behavior of two-dimensional and three-dimensional boundary roughness element types.

With respect to the data presented in Figure 3-11, the quasi-smooth flow boundary condition occurs for rougher boundary materials when a sufficiently high $R_h/k$ condition, referred to as relative submergence or the boundary roughness elements, is reached and $n$ becomes constant. For $R_h/k$ values below the quasi-smooth flow limit, the constant-$n$ assumption is not appropriate. According to the data presented in Figure 3-11, the level of relative submergence required to produce a quasi-smooth flow condition varies with the boundary roughness characteristics, which are partially described by $k$ and $R_h/k$. Manning (1889) reported relatively constant $n$ values for numerous river channel sections. The river channel sections most likely featured sufficiently high $R_h/k$ values to validate a constant $n$ assumption.

Subcritical vs. Supercritical Flow

Nineteen of 35 metal lath lining data points featured supercritical flow conditions and were dispersed over the range of $Re$ tested. Three of 7 flow conditions corresponding to the steepest channel slope for the block and trapezoidal roughness produced supercritical flow. Although the data are not specifically identified as sub- or supercritical flow in Figure 3-11, the consistent trends in the data sets indicate that $n$ is relatively independent of $Fr$ over the range of $Fr$ values tested. For the entire data set (all four boundary roughness data sets) $Fr$ ranged from
0.33 to ~1.54. These results concur with Chow (1959), who stated that for small $Fr (Fr < 3)$, the effect of gravity on flow resistance is negligible.

CONCLUSIONS

The behavior of Manning’s $n$ (and $f$) in a prismatic (rectangular) open channel flow as a function of $Re$, $R_b$, $k$, $S_o$, and $Fr$ was evaluated in the laboratory in an effort to characterize their constant and/or variable nature for various uniform channel lining roughnesses ranging from smooth to relatively rough. The results of this study are intended to provide additional insight into the appropriateness of the constant Manning’s $n$ assumption, relative to the specific roughness materials tested. Based on the results of this study, the following conclusions are made:

1. In relation to $Re$, the $f$ and $n$ data from this study have similar characteristics to the data presented by Chow (1959). At a constant $S_o$, both $f$ and $n$ decrease with increasing $Re$. The $Re$-dependent $f$ data were bound by a material roughness-specific limiting curve consistent with Equation 3-3; the corresponding $n$ data were bound by a limiting constant $n$ value. Chow (1959) suggested that the $f$-data bounding curves are consistent with a smooth surface condition, analogous to the Prandtl-von Kármán smooth pipe wall boundary condition, or a quasi-smooth boundary flow condition, which describes a condition where the voids between boundary roughness elements are filled with stable eddies, reducing the influence of the boundary roughness elements on flow resistance. The constant-$n$ assumption is appropriate for smooth and quasi-smooth flow conditions. For rougher boundary materials, $n$ can vary considerably for non-quasi-smooth flow conditions, which if not appropriately accounted for, could significantly increase the level of uncertainty associated with open channel flow stage-discharge calculations.

2. For a single boundary roughness material (characterized by $k$), flow resistance testing over a range of channel slopes produced a family of $S_o$-dependent curves (see Figures 3-9
and 3-10). The families of $f$ and $n$ data curves in Figures 3-9 and 3-10 do not necessarily confirm $S_o$ as a significant parameter influencing flow resistance behavior, but rather they serve as an indicator that there are likely additional system parameters that influence open channel flow resistance, which are not appropriately accounted for in an $f$ or $n$ vs. $Re$ analysis. The differences between the $S_o$-dependent curves for a single boundary roughness material increased as $k$ increased (i.e., the metal lath family of curves is more closely spaced than the curves for the block or trapezoidal corrugation boundary materials in Figures 3-9 and 3-10). Figures 3-9 and 3-10 also show that $f$ and $n$ increase with increasing $S$ for a given boundary roughness material.

3. Figure 3-11 shows that the $S_o$-dependent family of curves collapse relatively well to a single curve when $n$ is plotted with respect to $R_h/k$. This suggests that for two channels with a common cross sectional shape and boundary roughness material but differing slopes, the value of $n$ will be equal in both channels for flow conditions that produce the same $R_h$ values (i.e. same flow depth) and is also independent of the differences in $Q$, $V$, and $Re$ for the two slope conditions. The trapezoidal corrugations show a greater scatter in the $n$ data than do the blocks when plotted with respect to $R_h/k$; however, the $R_h/k$ relationship represents a significant improvement relative to the $Re$ relationship with respect to collapsing the data to a single curve. More research is needed to fully explain the scatter shown in the data.

4. According to Figures 3-10 and 3-11, the appropriateness of the constant Manning’s $n$ assumption or the existence of a quasi-smooth flow condition is dependent upon the boundary roughness and a specific value of $R_h/k$. There exists a minimum $R_h/k$ value for each boundary roughness material tested above which $n$ is essentially constant. The constant-$n$ minimum values of $R_h/k$ decrease as $k$ decreases (as the boundary becomes smoother). It is interesting to note that, despite the fact that the trapezoidal corrugation
and block elements had similar height dimensions (1.5 inches) used to quantify their $k$
values, the constant-$n$ minimum $R_h/k$ values differed appreciably (as shown in Figure 3-11). This suggests that simply using the vertical dimension or height of a boundary
roughness element, particularly for relatively rough boundary materials, does not
sufficiently characterize their equivalent roughness height ($k$). The height, width, length,
spacing, uniformity and surface texture, etc. will all influence the behavior of $n$. It is also
interesting to note that, despite the fact that the block and trapezoidal corrugations reach
the constant $n$ condition at differing values of $R_h/k$, the block and trapezoidal corrugation
boundary roughness materials converge to approximately the same constant $n$ values.

5. Consistent with the findings of Chow (1959), $n$ was found to be independent of $Fr$ for $Fr$< 3 (all test data from this study were less than $Fr = 3$). Ugarte and Madrid (1994)
reported that $n$ was $Fr$-dependent, but their test conditions were limited to relatively small
values of $R_h/k$ (large roughness elements and/or shallow flow depths) relative to the
current study.

In developing Equation 3-2 Manning’s (1889) primary objective was a simple open
channel flow equation with a roughness coefficient ($n$) that was solely dependent on $k$. Currently,
hydraulic engineering handbooks publish singular representative $n$ values (or a small range to
account for variations in material surface finish) per boundary material type (e.g., concrete, cast
iron, clay, etc.). Manning concluded that the constant $n$ assumption was “sufficiently accurate”
after applying Equation 3-2 to numerous data taken from rivers. The Task Force on Fiction
Factors in Open Channels organized by ASCE in 1957 recommended that this assumption can be
used with certain stipulations. One of which is that the flow condition must be designated as fully
rough where the friction factor is independent of $Re$; however, later studies (Limerinos 1970,
Jarrett 1984, Bathurst et al. 1981, Ugarte and Madrid 1994) suggest that $n$ can be influenced by
$R_h$, $k$, $S_e$, and $Fr$ indicating it is not constant even under flow conditions where the friction factor is independent of $Re$ (or “fully rough”) as described by the committee.

Based on the findings of this study, an engineer that desires to use Manning’s Equation for open channel calculations should take into consideration that the appropriateness of assuming material-specific constant Manning’s $n$ values for all stage and discharge conditions is not only limited to whether the flow is “fully rough” but is limited even further to smooth (physically smooth or quasi-smooth) boundary flow conditions. Additional research is needed to provide engineers with more comprehensive Manning’s $n$ data that better characterize the flow resistance behavior of common channel lining materials for design purposes to aid in the decision to apply a constant $n$ or some other method to determine a variable $n$. 
CHAPTER 4
OPEN CHANNEL FLOW RESISTANCE: THE HYDRAULIC RADIUS
DEPENDENCE OF MANNING’S EQUATION AND MANNING’S $N^2$

ABSTRACT

Manning’s equation, which is used to estimate head-discharge relationships in open channel flow applications, states that the mean channel flow velocity is inversely proportional to the Manning’s $n$ hydraulic roughness coefficient and proportional to the hydraulic radius raised to an exponent ($x'$) of $2/3$ (i.e., $R_h^{2/3}$). $n$ and $x'$ represent empirical coefficients used to correlate Manning’s equation with experimental data. In developing Manning’s equation, Manning evaluated the stage-discharge characteristics of various boundary roughness materials ranging from smooth cement to course gravels and reported unique values of $n$ and $x'$ for each boundary type. The $x'$ values ranged from approximately 0.65 (smoothest boundary tested) to 0.84 (roughest boundary tested). Manning chose $x' = 2/3$ as representative, compared it with field data, and suggested that it was sufficiently accurate. He also offered the caveat that the use of Manning’s equation should be limited to cases where its accuracy has been validated.

Chapter 3 of this document showed that Manning’s $n$ is not constant for all boundary materials and all stage-discharge conditions. This chapter evaluates the behavior of $x'$ with respect to constant $n$-assumptions for the four boundary roughness materials discussed in Chapter 3 (smooth acrylic sheeting, metal lath, trapezoidal corrugations, and blocks) and the boundary roughness materials analyzed by Manning (1889). Consistent with the results reported by Manning (1889), the result of this study found that the $x'=2/3$ assumption is appropriate for smooth boundaries (e.g., acrylic and pure cement) and for rougher boundary materials when a quasi-smooth boundary condition exists. The quasi-smooth boundary condition describes a

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2 Coauthored by Tyler G. Allen, P.E.; Blake P. Tullis, Ph.D., P.E.
condition where the voids between the boundary roughness elements are filled with stable eddies, which effectively reduces the influence of the boundary roughness elements on flow resistance. For rougher boundary materials not in the quasi-smooth boundary flow condition, applying the constant Manning’s $n$ assumption results in $x'$ values in excess of 2/3. In order to accurately predict the stage-discharge relationship for rougher boundary conditions using Manning’s equation, either the $x'=2/3$ or a constant $n$ value assumption can be applied but not both; the other variable ($x'$ or $n$) must be treated as non-constant.

INTRODUCTION

Uniform flow head-discharge relationships for open channel applications correlate flow rate ($Q$) or mean channel velocity ($V$) to an energy gradient, taking into account the flow resistance associated with the channel cross-sectional shape and boundary roughness. Most open channel head-discharge or uniform-flow equations are in the form of Equation 4-1 (Chow, 1959).

\[ V = CR_h^{x'}S_e^{y'} \]  

In Equation 4-1, $R_h$ is the hydraulic radius [the flow area ($A$) divided by the wetted perimeter ($P$)], $S_e$ is the friction slope (equal to the channel slope $S_o$ for uniform flow conditions), $C$ is a flow resistance coefficient, and $x'$ and $y'$ are exponents. The Chezy (Equation 4-2), Darcy-Weisbach (Equation 4-3), and Manning (Equation 4-4) equations represent three common open channel flow head-discharge relationships derived from Equation 4-1.

\[ V = C \sqrt{R_h S_e} \]  

\[ V = \sqrt{\frac{8g}{f} \frac{R_h}{S_e}} \]  

\[ V = \frac{K_n}{n} R_h^{2/3} S_e^{1/2} \]
In Equations 4-2 through 4-4, \( g \) is the acceleration due to gravity; \( K_n = 1 \) for S.I. and \( K_n = 3.281^{(1)} \) for ES units; and \( C_r, f, \) and \( n \) are equation-specific hydraulic roughness coefficients. Equations 4-2 and 4-3 are identical (i.e., same \( x' \) and \( y' \) values), with the exception of the way the hydraulic roughness for flow resistance is quantified. Manning’s equation is significantly unique from the other two, with \( x' = 2/3 \) instead of \( 1/2 \). Manning (1889) made this change with the hope of developing a simplified open channel equation where the roughness coefficient \( (n) \) would be constant for a given channel lining material (i.e., independent of stage and discharge). Equation 4-4 is commonly applied in practice with the assumption that \( n \) remains constant for a given boundary roughness material.

