A Production Function for the Structural Steel Fabricating Industry of Utah

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A PRODUCTION FUNCTION FOR THE STRUCTURAL
STEEL FABRICATING INDUSTRY OF UTAH

by

Don Ward Thomas

A thesis submitted in partial fulfillment
of the requirements for the degree
of
MASTER OF SCIENCE
in
Economics

Approved:

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1965
ACKNOWLEDGMENT

I wish to express my sincere appreciation and thanks to Dr. Bartell C. Jensen, Chairman of my Graduate Committee, for his help, suggestions, and encouragement that have made the writing of this study possible.

I would also like to thank Dr. Reed R. Durtschi of my Graduate Committee for the help and moral support given to me.

Finally, a note of thanks to the other members of my Graduate Committee, Dr. B. Delworth Gardner and Dr. Donald V. Sisson, for their time and suggestions.
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INTRODUCTION

Origin and Nature of Problem

The steel industry, comprising one of Utah's largest payrolls, is a highly significant industry in the economic structure of the state. Furthermore, a highly specialized steel fabrication industry has developed as a subsidiary of the steel industry in Utah.

This thesis is part of a study under contract between Utah State University and the Office of the Utah State Planning Coordinator. The problem is one of finding the effect of changes in steel fabrication on Utah's economy.

Objectives

The objective is to derive as accurately as possible, the relationships which exist between the inputs necessary in the fabrication of structural steel and the final product. The basic inputs are: labor, capital, and raw materials. Raw materials are defined to include structural steel, rivets, paint, welding rods, etc. The output or final products of the industry are beams, girders, trusses, bridges, grandstands, etc. Once derived, these relationships will make it possible to analyze the factors necessary for future growth and what the impact will be on Utah's economy given changes in steel fabrication.

More specifically, the study is concerned with the derivation of a micro-economic production function for the structural steel
fabricating industry of Utah. However, since the basic engineering relationships for the industry are not available, a Cobb-Douglas form of the production function will be used to represent the input-output relationships. A general model of a Cobb-Douglas form is:

$$ Y = a X_1^b X_2^c \ldots $$

where $X_1, X_2 \ldots$ are the inputs; $a, b, \ldots$ are the parameters; and $Y$ is the output.

Statistical tests will be applied to several different models to determine which is the most significant. Once the appropriate production function has been derived, the implication of returns to scale will be noted and marginal productivity functions for the various inputs will be found. The functions will then be used to predict the impact of steel fabrication on employment, demand for raw materials, and other important variables.
Iron was probably discovered by Stone Age men who were looking for hard stones to make tools and weapons. They came upon meteorites, which were pieces of matter that had fallen to earth, and found that they could hammer these meteorites into various shapes.\textsuperscript{1} It would appear that men used meteorite iron for hundreds of years without thinking that there might be iron in the earth. The earliest discovery of iron-making was probably by means of a fire accidentally lighted where iron ore existed near the surface of the ground.\textsuperscript{2}

The next step was to make iron intentionally in a furnace. Primitive man mixed iron ore with charcoal in crude furnaces and learned to apply an artificial draft. The latter constitutes the first important step in the development of the iron and steel industry. This system was named the Catalan Forge, since it originated in Catalonis, Spain.\textsuperscript{3} With this ability, iron soon became man's chief tool.

One of the oldest methods of making steel was cementation. This method consisted of heating wrought iron in stone boxes with charcoal


\textsuperscript{3} \textit{Ibid.}
for long periods of time until some of the carbon was absorbed by the solid iron. By increasing the carbon content of the wrought iron it was possible to make a very hard edge on weapons and tools.

In the early 1780's, Benjamin Huntsman conceived the idea of making cemented steel in a crucible to improve the homogeneity of the metal. Huntsman's idea was to melt cemented steel in a clay crucible, skim off the slag, and pour the metal into a mold. This method produced a steel which was free from slag and dirt.