Chow (1959) stated that if the boundary roughness in a channel is uniform (i.e., the roughness is the same for the entire wetted perimeter over the length of the channel section) and the slope of the channel bottom is also uniform, then there is a possibility that Manning’s \( n \) could remain constant for all flow stages. The Task Force on Friction Factors in Open Channels (1963) organized by the American Society of Civil Engineers (ASCE) in 1957 indicates that a nearly constant \( n \) is applicable to channels where the friction factor is independent of Reynolds Number \( (Re=V4R/J/v, \) where \( v \) represents the kinematic viscosity) (the flow is designated as “fully rough” turbulence). More recently, Yen (2002) suggested that the constant \( n \) assumption is appropriate under certain conditions, and makes Equation 4-4 more convenient to use than Equations 4-2 and 4-3. Data have also shown (Limerinos 1971, Bray 1979, Bathurst et al. 1981, Jarrett 1984, Ugarte and Madrid 1994), however, that \( n \) is not always constant with stage and discharge even in channels where the stipulations listed by Chow (1959), the ASCE Task Force (1963), and Yen (2002) are met. These studies were performed in gravel bed streams or “steep” mountain streams with relatively rough natural channel boundaries [in some cases, the height of the roughness elements exceeded the flow depth (\( y \))]. The seemingly conflicting results of Bathurst et al. (1981), Jarrett (1984), and Ugarte and Madrid (1994) and the statements of Yen (2002), Chow
(1959), and the ASCE Task Force (1963) are all somewhat supported by the discussion in Chapter 3, which documents both variable and constant \( n \) flow regimes in a rectangular channel with a uniform boundary roughnesses.

In this chapter, the appropriateness of the constant \( n \) assumption, relative to the behavior of the other empirically-determined fitting parameter in Manning’s equation, \( x' \), is evaluated by applying a similar analysis method to that used by Manning (1889) in the development of Equation 4-4 to the data sets used by Manning (1889) and the Manning’s \( n \) data presented in Chapter 3 (acrylic, metal lath, trapezoidal corrugation, and the block channel boundary roughness materials). The results give the engineer further guidance as to the potential limitations of the use of Manning’s Equation (Equation 4-4) coupled with the assumption of a constant \( n \) value.

BACKGROUND

Chezy and Darcy-Weisbach Equations

The Chezy equation (Equation 4-2) was developed circa 1769 for uniform open channel flow. Two basic assumptions contributed to its derivation: (1) the force resisting the flow per unit area of the streambed is proportional to the square of the velocity and (2) the flow gravitational force is equal and opposite to the flow resistance force (Chow, 1959).

The Darcy-Weisbach equation was developed for pressurized pipe flow via dimensional analysis. \( f \) values, which vary with \( k/D \) (\( k \) is defined as an boundary material equivalent roughness height and \( D \) is the pipe diameter) and \( Re \), are presented for smooth-walled (non-profiled-wall) pipe in the Moody Diagram, which can be found in most hydraulic handbooks. Chow (1959) stated that if \( S_e \) represents the head loss per unit length of pipe or channel and if \( D \) were replaced by \( 4R_h \), then Equation 4-3 could be applied to open channel flow. The relationship between \( C_c \) and \( f \) are shown in Equation 4-5.
\[ C = C_c = \sqrt[3]{\frac{8g}{f}} \]  

(4-5)

Manning’s Equation

Aware of the variable nature of hydraulic roughness coefficient behavior with most open channel flow equations, including Equations 4-2 and 4-3, Manning (1889) presented an alternate open channel flow head-discharge relationship (Equation 4-4) intended to produce constant hydraulic roughness coefficients for given channel boundary materials (i.e., the roughness coefficient is independent of flow conditions). He assumed this equation would take the form of Equation 4-1 with \( y' = 1/2 \) as shown in Equation 4-6.

\[ V = C R_n^{y'} S_x^{1/2} \]  

(4-6)

The empirical basis for Equation 4-4 came from experimental data published by Bazin (1865), who hydraulically tested four different flow boundary materials [pure cement and 2-to-1 mix ratios of cement and fine sand, cement and small gravel (particle sizes ranging from 0.36 to 0.84 inches), and cement and large gravel (particle diameters ranged from 1.2 to 1.6 inches)]. After determining boundary roughness-specific constant values for \( C \) in Equation 4-6, Manning reported that the boundary roughness-specific average exponent \( x' \) values ranged from 0.6499 to 0.8395, with \( x' \) generally increasing with increasing boundary roughness. Manning assumed \( x' = 2/3 \) (a value most consistent with smoother boundary roughness materials) to be representative and considered the resulting equation, Equation 4-4, to be “sufficiently accurate” after applying the equation to numerous experiments. Recognizing the potential limitations of Equation 4-4, Manning (1889) suggested, that due to its empirical nature, the application of Equation 4-4 should be limited to situations where it has been tested and proven.
Equations for Variable Roughness Coefficients

Bathurst (2002) stated that some researchers have found success in using empirical formulas based on a power law relationship in the form of Equation 4-7 to describe hydraulic roughness coefficient variations.

\[ \frac{V}{V^*} = a \left( \frac{R_h}{k} \right)^b \]  

(4-7)

In Equation 4-7, \( V^* \) (shear velocity) = \( (gR_hS_e)^{1/2} \), \( a \) and \( b \) are empirical coefficients, and \( k \) is the equivalent roughness height, which Chow (1959) suggested is not necessarily equal to the height or even the average height of the roughness elements. \( k \) characterizes the effect of the roughness elements on the hydraulic roughness coefficient; however, it has limited physical meaning and its definition can vary by user. It is therefore another empirical coefficient, and its physical meaning depends on how it is defined for a particular equation. For example, in equations involving gravel beds, \( k \) is often defined as a representative \( D_r \) (the representative particle diameter of the channel boundary where \( r \) indicates the percentage of particles that are smaller than \( D_r \)). \( V/V^* \) is related to the standard hydraulic roughness coefficients as shown in Equation 4-8.

\[ \frac{V}{V^*} = \sqrt{\frac{8}{f}} = \frac{C_r}{\sqrt{g}} = \frac{K_\nu R_h^{1/6}}{n\sqrt{g}} \]  

(4-8)

It is important to note that Equation 4-7 is not a function of \( Re \) indicating that the data fit by the equation may be considered independent of \( Re \) and dependent only on relative submergence \( (R_s/k) \) which is indicative [according to the ASCE Task Force (1963)] of a “fully rough” turbulent flow regime.

Bray (1979) and Griffiths (1981) published power law relationships consistent with Equation 4-7 for rigid boundary gravel-bed rivers. Bathurst (2002) observed that, even though mountain streams may be characterized as gravel-bed rivers, these equations were relatively inaccurate when applied. Mountain streams are characterized by steep slopes and relatively low
According to Bathurst, one reason for these inaccuracies is that the relationships were developed by compiling data from many different river sites, fitting one curve to all the data, and then extrapolating these relationships to predict behaviors outside of the experimental data set. By gathering data for different flow conditions from the same river section and methodically grouping the data from similar sites, Bathurst (2002) showed that for the same type of channel (mountain streams), the data were best described by two significantly different relationships, suggesting that $a$ and $b$ are fairly site-specific parameters and are not solely dependent on a single channel type. He concluded that the differences between the coefficients in mountain streams were primarily related to variations in channel slope. Table 4-1 presents the coefficients for Equation 4-7 published in the referenced studies.

If Equation 4-7 is simplified and solved for $V$, as shown in Equation 4-9, the equation takes on the form of Equation 4-6 ($x' = b + 1/2$ and $C = a g^{1/2}/k^b$), which suggests a constant exponent $x'$ and a constant roughness coefficient $C$ for a given boundary roughness, provided that $a$ and $b$ are constant.

$$V = \left( \frac{a \sqrt{g}}{k^b} \right) R_s^{(1/2+b)} \delta_e^{1/2}$$  \hspace{1cm} (4-9)

Applying the coefficients from Table 4-1 to Equation 4-9 shows that the $x' = 2/3$ assumption made by Manning (1889) is not necessarily “sufficiently accurate” for all open channel flow conditions since the value of $x'$ can be boundary roughness specific, as illustrated by the data in Table 4-1. Manning (1889) warned of this himself in stating that empirical equations should be used with caution when they are applied outside of the boundaries of the data from which they are created. This study investigates the variation in $x'$ related to different boundary roughness types in a laboratory setting, where parameters are more easily controlled, to gain a better understanding of the appropriateness of the constant $n$ assumption applied to Manning’s equation.
Table 4-1. Published coefficients for power-law equation (Equation 4-7)

<table>
<thead>
<tr>
<th>Study</th>
<th>a</th>
<th>b</th>
<th>$x'$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bray (1979)</td>
<td>5.03</td>
<td>0.268</td>
<td>0.768</td>
<td>$D_{90}$</td>
</tr>
<tr>
<td>Griffiths (1981)</td>
<td>3.54</td>
<td>0.287</td>
<td>0.787</td>
<td>$D_{90}$</td>
</tr>
<tr>
<td>Bathurst (2002)</td>
<td>3.84</td>
<td>0.547</td>
<td>1.047</td>
<td>$D_{84}$</td>
</tr>
<tr>
<td>Bathurst (2002)</td>
<td>3.10</td>
<td>0.93</td>
<td>1.43</td>
<td>$D_{84}$</td>
</tr>
</tbody>
</table>

EXPERIMENTAL METHOD

The behavior of Manning’s $n$ for four different boundary roughness materials was investigated by conducting flow tests in a 4-ft wide by 3-ft deep by 48-ft long adjustable-slope, rectangular laboratory flume. The four boundary roughness materials tested included acrylic sheeting (see Figure 3-2); a low profile, commercially available expanded metal lath adhered to the acrylic flume walls and floor (see Figure 3-3); regularly spaced wooden blocks (see Figures 3-4 and 3-5); and trapezoidal corrugations oriented normal to the flow direction (see Figures 3-6 and 3-7). The wooden blocks, measuring 4 inches wide (normal to flow direction) by 3.5 inches long by 1.5 inches tall, with the top edges rounded (1-inch radius round-over), featured a painted exterior, and were assembled in a closely spaced, uniform pattern. The wooden trapezoidal corrugation elements were 1.5 inches tall, had a 1.5-inch wide top width and a 4.5-inch wide base, and were spaced 1.5 inches apart. The blocks and trapezoidal corrugation elements were attached to sheets of painted marine grade plywood that were attached to the flume floor and walls.

Water was supplied to the flume from a reservoir located adjacent to the laboratory and was metered using calibrated orifice flow meters located in the supply piping. Flow depths were measured using a precision point gage, readable to 0.008 inches, attached to a movable carriage located above the flume.

Manning’s $n$ can be directly calculated via Equation 4-4 when uniform flow exists in the channel and the flow depth ($y$) and flow rate ($Q$) are known. Due to the limited length of the
laboratory flume and the wide range of discharges and boundary roughness tests, uniform flow depths could not be achieved for all test conditions. For non-uniform flow conditions, a gradually varied flow (GVF) profile analysis technique was used, as discussed in Chapter 2. Figure 2-1 shows a plot of the \( n \) data calculated using the uniform flow and the GVF methods vs. \( y \) for the block boundary roughness. The plotted data show good agreement between the two methods.

The uniform flow data in Figure 2-1 show that, for the block roughness, \( n \) varies \( (0.087 \geq n \geq 0.048) \) with changes in uniform flow depth \( (0.13 \leq y \leq 0.9) \). Analysis of various truncated sections of a single GVF profile using the GVF \( n \) method also produced different predictive values for \( n \), suggesting that \( n \) is also variable with depth (and velocity) throughout a GVF profile. Based on the variable nature of \( n \) with \( y \) in the GVF profiles, the predicted normal depths \( (y_n) \) associated with the variable \( n \) values would also vary. Consequently, based on the good correlation between the uniform flow and GVF \( n \) data presented in Figure 2-1, \( y_{\text{average}} \), the average value of \( y \) in the measured GVF profile, was selected as the representative flow depth parameter in this analysis for calculating \( R_h, Re, V \), etc. For flow conditions where uniform flow developed, \( y_{\text{average}} = y_n \). The four boundary roughness materials were tested over a range of channel slopes and discharges.

**DISCUSSION AND RESULTS**

For Manning’s \( n \) coefficient to remain constant for a given channel lining material, independent of stage and discharge, the following two conditions must be met:

1. The mean flow velocity can be represented by an equation in the form of Equation 4-6.
2. \( x' \) will equal 2/3, independent of the channel lining material.

If these conditions are not met, then \( n \) must vary in order to match Equation 4-4 with the actual head-discharge relationship. Conditions 1 and 2 were tested by plotting \( \log(V/S_{0.5}) \) vs. \( \log(R_h) \) using data from Bazin (1865) and the current study. To satisfy Condition 1, the data
Table 4-2. Optimal $x'$ values

<table>
<thead>
<tr>
<th>Boundary Roughness Material</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bazin Study (1865)</strong></td>
<td></td>
</tr>
<tr>
<td>Pure Cement</td>
<td>Pure cement lining</td>
</tr>
<tr>
<td>Cement-Sand mix</td>
<td>2/3 cement, 1/3 fine sand mix</td>
</tr>
<tr>
<td>Small Gravel</td>
<td>Diameters ranging from 0.36-in to 0.84-in</td>
</tr>
<tr>
<td>Large Gravel</td>
<td>Diameters ranging from 1.2-in to 1.56-in</td>
</tr>
<tr>
<td>Laths of Wood (Corrugations)</td>
<td>0.36-in tall, 1.1-in wide, spaced 1.92-in apart, oriented normal to flume centerline</td>
</tr>
<tr>
<td><strong>Current Study</strong></td>
<td></td>
</tr>
<tr>
<td>Acrylic</td>
<td>Acrylic lining of flume boundary (see Figure 6-2)</td>
</tr>
<tr>
<td>Metal Lath</td>
<td>Commercially available expanded metal lath with a thickness of 0.125-in (see Figure 6-3)</td>
</tr>
<tr>
<td>Trapezoidal Corrugations</td>
<td>1.5-in in height, top width of 1.5-in, and bottom width of 4.5-in, spaced 1.5-in apart, oriented normal to flume centerline (see Figure 6-6)</td>
</tr>
<tr>
<td>Blocks</td>
<td>4.5-in by 3.5-in by 1.5-in tall, with the top edges rounded (1-in radius round-over) (see Figure 6-4)</td>
</tr>
</tbody>
</table>

should be well represented by a linear trendline of the form of Equation 4-10. In Equation 4-10, $C$ is equal to the y-intercept on the plot, and $x'$ is the slope. The corresponding $x'$ values are presented in Table 4-2.

$$\log \left( \frac{V}{S_e^{1/2}} \right) = C + x'\log(R_h) \quad (4-10)$$

The $x'$ values corresponding to Bazin’s data in Table 4-2 are consistent with those calculated and reported by Manning (1889) for the same data sets. The $r^2$ values [coefficient of determination applied to the linear relationship of $\log(V/S_e^{1/2})$ vs. $\log(R_h)$] in Table 4-2, which are all \(\approx 1.0\), indicate that $V$ is relatively well represented by Equation 4-6 and that Condition 1 is satisfied.

Condition 2, however, is not met according to the $x'$ data presented in Table 4-2, which varied with boundary roughness type. The smoother roughness boundary $x'$ values (e.g., pure cement, cement/sand mix, and acrylic) are approximately equal to the 2/3 value used by Manning.
Figure 4-1. Equivalent Manning’s $n$ coefficients ($n_c$, $n$, and $n_{opt}$) for Pure Cement (Bazin, 1865 data) (A), Large Gravel (Bazin, 1865 data) (B), and Block (C) roughness data.
$x'$ increased for the rougher boundaries (up to 1.16 for the blocks). The relevance of the different $x'$ values associated with Equation 4-6 to the assumption of a constant roughness coefficient is illustrated in Figure 4-1.

Figure 4-1 compares the roughness coefficients from three different versions of Equation 4-6: the Chezy (Equation 4-2) or Darcy-Weisbach (Equation 4-3) equation (where $x' = 1/2$), Manning’s equation (Equation 4-4) ($x' = 2/3$), and Equation 4-6 using the material roughness-specific $x'$ values presented in Table 4-2 that correspond to a constant $n$ value ($n_{opt}$). For convenience, the hydraulic roughness coefficient results in Figure 4-1 are all presented in terms of an equivalent $n$ value ($n_{eq}$). This was done by replacing $C$ in Equation 4-6 with $K_n/n_{eq}$ and noting that $K_n = 3.281^{(1-x')}$ for the individual boundary roughness materials.

Figure 4-1(A) presents $n_{eq}$ vs. $R_h$ for the pure cement lining data reported by Bazin (1865). The data show a variable $n_c$; $n$ and $n_{opt}$ are relatively constant and equal. The constancy of $n$ and $n_{opt}$ is due to the fact that Conditions 1 and 2 are both satisfied. The Manning’s $n$ data for the acrylic and the cement-sand mixture boundary conditions (not presented) had similar $x'$ values to the pure cement and behaved similarly.

Figures 4-1(B) and (C) present the data for the large gravel (Bazin, 1865) and the block roughnesses, respectively. These figures show examples where Condition 2 is not met and, therefore, Manning’s equation requires a variable $n$ value to match the results of the experimental data. While Manning’s equation (Equation 4-4) improves upon Chezy’s Equation (Equation 4-2) [i.e. the difference between the maximum and minimum $n_{eq}$ values decreases from 0.0124 ($n_c$ curve) to 0.008 ($n_c$ curve)], the roughness coefficient is not constant unless $x'$ of Equation 4-6 is optimized for these specific boundary roughness materials as evidenced in the $n_{opt}$ curve.

The results clearly indicate that either $x'$ or the boundary roughness coefficient ($n$, $f$, or $C$) must vary to accurately describe the hydraulic behavior of the stage-discharge relationship as $R_h$ varies. Although some research has suggested correcting Manning’s equation by changing $x'$...
(Blench, 1939), more recent research has focused on variable roughness coefficient predictive techniques (Limerinos 1970, Bray 1979, Griffiths 1981, Bathurst et al. 1981, Jarrett 1984, Ugarte and Madrid 1994, Bathurst 2002) for use in Equations 4-2, 4-3, and/or 4-4. Equation 4-9 shows that using the power law equation to determine a variable hydraulic roughness coefficient is basically the equivalent of changing the $x'$ value of Equation 4-6 and applying a constant roughness coefficient.

These power law equations are generally developed for a specific boundary roughness type with the underlying assumption that the equation applies to a range of roughness element sizes (generally characterized by $k$). For example, Bray (1979) and Griffiths (1981) present equations developed for channels with rigid gravel beds; Bathurst (2002) presents an equation for mountain streams. Each of these equations uses a $k$ value defined by specific gravel $D_r$ values. They assume that a single $x'$ value may apply to a range of roughness element sizes whose size can be characterized by a common $D_r$ value for a certain type of boundary roughness.

The $r^2$ value reported for the Bray (1979) and Griffiths (1981) equations are 0.355 and 0.591, respectively, suggesting that a considerable amount of scatter exists in the data. Griffiths (1981) attributes the scatter to inadequate descriptions of the channel reach and hydraulic variables, restrictions and errors in data collection procedures, irregularities in the alignments and channel cross sections, and the rugged bed topography.

Bathurst (2002) found that if the data were divided into groupings based on channel similarities, the scatter decreased significantly (increased $r^2$ values). This resulted in two equations with $x'$ values of 1.047 and 1.43, respectively. The difference in these two $x'$ values was attributed to differences in the channel slope: 1.047 for slopes < 0.8% and 1.43 for slopes > 0.8%.

The results from the current study (Table 4-2) suggest that roughness element size may have a significant effect on the value of $x'$. The Bazin (1865) gravel data produced $x'$ values
equal to 0.721 and 0.822 for the small and large gravel tests. The block data, which is somewhat representative of a rigid gravel or small cobble bed (flow can pass over and around the projecting roughness elements), produced an $x' = 1.16$, suggesting that $x'$ increases with increasing gravel or roughness element size. The smoothest boundary materials (acrylic, pure cement, and cement-sand mix) produced the smallest and relatively constant $x'$ values of 0.644, 0.676, and 0.684, respectively). The corrugated boundary roughness materials produced increasing $x'$ values with increasing corrugation size ($x' = 0.732$ for Bazin’s “laths of wood” and $x' = 0.968$ for the relatively larger trapezoidal corrugations).

For the roughness materials evaluated in this study, channel slope was not a significant factor of the $x'$ value (i.e., the data in Figure 4-1(C), which include multiple channel slopes, fall on a single curve). Although Bathurst (2002) points out differences between the channel geometries and typical boundary roughness materials used in flume studies and those found in natural mountain streams, both the Bathurst (2002) results and the current study indicate that $x'$ is dependent on more than simply the roughness material type or channel geometry. Therefore, an equation in the form of Equation 4-6, with a constant hydraulic roughness coefficient, will not accurately describe the stage-discharge relationship for a general boundary type classification such as gravel channels. $x'$ will vary with the size, density, spacing, and alignment of the boundary roughness elements.

The prospect of developing equations specific to the boundary roughness type as well as the size, density, and distribution of the individual roughness elements is a somewhat daunting task. Manning’s (1889) original intent was a single simple equation that would produce “sufficiently accurate” results considering the information available. It is interesting that Manning’s $x' = 2/3$ and his boundary-specific, constant $n$ assumption have withstood the test of time for so long considering the resulting range of required $x'$ values determined in this and other
Figure 4-2. Plot of $\log(V/S_{e}^{1/2})$ versus $\log(R_{h})$ data for acrylic, metal lath, block, and trapezoidal corrugation boundary roughness materials studies required to support a constant $n$ value. A closer look at the data gives insight to the longevity and relative reliability of Manning’s equation.

The $x'$ values reported in Table 4-2 represent the data with a single optimized head-discharge curve. Figure 4-2 presents $\log(V/S_{e}^{1/2})$ vs. $\log(R_{h})$ plots for the acrylic, metal lath, blocks, and trapezoidal corrugation channel lining material data. With the exception of the data for the smooth acrylic, the data in Figure 4-2 are better represented by two linear trend lines, each with a different slope ($x'$), as described by Equation 4-10. Consistent with Manning’s equation (Equation 4-4), the acrylic data correlated well with the $x' = 2/3$ trend line slope represented on the plot by a dashed line. The metal lath and the trapezoidal corrugation data sets both exhibit variable dependence on $R_{h}$ as shown by the two distinct trend lines of differing slope corresponding to the “higher” and “lower” $R_{h}$ data ranges. $x'$ values for the higher $R_{h}$ data ranges
(metal lath and trapezoidal corrugation data) are reasonably represented by \( x' = 2/3 \) (Manning’s equation). The smaller \( R_h \) data ranges for both data sets require \( x' > 2/3 \) to match the experimental data (e.g., \( x' = 0.9 \) for the metal lath and \( x' = 1.25 \) for the trapezoidal corrugations are required to better match the larger \( R_h \) experimental data). The block data correspond to a single linear trend line with \( x' = 1.2 \). This result, however, may be due only to the fact that sufficiently high \( R_h \) values could not be achieved in the test facility to identify a range of \( R_h \) where the \( x' = 2/3 \) is appropriate. Note that the higher \( R_h \) block data (top 7 to 8 data points) are beginning to deviate slightly from the trend line. In summary, the acrylic boundary (over the full range of \( R_h \)) and the metal lath and trapezoidal corrugation channel lining materials at larger \( R_h \) values produced an \( x' = 2/3 \). For all other conditions, including the block channel lining material, alternate \( x' \) values were required in order to fit the data for each of the roughness materials.