The so-called steel age, however, was not initiated until the 1850's when the Bessemer Conversion Process was first introduced. With the new method, steel was produced in tons instead of in pounds. Bessemer steel is made in a so-called converter. This converter is filled with molten iron and powerful blasts of air from holes in the bottom of the converter rush up through the iron. The air causes the impurities in the iron to be oxidized, thus converting iron into steel. This invention made possible the production of low-cost steel and in many ways aided the industrial development of the United States.

The open hearth furnace, the principle method of steel-making today, accounts for nine out of ten tons of steel produced in the United States. The open hearth furnace is like a large oven.

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4Ibid., p. 9.
Limestone and steel scraps are put into the furnace and after the limestone and scrap are melted, liquid iron is added. The mixture of liquid iron, scrap, and limestone is cooked under flame for 8 to 10 hours. This cooking process converts the mixture into steel. When the steel is ready to leave the furnace it is poured into a huge ladle. The molten steel then flows from holes in the bottom of the ladle into molds. When the molds are lifted, red-hot blocks of steel called ingots remain.

Open hearth furnaces currently produce more steel than any other type of process. The average open hearth furnace produces about 130 tons of steel per charge of limestone, scrap, and molten iron.

A final process which is very important in making steel alloys is the electric furnace. These furnaces are useful in making steel alloys because the heat is regulated much more precisely. Electric furnaces employ only steel scrap, which is melted by electric currents. After melting, various alloying elements are added with the steel and cooked until they are blended into an alloy. There are many different kinds of steel alloys. Each is made to do a special job that plain steel cannot do. Electric furnaces produce about 7 percent of all the steel made in the United States. Most of this is in the form of an alloy.

After the molten iron has been refined and solidified in ingot form, it is then mechanically worked into various shapes and ultimately into manufactured products.

---

8The limestone is used in the open hearth furnace to soak up impurities which form a scummy slag and float on top of the steel.


10Ibid., p. 19.
The four main methods that are used in converting steel into manufactured products are: Casting, which is a process of pouring molten steel into molds of desired shapes and sizes; drawing, which is used in producing wire and bars; forging, which is working the hot metal by hammering and pressing; and rolling, which includes the forming of blooms, billets, slabs, strip, bars, plates, sheets, rails, structural shapes, tubing, and pipes.11

Structural Steel

Rolling mills convert hot steel ingots into various shapes for different uses. Rolling is the process by which structural steel is made. Before the ingots can be taken to the rolling mill they must be at a uniform temperature of 2,200 degrees Fahrenheit.12 This condition is obtained by means of a soaking pit, where the ingots are placed for six to eight hours or until the temperature for roller mill conditions is met.

The rolling mill process passes the hot ingots between a series of steel rolls containing various shaped grooves with projecting collars which shape the hot plastic metal. Rolling not only produces the desired shapes, but greatly improves the quality of steel. In the unrolled form, the ingot is a weak mass of crystals which are overlapped and elongated during the rolling process producing greater strength.13

13Ibid.
The products of rolling mills that are used as structural steel members are known as sections, and are designated by the shapes of their cross sections. The most commonly used sections are the American Standard beam and Channel sections, wide flange sections, H-sections, angles, tees, zees, plates, and bars.\textsuperscript{14}

\begin{figure}[H]
\centering
\includegraphics[width=\textwidth]{steel_sections.png}
\caption{Structural steel sections}
\end{figure}

The middle of the eighteenth century saw exploratory uses of iron and steel to support wood and masonry structures.\textsuperscript{15} With the advancement of structural steel it has become one of the most important materials used in construction of buildings, bridges, ships, etc. It possesses strength, ductility, as well as many other desirable properties. Ingredients which affect the properties of structural steel include carbon, which increases the strength and hardness but lowers ductility; phosphorus, which increases strength but makes it brittle.


\textsuperscript{15}Fisher, \textit{The Epic of Steel}, p. 102.
when cold; sulphur, which decreases strength, ductility, and causes rapid corrosion; manganese, which increases hardness and decreases corrosion; and nickel, which is used where exceptional strength is required.16

**Fabricating Structural Steel**

From the rolling mill the shapes are brought to the fabricating shop, where they are cut to proper length, holes are punched to permit riveting or bolting, and surfaces are painted after the structures are assembled by welding, riveting, or bolting.