These results suggest that Conditions 1 and 2 are met when either the roughness boundary itself is smooth (e.g. the acrylic and cement boundaries) or at higher \( R_h \) values for rougher boundaries. This finding is consistent with the quasi-smooth boundary condition theory discussed in Chapter 3, where stable eddies form between the roughness elements of the rougher boundaries, creating a quasi-smooth flow condition above the roughness elements, and reducing the effect of the specific boundary roughness geometry on the flow resistance. The acrylic and cement boundaries represent smooth-flow boundary conditions. When \( x' \) for the rougher boundary materials is equal to 2/3 (larger \( R_h \) values), the flow condition is consistent with the quasi-smooth boundary condition. When channels lined with rougher boundary materials operate outside of the quasi-smooth flow condition, then Conditions 1 and 2 are no longer met, \( x' \neq 2/3 \), and/or \( n \) cannot be considered constant. The longevity and relative reliability of the use of Manning’s equation (Equation 4-4) with boundary-specific constant \( n \) values suggests that many of the channels used in practice have relatively smooth flow boundaries (e.g., cement-lined channels) or that they may commonly operate in the quasi-smooth flow condition.
CONCLUSIONS

When applying Manning’s equation, the assumption is often made that \( n \) is a constant value, independent of flow depth and discharge for a given channel lining material. An inspection of the experimental data from the current study and from Bazin’s (1865) showed that the applicability of the constant \( n \) assumption diminishes as the roughness of the boundary increases. To produce a constant \( n \) value for a given boundary roughness material at all flow conditions, the mean velocity must be well represented by an equation in the form of Equation 4-6 and the representative \( x’ \) coefficient must equal 2/3. This study evaluated these two conditions for a range of boundary roughness materials and produced the following conclusions:

1. The data showed that Equation 4-6 provided a relatively good overall fit to the data for each of the lining materials tested.

2. Only the smooth boundary materials (e.g., acrylic sheeting and pure cement) produced an \( x’ = 2/3 \), based on the Equation 4-6 relationship for the entire range of \( R_h \) tested. \( x’ \) was found to be a unique value for each boundary material tested, ranging from 0.644 (acrylic sheeting) to 1.16 (blocks), with the \( x’ \) value increasing with increasing boundary roughness.

3. Relative to the other hydraulic roughness coefficients (\( C_c \) and \( f \)), Manning’s \( n \) exhibited less variability with respect to changes in \( R_h \) (see Figure 4-1). \( n \) approaches or becomes constant as \( R_h \) increases. Based on the range of flow conditions tested (in a rectangular flume), the range of \( R_h \) values over which \( n \) is constant decreases as the roughness of the boundary material increases. For very smooth boundaries (e.g., acrylic sheeting), \( n \) was approximately constant over the entire range of \( R_h \) tested.

4. The value of \( x’ \) that corresponds to Equation 4-6 not only varied with boundary roughness material type as reported in conclusion 2, it also varied with \( R_h \) for a given boundary roughness material. The metal lath and trapezoidal corrugation data in Figure
4-2 show that two separate linear curves that correspond to different \( x' \) (Equation 4-6) values are required in order to best match the experimental data. This means that over the range of \( R_h \) values tested, a constant \( n \) value cannot be applied to these boundary roughness materials when using Manning’s equation (Equation 4-4) with \( x' = 2/3 \).

Manning’s equation with a constant \( n \) value gives a good representation of the data at larger \( R_h \) values where quasi-smooth-type flow conditions exist. The block data also showed evidence that at larger \( R_h \) values there would be a shift in the \( x' \) value. Sufficiently high \( R_h \) data for the blocks were not obtainable with the experimental test setup to confirm the high \( R_h \) block \( x' \) value.

5. The results of this study show that Manning’s \( n \) will not likely be a constant value for canals, streams, and rivers with rough boundaries such as large gravels and cobbles unless the \( R_h \) is sufficiently large. The limiting \( R_h \) above which quasi-smooth flow conditions exist and \( n \) becomes constant will be specific for each boundary roughness type and must be determined by testing.

In summary, Manning’s Equation (Equation 4-4) coupled with a constant \( n \) assumption has its place and as suggested by the ASCE Task Force on Friction Factors in Open Channels (1963) should continue to be used by Engineers who prefer it; however, the engineer should consider additional stipulations as laid out by the conclusions of this study. Not only should the flow in the channel be considered to be of “Fully Rough” turbulence (friction factor is independent of \( Re \)) but the flow condition should also be such that it has reached a quasi-smooth flow condition as described in Chow’s Open Channel Hydraulics (1959). More data is required to define exactly when that condition might occur and it is stipulated that this condition will be dependent on the roughness in the channel.
Otherwise, the engineer may need to consider other methods or the use of sets of equations to determine the relationship between discharge and head in open channels. For example, Manning’s equation might still be applied but must be accompanied by an equation which would determine a variable $n$ value like that of the form of Equation 4-7. The engineer should also take into consideration that the accuracy of any such equation of this nature is most likely dependent on a relatively narrow set of parameters describing the roughness boundary for which it was prescribed.
CHAPTER 5
OPEN CHANNEL FLOW RESISTANCE: COMPOSITE CHANNELS

ABSTRACT

A composite channel in open channel flow describes a condition where different roughness materials line different parts of a channel cross section. Some examples of composite channels include concrete rectangular or trapezoidal channels where the channel invert has been covered with sand and/or gravel as a result of sediment transport; vegetation can also be present in the channel invert. Fish passage culverts, where the culvert invert is typically representative of the natural channel (e.g., bottomless culverts) and the walls are fabricated from concrete or corrugated metal, are another example of composite roughness channels. Most open channel flow problems are solved using Manning’s equation. Estimating the head-discharge relationship for composite channels poses a unique challenge due to the fact that Manning’s equation is a one-dimensional head-discharge relationship that is being applied to what are very likely three-dimensional flow problems. Ideally, a representative Manning’s $n$ hydraulic roughness coefficient would be defined that accounts for the three-dimensional nature of the composite channel flow condition.

A literature search produced a list of 16 different relationships that have been proposed for estimating representative composite channel $n$ values, referred to as $n_c$, which are dependent upon the $n$ values of the individual channel lining materials, referred to as $n_i$, comprising the composite channel boundary geometry. The degree to which these relationships have been evaluated against experimental composite channel data is limited. In this study 12 different composite channel configurations were tested in a rectangular laboratory flume, using combinations of the boundary roughness materials evaluated in Chapters 3 and 4 (acrylic

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3 Coauthored by Tyler G. Allen, P.E.; Blake P. Tullis, Ph.D., P.E.
sheeting, metal lath, blocks, and trapezoidal corrugations). The composite roughness configurations were categorized into three different channel types: Type I featured rougher walls and a smoother floor, Type II featured smoother walls and a rougher floor, and Type III featured rough walls and floor. The 16 different \( n_e \) relationships, which use a weighted average of the \( n_i \) values based on a corresponding flow subarea and/or wetted perimeter to each roughness material comprising the composite roughness boundary, were evaluated along with different methods for evaluating \( n_i \). It was determined that for hydraulically rougher boundary roughness materials where \( n \) varies with flow conditions (e.g., \( n \) varied with \( R_e/K \) for all of the materials tested except for the smooth acrylic sheeting) the variation in \( n_i \) should be applied to the \( n_e \) relationships. In general, some of the relationships performed worse than the others, but no relationship proved to be consistently more accurate than the other predictive relationships for all composite channel configurations. The predictive error, which was represented by the root-mean-square (RMS), ranged from approximately 5 to 90%, with the majority of the methods producing RMS values in the range of 5 to 20%.

Based on the fact that the more complicated \( n_e \) predictive methods did not produce more accurate results than the simpler \( n_e \) predictive methods, the simpler \( n_e \) predictive methods are recommended, namely the Horton method, keeping in mind that the level of uncertainty can still be significantly high. It should also be noted that, even though the range of hydraulic roughness boundary materials (\( n_i \)) was broader, the number of composite roughness geometries tested (12) was larger, and the number of \( n_e \) relationships evaluated was significantly larger than previous studies. The applicability of the test results to channels with cross sections that are different from the one tested in this study, as well as to composite roughness geometries that feature irregular roughness element patterns (the individual boundary roughness elements used in this study all feature uniform roughness element patterns) has not been determined. Until such time as more accurate data are available, the results from this study are recommended as a first-order
approximation for composite roughness problems in practice. The inclusion of a reasonable factor of safety is also recommended.

INTRODUCTION

It is not uncommon in open channel flow field applications for the wetted perimeter of a cross section to be made up of more than one roughness material (e.g., concrete channels with the invert covered with sediment, gravel, and/or vegetation, or buried-invert culverts). Yen (2002) referred to such channels as composite channels. The composite channel flow resistance will be a function of the combined effects of the individual flow boundary roughness materials. The most commonly used open channel flow equations (Manning, Chezy, Darcy-Weisbach), however, are one-dimensional and are limited to a single, representative hydraulic roughness coefficient. Yen (2002) published 16 different composite Manning’s $n (n_e)$ relationships (see Table 5-1) as possible candidates for use with Manning’s Equation (Equation 8-1) to predict flow resistance in composite channels.

$$V = \frac{K_n}{n} R_h^{2/3} S_e^{1/2}$$  \hspace{1cm} (5-1)

In Equation 5-1, $V$ is the mean velocity, $K_n = 1.0$ (1.49 for ES), $R_h$ is the hydraulic radius [the ratio of the flow area ($A$) to the wetted perimeter ($P$)], and $S_e$ is the friction slope, which at uniform flow is equal to the channel slope ($S_o$). The $n_e$ relationships published by Yen (2002) are based on various techniques for weighting the resistance of the individual boundary roughness materials in the channel cross section. This is accomplished by partitioning $A$ and/or $P$ (resulting in component $A_i$ and/or $P_i$ values) between the boundary roughness materials and applying the individual $n$ values of the boundary roughness materials, referred to as component $n$ values ($n_i$), to each partitioned section. The result is a single, representative $n_e$ value that then is applied to Equation 5-1.
Previous studies compared relatively small subsets of the $n_e$ relationships listed in Table 5-1 (Pillai 1962, Cox 1973, Flintham and Carling 1992); in total, the performances of 5 of the 16 $n_e$ relationships presented by Yen (2002) have been evaluated using experimental data. Yen (2002) stated that the amount of published data available for composite channels is limited and therefore it is yet to be determined which of the 16 predictive $n_e$ relationships is best suited for use. The current study provides an expanded experimental data set for evaluating the performance of the 16 $n_e$ relationships using combinations of the four boundary roughness materials (acrylic sheeting, metal lath, trapezoidal corrugations, and blocks) discussed in Chapter 3.

The $n_i$ values for the individual boundary roughness materials of the current study range from $n_i = 0.0096$ for the smooth acrylic sheeting to $n_i = 0.033$ to 0.086 ($R_h$ or $R_h/k$ dependent) for the blocks. This range of $n_i$ values exceeds the range of hydraulic roughness values evaluated in the previous studies (Pillai 1962, Cox 1973, Flintham and Carling 1992). The composite-channel flow resistance testing of the current study includes 12 different composite-channel lining combinations of the individual lining materials.