Before the days of industrial combinations, structural companies were operated as single, independent units. Possibly, many of them began with a drill press and a chain hoist. With increasing profits and volume of work, the business has extended to include punches, shears, and riveters, until the shops were equipped to fabricate beam work, plate girders, columns, and trusses.17

The operations within the shop require the movement of steel sections, which is accomplished by large overhead cranes. The following are the necessary steps in the fabrication of structural steel:

**Receiving materials**

The material received from the mills is unloaded and sorted in the receiving yard. The mill invoices are compared with the original mill orders to check specifications. Then each piece is measured and


inspected for defects. If some of the structural steel sections have chambers, they must be straightened.\textsuperscript{18}

There are certain sections that are used generally on all types of structures. These materials are kept on inventory and are known as stock materials.\textsuperscript{19} Some of these sections are sold directly, therefore, the fabricating shops act as retail outlets for the rolling mills.

**Laying out**

The laying out process is marking the steel directly for punching and shearing, and is accomplished by the use of "templets" which are made from engineers' drawings.\textsuperscript{20} In a shop equipped with modern punching machines, the laying out process is reduced since the machines can be programmed to punch most of the holes.

**Cutting and holes**

The shops save cutting expenses by ordering materials already cut to length from the roller mills. When cutting is necessary, however, it is accomplished by: Shears, which cut plates and angles by a single stroke of a blade that comes down against a die; and sawing, which cuts either by means of a circular saw, or flame. The latter includes cutting by an electric arc, acetylene, or other gas flames.\textsuperscript{21}

Holes are cut into steel when bolts or rivets are used, and are made by drilling or punching. Drilling is preferable because it does

\textsuperscript{18}Ibid., p. 208.


\textsuperscript{20}Ibid., p. 221.

\textsuperscript{21}Ibid., pp. 246-251.
not damage the metal around the hole. Punching is, however, the most commonly used method because of its low cost. There are limitations to the thickness of materials punched.\textsuperscript{22}

\textbf{Assembling}

This is the process of fitting the individual sections into a complete structure. The sections are assembled then riveted, bolted, or welded. Hot rivets are passed through the holes in the steel and the plain end is pressed down to form a second head. As the rivets cool, there is a slight shrinkage in length and the two plates are drawn tightly together.\textsuperscript{23} Welding, in addition to reducing construction noise, has the following advantages: It makes very rigid frames, is easy to connect new work to existing structures, and has an economic advantage since holes are not required.\textsuperscript{24} Welding relative to riveting and bolting is increasing in importance.

\textbf{Inspecting, painting, and shipping}

The inspection is generally done by companies specializing in steel inspection. They are employed by purchasers to check the quality of the workmanship, materials, etc. If the structure is accepted, then it is cleaned by gasoline or sandblasting, and painted to prevent corrosion. The structures are shipped by truck or railroad to the construction site.

\textsuperscript{22}Ibid., pp. 223-224.

\textsuperscript{23}Harry Parker, \textit{Simplified Design of Structural Steel}, p. 131.

\textsuperscript{24}Ibid., p. 156.
Since most contracts are let under bid the following costs must be considered in determining bid prices: the mill cost of raw materials, shipping costs from mill to fabricator, cost of shop drawings and templets, shop fabricating costs, cost of shipping fabricated work from shop to site, erection costs if called for, and overhead and profit.\textsuperscript{25} The engineer can aid in lowering the cost of fabrication by making simple designs so there is as little moving of materials as possible, and a minimum of fabrication.

The following is a flow diagram of a structural steel fabricating shop:

Several years following the advent of the Mormons into Utah, significant deposits of iron and coal were discovered in the area now known as Cedar City. Within a short time a colonizing company built a crude blast furnace and in the year 1852 produced the first pig iron west of the Missouri River.

This operation, however, because of Indian uprisings, flash floods, windstorms, and other such events was not a commercial success and, in 1859, was finally abandoned. In the seven years of operation, an estimated 25 tons of pig iron was produced.

The next venture at iron making in Utah was undertaken by the Great Western Iron Manufacturing Company at Irontown. In 1868 it began operations with a daily capacity of 2,400 pounds of pig iron. This operation, like its predecessor, was not a financial success and in 1893 ceased operations.

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2. Dr. Walther Mathesius, "The Growth of Western Steel," addressed to a Joint Meeting of the American Society for Metals and the American Institute of Mining and Metallurgical Engineers, September 24, 1951, Los Angeles, California, typescript, Department of Economics, Utah State University.


4. Dr. Walther Mathesius, "The Growth of Western Steel."
With its accompanying needs for steel and steel products, World War I brought about the creation of the Utah Iron and Steel Company in 1915. Its plant, located at Midvale, Utah, had a single open-hearth furnace with a daily capacity of 150 tons of steel. At the close of the War, however, the cancellation of government contracts brought about financial ruin and forced the company, which had expanded on the basis of the government's need for steel, to cease operations.\(^5\)

The next important development of the steel industry in Utah came in 1941-42 when the government, as a result of World War II, decided to increase the steel producing facilities of the West. This was done to guard against a shortage of steel supplies to the Pacific Coast shipbuilders in the event the Panama Canal were to be closed from enemy attacks.\(^6\)

The new mill was constructed near Provo, Utah. This site possessed adequate transportation facilities and was close to sources of both iron ore and coal. The plant was nearly completed by the end of 1943 and the first open-hearth steel was produced in January, 1944.\(^7\) The new plant cost more than $200 million and had a rated capacity of 1,150,000 net tons of pig iron and 1,283,400 tons of steel ingots per year.\(^8\)

At the end of the war the plant was virtually closed and the

\(^5\)Growth of the Iron and Steel Industry of Utah, p. 4.

\(^6\)Ibid., p. 5.


\(^8\)ElRoy Nelson, p. 109.
facilities were offered for sale by the government. U. S. Steel purchased the plant in June, 1947, for $47.5 million with the stipulation that an additional $18.6 million be spent in conversion to peacetime operations.\textsuperscript{9} The plant's capacity was subsequently increased and altered for commercial production.

The Utah Division of United States Steel, known as the Geneva plant, produces primarily strips and plates which are shipped to the West Coast for further processing and final marketing. Only about 15 percent of their output is retained in the Mountain West. While the steel market on the West Coast has shown considerable expansion in the past two or three decades, the market in the Intermountain West has remained rather stable, absorbing only a small fraction of the steel produced in Utah.\textsuperscript{10}

\textbf{Utah's Steel Fabrication Industry}

With the completion of the Geneva plant in Utah, a steel fabrication industry was soon established to take advantage of the close source of raw materials.

The first satellite industry to be started in Utah as a result of this availability of steel, is being promoted by the Structural Steel and Forge Company, which purchased the government-owned vanadium plant in Salt Lake City. When completed, the new plant will employ 100 men. The business will be devoted to the fabrication of steel to be supplied by the Geneva Steel Plant.\textsuperscript{11}

\textsuperscript{9}Ibid., p. 110.

\textsuperscript{10}Ibid.

\textsuperscript{11}The newspaper clipping collection of Leonard J. Arrington, Department of Economics, Utah State University.
However, since certain kinds of steel used by Utah's fabricating industry originate in far parts of the nation, the price and freight cost advantages to Utah's firms of using Geneva's products is in part offset by the high cost of materials that must be shipped from distant points. Therefore only in the manufacture of products geared to the use of Geneva's output is there a cost advantage to Utah firms.12

Products that are available through Utah's steel fabrication firms include pressure tanks, filters, structural steel, rail car wheels, decorative iron work, etc. A number of these products are used nationally and internationally, but most are used locally.13