According to Flintham and Carling (1992), the accuracy of $n_e$ relationships should be dependent upon two factors: (1) the method used to partition the channel cross-sectional flow area into the subareas directly influenced by each roughness material lining the boundary and (2) an accurate determination of the $n_i$ values. The influence of the flow area partitioning technique on $n_e$ was found by Flintham and Carling (1992) to be relatively negligible when compared to the significance of the $n_i$ values selected. This study examines the behavior of $n_i$ (the dependence of $n_i$ on $R_h$ in a uniformly lined channel) and the influence of $n_i$ on the $n_e$ relationships.
Table 5-1 Composite Channel $n_z$ Relationships

<table>
<thead>
<tr>
<th>Name</th>
<th>$n_z$ Relationship</th>
<th>Secondary Assumptions</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean velocity assumption</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horton</td>
<td>$\left[ \frac{\sum (n_i^{3/2} P_i)}{P} \right]^{2/3}$</td>
<td>$S_{ei} = S_e$</td>
<td>(5-2)</td>
</tr>
<tr>
<td>Colebatch</td>
<td>$\left[ \frac{\sum (n_i^{3/2} A_i)}{A} \right]^{2/3}$</td>
<td>Same as Horton but adjusted by a factor of $C = \frac{R_{real}}{R_{base}}$</td>
<td>(5-3)</td>
</tr>
<tr>
<td><strong>Total force assumption</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pavlovski</td>
<td>$\sqrt{\frac{\sum (n_i^2 P_i)}{P}}$</td>
<td>Yen (2002): $V_i/V = (R_i/R_h)^{1/6}$ or Flintham &amp; Carling (1992): $V_i = V$ and $R_i = R_h$</td>
<td>(5-4)</td>
</tr>
<tr>
<td>Total F2</td>
<td>$\sqrt{\frac{\sum (n_i^2 A_i)}{A}}$</td>
<td>$V_i/V = (R_i/R_h)^{2/3}$</td>
<td>(5-5)</td>
</tr>
<tr>
<td>Total F3</td>
<td>$\sqrt{\frac{R_h^{1/3}}{P} \sum \frac{n_i^2 P_i}{R_h^{1/3}}}$</td>
<td>$V_i = V$</td>
<td>(5-6)</td>
</tr>
<tr>
<td>Total F4</td>
<td>$\sqrt{\frac{\sum (n_i^2 P_i R_i^{2/3})}{P R_h^{2/3}}}$</td>
<td>$V_i/V = (R_i/R_h)^{1/2}$</td>
<td>(5-7)</td>
</tr>
<tr>
<td><strong>Total flow assumption</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lotters</td>
<td>$\frac{p R_h^{5/3}}{\sum (P_i R_i^{7/3} / n_i)}$</td>
<td>$S_{ei} = S_e$</td>
<td>(5-8)</td>
</tr>
<tr>
<td>Lotters II</td>
<td>$\frac{\sum (p R_i^{5/3})}{\sum (P_i R_i^{7/3} / n_i)}$</td>
<td>-</td>
<td>(5-9)</td>
</tr>
<tr>
<td>Total Q1</td>
<td>$\frac{A}{\sum (A_i / n_i)}$</td>
<td>$S_{ei}/S_e = (R_i/R_h)^{4/3}$</td>
<td>(5-10)</td>
</tr>
<tr>
<td>Total Q2</td>
<td>$\frac{p}{\sum (P_i / n_i)}$</td>
<td>$S_{ei}/S_e = (R_i/R_h)^{10/3}$</td>
<td>(5-11)</td>
</tr>
<tr>
<td>Total Q3</td>
<td>$\frac{p R_h^{7/6}}{\sum (P_i R_i^{7/6} / n_i)}$</td>
<td>$S_{ei}/S_e = (R_i/R_h)$</td>
<td>(5-12)</td>
</tr>
<tr>
<td><strong>Total shear velocity assumption</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAD</td>
<td>$\frac{\sum (n_i A_i)}{A}$</td>
<td>$V_i/V = (R_i/R_h)^{7/6}$</td>
<td>(5-13)</td>
</tr>
<tr>
<td>HDM</td>
<td>$\frac{\sum (n_i P_i)}{P}$</td>
<td>$V_i/V = (R_i/R_h)^{1/6}$</td>
<td>(5-14)</td>
</tr>
<tr>
<td>Total U*1</td>
<td>$\frac{\sum (n_i P_i / R_i^{1/6})}{P / R_h^{1/6}}$</td>
<td>$V_i = V$</td>
<td>(5-15)</td>
</tr>
<tr>
<td>Total U*2</td>
<td>$\frac{\sum (n_i P_i / R_h)}{P / R_h}$</td>
<td>$V_i/V = (R_i/R_h)^{2/3}$</td>
<td>(5-16)</td>
</tr>
<tr>
<td>Total U*3</td>
<td>$\frac{\sum (n_i P_i / R_h^{1/3})}{P / R_h^{4/3}}$</td>
<td>$V_i/V = (R_i/R_h)^{1/2}$</td>
<td>(5-17)</td>
</tr>
</tbody>
</table>
BACKGROUND

Component n Values ($n_i$)

Chow (1952) stated that the most difficult task in the use of Equation 5-1 is assigning a roughness coefficient ($n$) value and that the inexact methods for doing so range from guesswork to empirical relationships. Although the $n_e$ relationships in Table 5-1 are fundamentally based on the channel geometry and the distribution of the hydraulic roughness boundary materials over the wetted perimeter, $n_e$ has an inherent level of uncertainty due to the uncertainty associated with specifying $n$. The relationship between $n$ (or $n_i$) and $Re$, $R_f/k$, and other factors was discussed in Chapters 3 and 4.

Manning’s objective in developing the one-dimensional, open channel flow equation (Equation 5-1) was to find a relationship where the hydraulic roughness coefficient ($n$) would be constant (dependent only on $k$ and independent of the flow conditions). After evaluating Equation 5-1 (using the boundary roughness-specific constant $n$ assumption) using numerous experimental data sets, Manning (1889) concluded that the equation is “sufficiently accurate.” Chow (1952) stated that, in general, $n$ is not constant but rather decreases with increasing stage for most streams, a fact that was confirmed in Chapters 3 and 4. Other studies have shown that $n$ can vary with stage, discharge, and slope in certain uniformly lined channel applications (Limerinos 1970, Bray 1979, Bathurst et al. 1981); Yen (2002) recommended that $n$ may be considered nearly a constant and almost independent of flow conditions. These apparent contradictions suggest that some level of uncertainty still exists regarding the appropriateness of the constant $n$ assumption and Manning’s Equation.

The $n$ (or $n_i$) data for this study are determined in the rectangular test flume, uniformly lined with each boundary material separately. The $n_i$ data, the constant and/or variable nature of which depends in part upon the boundary roughness ($k$) and $R_f$, are presented in Figure 5-1 for the smooth acrylic sheeting, metal lath sheeting, blocks, and trapezoidal corrugations.
The acrylic sheeting Manning’s $n$ data in Figure 5-1, which represents the smoothest boundary roughness material tested, remain relatively constant over the full range of $R_h$ tested. The metal lath and trapezoidal corrugation $n$ values vary with $R_h$ over the lower 20-30% of the data range and are relatively constant above that limit. The block data varies over the full range of $R_h$ tested; however, the fact that the block $n$ data appears to be approaching a constant value suggests that the absence of a constant $n$ range in the experimental data set is likely due more to flow capacity limitations than boundary roughness characteristics. These same boundary roughness materials are used to create the composite channel linings in the current study; the data in Figure 5-1 are used to generate the $n_i$ values used in evaluating the $n_e$ relationships in Table 5-1. The boundary roughness materials are identified in this chapter as follows: A (acrylic

![Graph](image)

Figure 5-1. Manning’s $n$ data from channels with uniform roughness materials
sheeting), B (metal lath), D (blocks), and E (trapezoidal corrugations). For all composite roughness test configurations, common roughness materials are used for the walls and a different roughness material is used on the floor.

**Composite Manning’s n (n_c) Equations**

The 16 $n_c$ relationships listed in Table 5-1 are divided into four groups based on the main assumption used in their derivation. These assumptions are as follows:

- **The Mean Velocity assumption**: The mean velocity in the cross-sectional flow subarea associated with each boundary roughness material is equal to the mean velocity of the entire channel cross section.

- **The Total Force assumption**: The sum of the forces resisting the flow in each subarea is equal to the total force resisting the flow in the channel.

- **The Total Discharge assumption**: The sum of the subarea discharges is equal to the total channel discharge.

- **The Total Shear Velocity assumption**: The weighted sum of the shear velocities of each subarea is equal to the total shear velocity of the channel.

Secondary assumptions are also typically required for the derivation of these equations. The secondary assumptions for each relationship, where applicable, are also listed in Table 5-1.

The $n_c$ relationships in Table 5-1 are dependent on the way in which subareas of the channel cross-sectional flow area are apportioned to each boundary roughness material comprising the composite wetted perimeter. In Equations 5-2 through 5-17, $R_{ni}$ is equal to the ratio of $A_i$ to $P_i$ ($R_{ni} = A_i/P_i$) and the subscript “i” denotes the different subareas of the channel cross section associated with each roughness material comprising the wetted perimeter.
Komora (1973) recommended that the cross-sectional flow area of a composite-channel be subdivided by curves that intersect the cross-sectional velocity contours at right angles as depicted in Figure 5-2. This requires detailed velocity data that for most practical applications will likely not be available. To avoid this complication, Colebatch (1941) recommended using a straight line to bisect the angle at the point of the boundary roughness change (e.g., In Figure 5-2, the 45°-angled lines from the corner separate the flow subareas in the rectangular channel featuring different boundary roughness materials on the floor and walls). Flintham and Carling (1992) compared both methods to their data set and concluded that there were no obvious advantages with either subarea delineation method. For convenience, the angle bisection method is used throughout this study for the \( n_e \) equations.

Wherever the subarea dividing line is drawn, it is assumed that shear stress is equal to zero along that boundary (though not necessarily true). Consequently, only wetted perimeters corresponding to physical channel boundaries \( P_i \) are included in flow resistance calculations, as shown in Figure 5-2.
shown in Figure 5-2. Flow boundaries between adjacent subareas are not included as part of the $P_i$ dimension (Yen, 2002).

**Previous Studies**

Three published studies that evaluated the effectiveness of various subsets of the $n_e$ relationships in Table 5-1 are reviewed for this study. Each study featured a unique set of composite channel boundary roughness materials and configurations. The experimental composite-channel results were compared with the predictive $n_e$ relationships.

Pillai (1962) studied composite roughness flow resistance in both rectangular and trapezoidal channels and evaluated the Horton (1933), Pavlovskii (1931), and Lotter (1933) $n_e$ relationships using two different boundary roughness materials described as (1) smooth cement with fine sand and (2) cement plastered with gravel that passes a 1/2-inch sieve and was retained on a 1/4-inch sieve. Pillai (1962) selected $n_i$ as the average experimental $n$ value ($n_{\text{average}}$) for each boundary roughness material, whose values were reported as 0.009836 (cement and fine sand mix) and 0.0178 (cement and gravel). Of the three relationships evaluated by Pillai, the Lotter relationship was the only one requiring subarea delineation. Lotter’s relationship was only applied to the trapezoidal channel data where the subareas were divided using vertical lines originating at the corners of the channel cross-section. Pillai (1962) concluded that the Horton relationship performed the best and that the Lotter relationship gave inconsistent results.

Cox (1973) conducted composite roughness testing in a rectangular channel using the bisecting angle method for subarea delineation. Two roughness materials were tested, a plastic coated plywood ($n = 0.0095$) and crushed limestone particles that passed a No. 4 sieve and were retained on a No. 8 sieve ($n_{\text{average}} = 0.0165$). Cox compared the Horton (1933), Colebatch (1941), and LAD relationships and recommended the LAD and Colebatch relationships over the Horton.
Flintham and Carling (1992) studied composite roughness in a trapezoidal channel using the bisecting angle method for subarea delineation. Three roughness materials were tested: plywood, 0.24-inch diameter gravel, and 0.55-inch diameter gravel. The reported average Manning’s $n$ values for the 0.24-inch and 0.55-inch gravels were 0.019 and 0.022, respectively (the plywood $n$ was not published). Flintham and Carling (1992) were the only ones to use boundary material-specific variable $n_i$ values in their analysis. They concluded that, with respect to the boundary roughness materials tested, using the varying $n_i$ values improved the accuracy of the predictive relationships relative to using average $n$ values. Their study was limited, however, to channel roughness configurations where the floor roughness exceeded the sidewall roughness. Flintham and Carling (1992) evaluated the Horton (1933), Colebatch (1941), Pavlovski (1931), and Lotter (1933) methods. They concluded that the Pavloski relationship is the most accurate, the Horton and Colebatch relationships are satisfactory, and the Lotter relationship performs poorly.

Four of the five relationships that are evaluated in the three different studies were identified at least once as a “best performer,” but consensus was not achieved regarding an overall best method. The Lotter relationship, on the other hand, was singled out in each study as “not recommended for use.” In the current study, all 16 predictive $n_r$ relationships are compared against the experimental data set developed in this study. The number of boundary roughness materials tested in the current study (four), exceed the number of roughness materials tested in any of the three previous studies. The diversity in composite roughness channel lining configurations and the hydraulic roughness characteristics of the boundary roughness materials used in the current study are also broader than those used in the previous studies.
EXPERIMENTAL SETUP

All composite roughness testing was conducted in a 4-ft wide by 3-ft deep by 48-ft long rectangular flume. Flow was supplied to the flume through either an 8-inch or a 20-inch diameter supply pipe, each containing a calibrated orifice flow meter.

Four boundary roughness materials were used in this study: the acrylic flume walls and floor were used as a smooth surface (see Figure 3-2), a commercially available metal lath sheeting material measuring 1/8-inch in height (see Figure 3-3), wooden blocks measuring 3.5-inches long (in the flow direction) by 4.5-inches wide by 1.5-inches tall with a 1-inch radius rounded top edge (Figures 3-4 and 3-5), and 1.5-inch tall trapezoidal corrugations measuring 4.5-inches wide at the base and 1.5-inches wide at the top (Figures 3-6 and 3-7). The blocks were attached to a plywood base in a staggered pattern with a 1.85-inch spacing between blocks as shown in Figure 3-5. The trapezoidal strips were also attached to a plywood base and oriented perpendicular to the flow direction at a spacing of 1.5 inches as shown in Figure 3-7. The acrylic, metal lath, block, and trapezoidal corrugation roughness materials are hereafter identified as boundary roughness materials A, B, D, and E, respectively.

Manning’s $n$ data for each boundary roughness material were determined as described in Chapter 2. The $n$ data for Material A was relatively constant ($n_{\text{average}} = 0.0096$), as shown in Figure 5-1. The $n$ data for materials B, D, and E varied with $R_h$ (see Figure 5-1) and trend line functions were used to represent $n_i$ in the $n_c$ calculations.

Twelve different composite channel geometries were created through various combinations of the materials A, B, D, and E. In all cases, the channel sidewalls featured common boundary roughness materials while the floor featured another. The three-letter notation for the composite roughness configurations represents the sidewall, floor, and sidewall boundary roughness materials. The following combinations were tested: ABA, BAB, ADA, DAD, BDB, DBD, AEA, EAE, BEB, EBE, EDE, and DED. An example of the BDB composite roughness
configuration (metal lath on the sidewalls and wooden blocks on the floor) is shown in Figure 5-3(A). The same procedure discussed in Section 6-4 to determine $n_i$ for the uniform channel roughness lining tests was also used to determine the experimental composite $n_e$.

The composite roughness channel configurations were also categorized into three channel types. A Type I channel represents a channel where the floor roughness exceeds the wall roughness (e.g., ABA, ADA, BDB, AEA, BEB); Type II represents a channel where the wall roughness exceeds the floor roughness (e.g., BAB, DAD, DBD, EAE, EBE); and Type III represents a channel where the walls and floor both featured “large roughness element” boundary materials of different types (e.g., EDE, DED).

EXPERIMENTAL RESULTS

Optimization of the $n_e$ Relationships

The results of the comparison between the experimental $n_e$ data and the 16 $n_e$ relationships in Table 5-1 are quantified using the root mean square (RMS) (Equations 5-18 and 5-19). Doubling the RMS represents a 95% confidence interval.

![Figure 5-3: Examples of composite roughness channel types: Type I (BDB) (A), Type II (DAD), and Type III (EDE)](image)
\[ \text{RMS} = \sqrt{\frac{\sum PE^2}{\text{samples}}} \]  

(5-18)

\[ PE = \frac{\text{predicted} - \text{measured}}{\text{measured}} \times 100 \]  

(5-19)

In Equations 5-18 and 5-19, \( PE \) is the percent predictive error and “samples” represents the total number of data points in the set. The bias is the mean value of \( PE \). RMS values of each equation were calculated for both the individual composite channel configurations (EAE, BDB, etc.) and each of the composite channel types (Types I, II, and III). The bias of each equation is also determined.

Flintham and Carling (1992) emphasized the sensitivity of the specific \( n_i \) values assigned to represent the individual roughness boundaries in a composite roughness channel when calculating \( n_e \). Three different methods for determining \( n_i \) are used in the current study in an effort to investigate the influence of the \( R_h \) dependence of \( n_i \) on \( n_e \). Method 1 assumes a constant \( n_i \) value for each material that corresponds to the large-\( R_h \) constant \( n_i \) values shown in Figure 5-1 instead of the average \( n_i \) value as used by Flintham and Carling (1992). The constant \( n_i \) value for material D is estimated by extrapolating the experimental data trend to larger \( R_h \) values (\( n_i = 0.0335 \)). Method 2 assumes that the \( n_i = F(R_h) \) relationships for the composite channel subareas are equal to the \( n = F(R_h) \) relationships for the uniformly lined channel data (i.e., \( n_i \) for each subarea is calculated based on \( R_{hi} \) for that subarea). Method 3 is similar to Method 2 except that \( n_i \) for each subarea is calculated using \( R_h \) (the total channel hydraulic radius) rather than \( R_{hi} \) [i.e., \( n_i = F(R_h) \)]. The RMS values for the trend line functions used to predict the variable \( n_i \) relationships for the \( n \) vs. \( R_h \) data presented in Figure 5-1 for each boundary roughness material (A, B, D, and E) are 4.5%, 2.43%, 3.04%, and 4.29%, respectively.

The resulting total RMS values based on a combined data set from all composite channel configurations (e.g., ADA, BEB, etc.) in each channel type (Channel Types I, II, or III) of the
individual relationships in Table 5-1 are presented in Table 5-2 according to channel type and method, or combination of methods, applied to determine \( n_e \). Similar to the findings of previous studies, the Lotter relationship preforms inconsistently with respect to its ability to match the experimental data from the current study. The inconsistent results are shared by all the relationships within the Total Flow Assumption \( n_e \) group and, as a result, the outcome for the Total Flow Assumption relationships will be discussed separately from the other relationships.

It is clear that a significant improvement is made to the predictive abilities of the \( n_e \) relationships by applying variable \( n_i \) (Method 2 or Method 3) where appropriate (see Table 5-2 and Figure 5-4 (A, B, and C). This was a somewhat obvious or a foregone conclusion, given the results of the analysis presented in Chapters 3 and 4. Not so obvious, however, are the results of the Type II channel, where the accuracy of the relationships decrease when accounting for \( n_i \) variability via Method 2 or Method 3. Figure 5-4 (C) shows that at lower \( R_h \) values, too much emphasis is given to the channel wall roughness when calculating \( n_e \). The reasons for this are likely related to the way the channel is divided into subsections (the values of \( P_i, A_i, \) and/or \( R_{hi} \)) and the net effect of the assigned subsection parameters, along with \( n_i \), on predicting the contribution of the sidewall hydraulic roughness on the overall composite flow resistance of the channel. It is also possible that the hydraulic roughness characteristics of boundary roughness elements are location dependent. Even for a uniformly lined channel, the flow resistance associated with the walls may well differ from the flow resistance produced by the channel floor.

It is important to note that, regardless of the technique used to estimate \( n_e \) based on \( n_i \), an empirically-based one-dimensional equation [Manning’s equation (Equation 5-1)] is still being used in an attempt to solve a three-dimensional flow problem. As shown in Figure 5-4 (B), applying Method 1 to the floor and the walls of the channel under-predicted \( n_e \) values; applying
Table 5-2. Summary of RMS Values Based on Combined Data Sets for all 12 Composite Roughness Test Configurations

<table>
<thead>
<tr>
<th>$n_i$ method*</th>
<th>Mean Velocity</th>
<th>Total Force</th>
<th>Total Flow</th>
<th>Total Shear Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Walls</td>
<td>Floor</td>
<td>Horton</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CHANNEL TYPE I</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>25.6%</td>
<td>24.5%</td>
<td>24.8%</td>
<td>26.3%</td>
</tr>
<tr>
<td>2</td>
<td>7.8%</td>
<td>6.9%</td>
<td>7.3%</td>
<td>7.2%</td>
</tr>
<tr>
<td>1</td>
<td>7.9%</td>
<td>6.9%</td>
<td>7.3%</td>
<td>7.2%</td>
</tr>
<tr>
<td>3</td>
<td>6.5%</td>
<td>6.5%</td>
<td>6.3%</td>
<td>7.1%</td>
</tr>
<tr>
<td>1</td>
<td>6.5%</td>
<td>6.5%</td>
<td>6.3%</td>
<td>7.1%</td>
</tr>
<tr>
<td>CHANNEL TYPE II</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>13.0%</td>
<td>12.9%</td>
<td>14.4%</td>
<td>12.3%</td>
</tr>
<tr>
<td>2</td>
<td>52.0%</td>
<td>29.1%</td>
<td>88.0%</td>
<td>55.4%</td>
</tr>
<tr>
<td>1</td>
<td>11.9%</td>
<td>11.4%</td>
<td>15.1%</td>
<td>11.0%</td>
</tr>
<tr>
<td>3</td>
<td>32.3%</td>
<td>18.9%</td>
<td>53.6%</td>
<td>32.7%</td>
</tr>
<tr>
<td>1</td>
<td>11.9%</td>
<td>11.4%</td>
<td>15.1%</td>
<td>11.0%</td>
</tr>
<tr>
<td>CHANNEL TYPE III</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>24.3%</td>
<td>24.2%</td>
<td>24.2%</td>
<td>24.3%</td>
</tr>
<tr>
<td>2</td>
<td>7.5%</td>
<td>5.5%</td>
<td>8.3%</td>
<td>5.9%</td>
</tr>
<tr>
<td>1</td>
<td>6.9%</td>
<td>6.6%</td>
<td>6.8%</td>
<td>6.6%</td>
</tr>
<tr>
<td>3</td>
<td>5.3%</td>
<td>5.0%</td>
<td>5.4%</td>
<td>5.1%</td>
</tr>
<tr>
<td>1</td>
<td>5.5%</td>
<td>5.5%</td>
<td>5.5%</td>
<td>5.5%</td>
</tr>
</tbody>
</table>

* Method 1: $n_i$ = constant, Method 2: $n_i = f(R)$, and Method 3: $n_i = f(R)$
Figure 5-4. Examples of experimental and Horton-relationship $n_r$ vs. $R_h$ data for Type I, II, and III composite roughness channel along with the corresponding experimental $n_t$ vs. $R_h$ data [(A) Type I (DBD), (B) Type II (DBD), (C) Type III (DED)]
either Method 2 or Method 3 to both the floor and the walls of the channel produces \( n_e \) values that over-predict the measured values. As a result, the analysis is repeated with Method 1 applied to the channel walls and either Method 2 or 3 to the floor of the channel. Figure 5-4 (B) shows that, by applying Method 3 to the floor and Method 1 to the walls, the predicted \( n_e \) values more closely follow the trend of the experimental data over the range of \( R_h \) tested. They do not, however, provide a relatively good estimate of the measured \( n_e \) data. For some of the equations, the \( RMS \) values increase when using a combination of methods approach. For cases where the combination of methods result in an improvement (i.e., reduction in \( RMS \) values), the improvements are only modest [e.g., Type I and III channels as in Table 5-2 and Figure 5-4 (A and C)]. With respect to the Type II channel, the combination of methods provides an improvement only for the lowest \( R_h \) values tested, relative to Method 1. In general, it can be concluded that where data are available, a variable Manning’s \( n \) should be applied to the \( n_f \) of the floor of the channel. A constant \( n_i \) may be applied to the walls of the channel with little change in predictive error; in fact in most cases it improves \( n_e \) predictions.