Transportation costs become an important factor in the total marketing costs of fabricated products. By avoiding high transportation costs, a local industry has a certain economic advantage over a similar industry competing from a distant location. Most of Utah's steel fabricators, however, report their market to be "the Intermountain region" unless they have patent rights which virtually place them in the position of a monopolist for a particular product.14

**Structural Steel Fabrication Firms**

The structural steel fabrication industry of Utah is rather specialized, and produces most of the fabricated steel used in the


13*Salt Lake Tribune* (Salt Lake City, Utah), January 15, 1958.

construction industries of Utah and the Intermountain region. There are fifteen companies in the state that fabricate structural steel. These companies are all located in the Ogden, Provo, and Salt Lake City area. Together they employ about 890 workers with an annual payroll of $4,797,266.\textsuperscript{15} Products manufactured by the structural steel fabrication industry in Utah are:\textsuperscript{16}

<table>
<thead>
<tr>
<th>Item</th>
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<th>Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>trusses</td>
<td>cashier cages</td>
<td>boxing rings</td>
</tr>
<tr>
<td>building frames</td>
<td>tool cribs</td>
<td>basketball hoops</td>
</tr>
<tr>
<td>guard rails</td>
<td>car transfers</td>
<td>lamp posts</td>
</tr>
<tr>
<td>bleachers</td>
<td>bridges</td>
<td>street lighting lamps</td>
</tr>
<tr>
<td>grandstands</td>
<td>switches</td>
<td>brackets</td>
</tr>
<tr>
<td>flag poles</td>
<td>frogs</td>
<td>knock-down basket floors</td>
</tr>
<tr>
<td>stairs</td>
<td>swings</td>
<td>fence posts</td>
</tr>
<tr>
<td>railings</td>
<td>sliders</td>
<td>window guards</td>
</tr>
<tr>
<td>fire escapes</td>
<td>teeter-totters</td>
<td>machine guards</td>
</tr>
<tr>
<td>trick bars</td>
<td></td>
<td>ornamental products</td>
</tr>
</tbody>
</table>

A brief description of some of the more important structural steel fabricating firms is presented in the remaining pages of this chapter.

**Industrial Steel Company**

Before 1929 the Industrial Steel Company was known as the Builders Steel Company. In 1929, after bankruptcy, it was purchased by its present owners at a creditors sale. The firm was then operated as a partnership until 1944 when it was incorporated. The plant is located at Sixth South and Fourth West Street in Salt Lake City.

The company fabricates steel structures, and wholesales steel to other smaller consumers. This warehousing of steel has become a major part of the business during the past few years. Officials of the

firm say that warehousing is a profitable business for both buyer and seller. Buying in large quantities from the producer enables the wholesale warehouse to purchase at much lower rates. After different fees are added for service and commission, the consumer buys for less than if he were to make small purchases directly from the mill.

The steel work incorporated in many structures throughout the area has been contracted by this firm. Some contracts have been as far away as Sacramento, California, but their primary market is the Intermountain area and particularly Utah. Depending upon the number of contracts, employment varies between 25 and 100 workers. 17

Allen Steel Company

The Allen Company was organized in January, 1947, by Mr. Robert B. Allen and is located at 1340 South First West, Salt Lake City.

The company purchases their structural and reinforcing steel from the Geneva plant, the Colorado Fuel and Iron Company, and other mills. Part of this steel is fabricated into beams, trusses, columns, and other products for use in industries. The balance of the steel is sold directly through the company's warehousing operations.

The plant size has been increased regularly. The company has approximately 4,000 square feet of office space, 10,000 square feet of enclosed fabricating space, and an outdoor yard and storage area of approximately one acre. They employ between 100 and 249 workers. 18

17"The Steel Fabricating and Steel-Using Industries of Utah," p. 22.

18Ibid., p. 43.
Empire Steel Company (American Steel Company)

The Empire Steel Company incorporated in April, 1950, was an outgrowth of the Ellis Steel Company, a fabricating firm organized in 1943. It is located at 830 South Sixth West in Salt Lake City. The fabricating shop is 78 feet wide and 204 feet in length.