Comparison of \( n_e \) Relationships

A comparison of the predictive accuracies of the various composite roughness \( n_e \) relationships listed in Table 5-2 shows that no single composite roughness \( n_e \) relationship performs appreciably better than the rest. Table 5-3 also shows that there is moderate scatter in the accuracy of each of the predictive \( n_e \) relationships over the range of composite roughness boundary configurations tested. For example, Colebatch \((RMS = 3.27\%)\) performs better than Horton \((RMS = 5.98\%)\) in the ADA composite channel; the opposite is true in the AEA composite channel where Horton \((RMS = 5.87\%)\) performs better than Colebatch \((RMS = 8.80\%)\). The \( RMS \) values based on the collective data from all the channel configurations (“Total \( RMS \)” reported in Table 5-3) show that, from a broad perspective, neither relationship (Horton nor Colebatch) is
### Table 5-3. Total RMS and Bias for $n$, Relationships Using Method 3 on the Walls and Method 1 on the Floor

<table>
<thead>
<tr>
<th>Config.</th>
<th>Horton</th>
<th>Colebatch</th>
<th>Pavlovski</th>
<th>Total F2</th>
<th>Total F3</th>
<th>Total F4</th>
<th>Lotters</th>
<th>Lotters II</th>
<th>Total Q</th>
<th>Total Q2</th>
<th>Total Q3</th>
<th>LAD</th>
<th>HDM</th>
<th>Total $U^*$</th>
<th>Total $U^{*2}$</th>
<th>Total $U^{*3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABA RMS</td>
<td>-7.6%</td>
<td>-5.1%</td>
<td>-6.9%</td>
<td>-4.6%</td>
<td>-7.5%</td>
<td>-5.4%</td>
<td>-8.4%</td>
<td>-7.0%</td>
<td>-9.0%</td>
<td>-13.0%</td>
<td>-8.7%</td>
<td>-5.7%</td>
<td>-8.4%</td>
<td>-8.7%</td>
<td>-7.3%</td>
<td>-7.7%</td>
</tr>
<tr>
<td>ADA RMS</td>
<td>-4.9%</td>
<td>-0.2%</td>
<td>-2.3%</td>
<td>1.6%</td>
<td>-3.6%</td>
<td>0.2%</td>
<td>-20.7%</td>
<td>-19.3%</td>
<td>-26.0%</td>
<td>-37.0%</td>
<td>-24.5%</td>
<td>-2.7%</td>
<td>-8.5%</td>
<td>-9.3%</td>
<td>-5.8%</td>
<td>-6.7%</td>
</tr>
<tr>
<td>BDB RMS</td>
<td>6.0%</td>
<td>3.3%</td>
<td>3.9%</td>
<td>3.6%</td>
<td>4.7%</td>
<td>3.1%</td>
<td>21.3%</td>
<td>20.0%</td>
<td>26.4%</td>
<td>37.1%</td>
<td>25.0%</td>
<td>4.5%</td>
<td>9.4%</td>
<td>10.2%</td>
<td>6.9%</td>
<td>7.7%</td>
</tr>
<tr>
<td>AEA RMS</td>
<td>0.6%</td>
<td>5.2%</td>
<td>3.0%</td>
<td>6.9%</td>
<td>1.8%</td>
<td>5.5%</td>
<td>-10.2%</td>
<td>-8.8%</td>
<td>-14.5%</td>
<td>-24.4%</td>
<td>-13.2%</td>
<td>3.1%</td>
<td>-2.4%</td>
<td>-3.2%</td>
<td>0.1%</td>
<td>-0.8%</td>
</tr>
<tr>
<td>BEB RMS</td>
<td>5.9%</td>
<td>8.8%</td>
<td>7.4%</td>
<td>10.5%</td>
<td>6.6%</td>
<td>9.3%</td>
<td>11.2%</td>
<td>9.9%</td>
<td>15.1%</td>
<td>24.6%</td>
<td>13.9%</td>
<td>6.9%</td>
<td>5.5%</td>
<td>5.8%</td>
<td>5.5%</td>
<td>5.3%</td>
</tr>
<tr>
<td>BAB RMS</td>
<td>-3.0%</td>
<td>0.8%</td>
<td>-1.3%</td>
<td>2.0%</td>
<td>-2.3%</td>
<td>0.8%</td>
<td>-9.7%</td>
<td>-8.2%</td>
<td>-12.5%</td>
<td>-20.0%</td>
<td>-11.6%</td>
<td>-0.7%</td>
<td>-5.1%</td>
<td>-5.6%</td>
<td>-3.1%</td>
<td>-3.8%</td>
</tr>
<tr>
<td>EDE RMS</td>
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<td>6.5%</td>
<td>6.3%</td>
<td>7.1%</td>
<td>6.4%</td>
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<td>12.5%</td>
<td>11.4%</td>
<td>15.4%</td>
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</tr>
<tr>
<td>EBA RMS</td>
<td>2.4%</td>
<td>-7.3%</td>
<td>10.0%</td>
<td>-1.5%</td>
<td>15.1%</td>
<td>1.8%</td>
<td>-23.8%</td>
<td>-22.9%</td>
<td>-21.7%</td>
<td>-18.8%</td>
<td>-22.2%</td>
<td>-12.0%</td>
<td>-4.2%</td>
<td>-2.3%</td>
<td>-8.8%</td>
<td>-7.4%</td>
</tr>
<tr>
<td>DAD RMS</td>
<td>17.9%</td>
<td>19.7%</td>
<td>20.5%</td>
<td>17.5%</td>
<td>24.0%</td>
<td>17.4%</td>
<td>34.2%</td>
<td>33.5%</td>
<td>32.3%</td>
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notably better than the other for any of the channel types (I, II, or III). The Total Velocity assumption group has a slight advantage over the other groups based on consistency of predictive accuracy for the three different composite channel types. The Total Discharge assumption group gives inconsistent results. The results for the individual relationships fluctuate, to a certain extent, with both the channel configuration and channel type, as shown in Table 5-3.

Based on the total RMS values for Channel Type I, on average the LAD, Horton, Colebatch, Pavloski, Total F3, Total F4, Total U*2, and Total U*3 predictive relationships perform the best (all within 1% of one another), with the LAD relationship producing a slightly smaller RMS value than the others. For the Type II channel, the Total F2, Horton, Colebatch, Total F4, HDM, Total U*1, Total U*2 and Total U*3 predictive relationships perform the best (all within 1.0% of one another), with Total F2 being slightly better than the others. For the Type III channel, all of the predictive $n_e$ relationships perform essentially the same, with the Total Q relationship producing a slightly smaller RMS than the other relationships.

The results of the data presented in Table 5-3 show that no obvious advantage exists in using the more complicated subarea dividing-based $n_e$ relationships over the simpler to use relationships that only use $P_i$ as the weighting parameter for the $n_i$ in the channel. Because $P_i$ is the sole weighting parameter in these relationships and Method 3, which uses the total hydraulic radius ($R_h$) of the channel instead of $R_{hi}$, has been shown to work as well as or better than the other methods, there is no need to divide the cross section of the channel into subareas. There is one such relationship per assumption group: the Horton relationship (Mean Velocity assumption group), Pavlovski’s relationship (Total Force assumption group), Total Q2 (Total Flow assumption group), and the HDM relationship (Total Shear Velocity assumption group). Of those relationships, Horton is the most consistent when considering all three channel types (I, II, and III).
It is important to remember that the data in this study were collected in a channel with a simple and uniform cross section (rectangular). In addition, although the range or boundary roughness materials are varied appreciably in this study, it should be noted that a high level of roughness element uniformity existed for each composite roughness boundary material (no random roughness elements within a given boundary roughness material) in relation to itself. The extent to which these results can be applied to other types of composite roughness channels that feature different channel cross sections and variation in the degree of component boundary roughness element uniformity has yet to be determined. In the absence of better information, however, the data from this study can be used as a first-order approximation for other composite roughness channel applications. It is also important to note that, based on the variability in the RMS values in Table 5-3 for the individual composite roughness geometries (e.g., ADA, etc.), the predictive $n_c$ values associated with any of the relationships listed in Table 5-2 should be considered approximate. This is especially true when looking at the Total RMS (presented in Tables 5-2 and 5-3), which is based on a compilation of all of the data from composite channel configurations in a single channel type (Type I, II, or III channels).

CONCLUSIONS

The conclusions associated with composite roughness open channel flow resistance in a rectangular flume that result from this study include the following:

1. It is important to note that, regardless of the technique used to estimate $n_c$ based on $n_r$, in general, composite roughness open channel flow conditions represent three-dimensional flow problems that we are attempting to solve with the empirically-based one-dimensional Manning’s equation (Equation 5-1). The likelihood of finding a robust $n_c$ prediction method that will work with Equation 5-1 for solving a wide range of
composite roughness channel configurations is low due to the complex nature of the problem.

2. Where data are available, a variable Manning’s $n$ (the appropriate $n$ value for a given flow condition) should be used on the channel floor. A constant $n_i$ related to larger flow depths in the channel may be applied to the walls of the channel with little negative impact to the predictive error; in fact, in most cases it improves $n_e$ predictions.

3. The Total Velocity assumption relationships, as a group, perform more consistently than the other groups as a whole; however, there are only two equations in the Total Velocity group compared to the four or five equations of the others. The Total Flow assumption relationships, as a group, perform inconsistently relative to the other relationships.

4. Based on the data obtained for this study, there is no evidence that a single $n_e$ equation has a clear advantage over the rest. Taking into consideration the results from all three channel types, the most consistent equations (those which were within 1% of the lowest $RMS$ of each channel type) are Horton, Colebatch, Total F4, Total $U^*2$, and Total $U^*3$.

5. Of these equations, there is no conclusive evidence that the more complex $n_e$ equations will produce better results than the most simple equation (Horton’s equation).

6. Due to the inconsistent results of the $n_e$ equations in their ability to predict $n_e$ for channels where the wall is relatively rough in comparison to the floor of the channel (Type II channels), it is recommended that further studies be conducted to examine the difference between the resistance provided by a specific roughness material, whether it be on the wall of the channel or on the floor of the channel. Relating to this study, it would be of worth to study the difference between a rectangular channel and channels of other shapes where the flow would interact with non-vertical sidewalls.
In summary, for those who wish to use any of the composite roughness $n_e$ relationships presented by Yen (2002), while a constant $n_i$ associated with larger flow depths in the channel may be used for the walls of a channel, when data are available, a variable $n_i$ value should be used for the roughness comprising the floor of a channel (this generally applies to channels where the boundaries are lined with larger roughness elements along the boundary). When data are not available describing a variable $n_i$ value (probably the case in most applications), the user should recognize the considerable error that may occur in predicting $n_e$ values; although, the error will decrease relative to an increasing flow depth in the channel.

Also, while this study does not specifically single out an $n_e$ relationship and recommend it for use in all channels due to the specific nature of the channel used in the study (rectangular channel of a single size), it does point out that there is no evidence that the more complex relationships perform at a consistently higher level than simplest relationships (the equations which weight $n_i$ based solely on $P_i$). As pointed out in conclusion 5, the Horton relationship (Table 5-1) performs better than the rest of the simple $P_i$ based relationships and is also included among the best overall performers of the 16 relationships (based on the data presented in this study).
Chapter 5 illustrates examples of when practicing engineers may need to determine when Manning’s $n$ may be applied as a constant value and when it needs to be considered variable with respect to composite roughness considerations. This is accomplished through a practical application of Manning’s equation to composite roughness relationships coupled with the comparison of 16 such relationships (see Table 5-1) compiled by Yen (2002) which he declared have not been decisively compared due to a lack of data. Regarding the comparison of the equations themselves, the more complex of the 16 equations presented by Yen used to determine $n_e$ produce no greater level of accuracy than the most simple of the relationships (those where only the wetted perimeter is used as the parameter from which the individual Manning’s $n$ values are weighted for each individual roughness material). The most consistent relationships are Horton, Colebatch, Total F4, Total U*$2, and Total U*$3. Of these, the Horton Equation (Equation 5-2) is the most simple to apply. The comparison of these equations, however, is only a portion of applicable knowledge that this study provides to those who wish to use Manning’s equation in open channel applications. The results of the comparison of these relationships also highlight the fact that to achieve the best results, where applicable and where data are available, the variability of $n_i$ should be accounted for.

The qualifying statement in the sentence above, “where applicable,” is important as is stressed through the conclusions of Chapters 3 and 4. In order to determine $n$ values for the different boundary roughness materials used in this study for the composite roughness relationship comparisons, the study channel was uniformly lined (the same roughness was installed on the floor and channel sidewalls) and tested. According to the ASCE Task Force on Friction Factors in Open Channels (1963) a constant $n$ is applicable in a channel if the friction
factor is independent of $Re$ for a given boundary roughness. The results in this study show that even though the data meet the criteria set by the ASCE Task Force (1963), Manning’s $n$ varies over lower ranges of $R_h$ with values of $n$ decreasing as $R_h$ increases. Also with increasing $R_h$, $n$ approaches and at some point becomes constant over a range of higher $R_h$ values (see Figure 5-1). The data (see Table 5-2) clearly show that the composite roughness equations yield the best results when a variable $n$ was applied to the roughness on the floor of the channel within the range of $R_h$ where a variable $n$ is applicable. It was different for the walls of the channel where relatively equal or better results are achieved by applying a constant $n$ value equivalent to the constant $n$ at the higher $R_h$ values rather than varying the $n$ value with respect to $R_h$. This is highlighted in the relationship comparisons for channels where the roughness is greater on the walls of the channel than on the floor (designated as Type II channels in Chapter 5).

In order to apply Manning’s $n$ coupled with Manning’s Equation (Equation 1-4) to specific applications such as the composite roughness equations, a method had to be established with which to predict $n$ values. Several different studies have produced $n$ predicting equations where $n = F(R_h, k, Fr, and S)$ (Limerinos 1970, Bray 1979, Griffiths 1981, Jarrett 1984, Bathurst 2002, Ugarte and Madrid 1994). These relationships are explored in Chapter 3 leading to the conclusion that the data in this study is best represented by a relationship including $R_h$ and $k$, ruling out $Re$, $Fr$, and $S$ as significant factors. Chapter 4 gives insight by way of $x'$ of Equation 4-4 into how Manning’s $n$ might be predicted and stresses important factors for consideration. The use of $x'$ and Equation 4-4 is one of convenience to simplify comparisons between Bray (1979), Griffiths (1981), and Bathurst (2002) relationships; Bazin’s (1865) data; and the data collected for this study. The connection between Equation 4-4 and the Power Law Relationship (Equation 4-7) (used to predict $n$ values intended for use in conjunction with Manning’s Equation where $x' = 2/3$) is laid out in Chapter 4. The Power Law Relationship was found to be a good predictor of Manning’s $n$ for both Bazin’s data and this studies data.
The conclusions from Chapter 4 indicate that the prediction of Manning’s $n$ values may be more specific than previously thought. Bray (1979), Griffiths (1981), and Limerinos (1970) each give a singular equation to be used on a whole range of sizes of a particular boundary roughness type (gravel roughness); however, the $r^2$ values reported for the Bray (1979) and Griffiths (1981) equations are relatively low (0.355 and 0.591 respectively). The $k$ value in these equations is represented by a single parameter representing the size of the gravel $D_r$. The data in this study coupled with Bazin’s (1865) data show that $x’$ is different with relatively high $r^2$ values not only for different roughness types but also differs within the same boundary roughness material type depending on the size of the roughness elements within each roughness type (see Table 4-2 and its corresponding discussion in Chapter 4). For instance, the $x’$ of the small gravel of Bazin’s data is different than the $x’$ of the large gravel of Bazin’s data. The same comparison might also be made with the block data (blocks were intended to simulate the same types of flows as gravel data with larger roughness elements) which yield a much higher $x’$ value than the gravels of Bazin’s data. The different $x’$ values indicate that for the best results a separate equation is needed for different sizes of roughness material even within a particular roughness element type. Other parameters of the boundary roughness not explored in this study such as spacing, uniformity and surface texture, etc. of the individual roughness elements may also have an effect and require separate equations. More research is needed on these parameters and their effect on Manning’s $n$ relationships.

The differences in $x’$ for each of the roughness materials studied brings the discussion back to a variable $n$ versus a constant $n$. Figure 4-2 shows that at some point the data for the metal lath and the trapezoidal roughness data are best fit by two separate lines, one with an $x’$ specific to the roughness (smaller $R_h$) and the other being $x’ = 2/3$ (larger $R_h$). The acrylic data are all relatively well described by a line where $x’ = 2/3$ and the block data shows a deviation towards higher $R_h$ values from a single $x’$ value to a separate line that is assumed may eventually
reach an $x' = 2/3$ value as well if more data were attainable. This gives insight as to why even though Manning (1889) saw the differences in $x'$ values himself in Bazin’s (1865) data he was able to use $x' = 2/3$ for his equation, use a constant $n$ assumption, apply it to 170 different experiments, and achieve results that were “sufficiently accurate”. Manning even stated that the greatest errors in these experiments were found at relatively small $R_h$ values, which is in agreement with the findings of this study. In chapter 3, the differences in $x'$ from an optimal value for the individual roughness materials to an $x'$ of 2/3 is tied to a theory described in Chow (1959). The theory describes a “quasi smooth” state of flow where the voids between the individual roughness elements are filled with stable eddies providing a relatively smooth boundary over which the flow above the elements moves relatively freely decreasing the resistance incurred by the individual roughness elements themselves. The conclusions from Chapter 3 tie the “quasi smooth” state of flow to the larger roughness materials where $x' = 2/3$ only at higher $R_h$ values, therefore, Manning’s equation may be applied assuming a constant $n$ value when the channel is relatively physically smooth (acrylic and cement lined channels) and when the flow in the channel has reached a “quasi smooth” state.

Robert Manning (1889) was well aware of the empirical nature of the equation, which now carries his name (Equation 1-4). He cautioned against applying Manning’s Equation outside of the range of data for which it had been tested. Chow (1959) indicates that the most difficult part of applying Manning’s equation [Equation (1-4)] is quantifying the roughness coefficient ($n$), describing the process as anywhere from guesswork to sound engineering judgment. While it is not the intention of this study to give exact parameters, it is the intention to further enhance the decision making abilities of an engineer who intends to apply Manning’s Equation (Equation 1-4). Especially in regards as to when the constant $n$ assumption might be appropriate versus some other method to predict a varying $n$. Any potential user should be aware that:
Manning’s $n$ is not necessarily a constant value even if the flow is as described by the ASCE Task Force on Friction Factors in Open Channels as “fully rough.”

Manning’s $n$ may be considered relatively constant, however, if the boundary of the channel is relatively smooth physically or if the flow in channel has reached a “quasi smooth” state.

Otherwise, Manning’s $n$ will vary to varying degrees depending on the specific boundary roughness material for which it is intended to represent.

For best results regarding the prediction of varying Manning’s $n$ values a separate equation should be used not only for a boundary roughness type, but for other parameters as well, size being an example of such a parameter as directly pointed out in this study.

Where no data are available it should be recognized that Manning’s $n$ may be relatively large at lower $R_h$ values but will decrease with increasing $R_h$ until at some point it reaches a constant value (the channel has reached a “quasi smooth” flow condition).

In regards to the specific practical application of these principles, the composite roughness relationships:

- There is not a single relationship which stands out as “the best” relationship in comparison to the others tested;

- however, it was discovered that the most complex of the equations is no better than the most simple to apply. The most simple equation with the most consistent performance is Horton’s Equation (Equation 5-2)

- For the best results, a varying Manning’s $n$ should be applied, where applicable, to the boundary roughness of the floor of the channel, while a constant $n$ consistent with the $n$ value of a roughness at relatively high $R_h$ values may be applied to the boundary
roughness of the walls (the channel tested was rectangular and further testing is needed to see what would apply to channels of a different shape).
REFERENCES


APPENDIX
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Dear Ms. Lamberton

I am preparing my dissertation, *A Study of the Variability Versus the Assumed Constancy of Manning’s n*, in the Civil and Environmental Engineering Department at Utah State University. I hope to complete my degree in the Spring of 2014.

Three chapters (Chapters 6, 7 and 8): The Behavior of Hydraulic Roughness Coefficients in Open Channel Flow, Open Channel Flow Resistance: the Hydraulic Radius Dependence of Manning’s Equation and Manning’s n, and Open Channel Flow Resistance: Composite Roughness, of which I am author in conjunction with the author of the compiled report, Dr. Blake Tullis, which appeared in your report, 2012, NCHRP Report 734, Pages 50-78, constitute an essential part of my dissertation research. I would like permission to reprint them as chapters in my dissertation. (Reprinting these chapters may necessitate some revision.) Please note that USU sends dissertations to ProQuest Dissertation Services to be made available for reproduction.

I will include an acknowledgment to the chapters of the report at the beginning of the dissertation, as shown below. This Copyright and permission information will be included in a special appendix. If you would like a different acknowledgment, please so indicate.

Please indicate your approval of this request by signing in the space provided, and attach any other form necessary to confirm permission.

If you have any questions, please call me at the number above or send me an e-mail message at the above address. Thank you for your assistance.

Tyler Allen, P.E.

I hereby give permission to Tyler Allen to reprint the requested report chapters in his dissertation, with the following acknowledgment:

Thanks to the Transportation Research Board and the National Cooperative Highway Research Program who provided funding for this research by way of the NCHRP Report 734 authored by Dr. Blake P. Tullis. This dissertation comprises specifically the NCHRP Report 734, *Hydraulic Loss Coefficients for Culverts*, Chapters 6, 7, and 8, Pages 50-78.

Signed

Date 5/1/14

Sharon C. Lamberton, Editor

CRP-TRB
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(May 2014)

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- Prepared Groundwater DWSP Plans submitted to the State of Utah Division of Drinking Water for compliance with the Drinking Water Source Protection Rule (R309-600)

Hydrologic Studies

- Conducted studies on localized hydrologic conditions for the purpose of predicting potential runoff for the design of detention basins, master planning, and FEMA flood mapping.

- Experience in preparing models including HEC-I, HEC-HMS, and Autodesk Storm and Sanitary Analysis to predict runoff.

FEMA Flood Mapping

- Prepared requests for Letter of Map Revisions (LOMRs) submitted to FEMA which include hydrologic studies and the implementation of hydraulic structures and conveyance channel improvements.
Hydraulic Analysis and Design

- Prepared hydraulic calculations for the design of detention basins, the outlet works to detention basins and dams, culverts, and open channel conveyance.
- Designed and prepared construction drawings for Detention Basins (including outlet works) and channel improvements and storm drain, sewer, and pressurized water systems.
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- Implemented software in design and analysis process including HEC-RAS, and AutoCAD Civil 3D, ArcGIS, and Autodesk Storm and Sanitary Analysis.

2003 to 2008: Utah Water Research Laboratory

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- Performed hydraulic analysis and calibrations of various types of flow meters and flow control apparatus.