The firm operates as a structural steel and reinforcing steel fabricator for commercial and industrial construction. The market in which the company makes its sales consists of Utah, Wyoming, and Colorado. Most of the contracts are in Utah and include churches, schools, commercial buildings, etc.

Raw materials are purchased from many different sources. Some steel is shipped direct from Geneva, from Pittsburgh and Torrance, California, and from the Colorado Fuel and Iron Company. Other material is obtained from local and West Coast warehouses. The raw material used are structural, including wide-range beams, bar size shapes, reinforcing rods and reinforcing mesh. They employ between 25 and 99 workers.19

Western Steel Company

This firm started operations in October, 1945, under the name of Western Steel Supply Company. In January, 1947, it was incorporated as the Western Steel Company. Offices are maintained in the Beason Building in Salt Lake City. A modern shop located at 651 West Seventeenth South was completed in 1949. The shop building is of steel construction, and is 220 feet long and 115 feet wide, with an outside crane area of 76 feet in length and 380 feet in width.

19 Ibid., pp. 45-46.
Production is in the area of reinforcing and structural steel fabrication.

Raw materials are obtained from various mills: angles, standard beams, plates, and channels are obtained from Geneva; wide flange beams from Pittsburgh and Chicago; bar size angles, reinforcing steel, channels, and other shapes from Colorado and the Pacific Coast.

The market area served includes Utah, Montana, Idaho, Wyoming, Nevada, Oregon, Washington, and western Colorado. They employ between 100 and 249 workers.20

Other firms

In addition to those companies described, there are in the state of Utah, eleven other structural steel fabricating firms: Cobusco Steel Products, 660 South West Temple, Salt Lake City; Commercial Shearing and Stamping Company, P. O. Box 2030, Salt Lake City; Gerstner Steel and Supply Company, Inc., P. O. Box 336, Salt Lake City; Monsey Iron and Metal Company, Inc., 750 South 3rd West, Provo; Ogden Iron Worker Company, Inc., 185-23 Street, Ogden; P. I. Street Corporation, 3100 South 11th West, Ogden; Provo Steel and Supply Company, 1400 South State, Provo; Steel Contractors, Inc., 6 Orange Street, Salt Lake City; Steel Engineers Company, 1526 South West Temple, Salt Lake City; Eimco Corporation, 545 West 7th South, Salt Lake City; and Taylor Steel Corporation, 1363 Major Street, Salt Lake City.21

20 Ibid., p. 56.

THE PRODUCTION FUNCTION FROM QUESTIONNAIRE DATA

Review of Production Function Theory

The production function is the economist’s way of stating symbolically that the output of a firm depends on its inputs. It is generally written as \( y = f(x_1, x_2, \ldots, x_n) \), which means that the total product, \( y \), depends on the amounts of the various inputs, \( x_1, x_2, \ldots, x_n \), used by the firm per unit of time.

Consider a process of production requiring two inputs. Let \( x_1 \) and \( x_2 \) be the respective quantities of the two inputs and \( y \) be the quantity of output. Then the production function can be written

\[ y = f(x_1, x_2). \]

This function provides a complete catalogue or quantitative description of the various quantities of the two inputs which can be employed to produce \( y \). Strictly speaking, we should think of this function as providing us with the largest output, \( y \), which can be produced by given \( x_1 \) and \( x_2 \). There are some production decisions which can be made on purely technical grounds without any knowledge of costs whatsoever. These decisions can be called engineering decisions as opposed to economic decisions. Thus, if a modification of the manner in which a process is performed allows the same output to be produced, and permits the quantity of at least one input to be reduced without requiring an increase in the quantity of any other input, then a decision in favor of the modification can be made on engineering grounds alone without
any knowledge of input prices. An action which saves on one input without altering any other requirement of a process will lower cost regardless of the price of that input.

The production function discussed above presupposes all such engineering decisions have been made. In constructing this function all methods, techniques, or processes which require more of one input and not less of any other input are rejected. Once all such engineering decisions have been made, we are left with the best engineering technology. But with this technology we are still left with a large number of input possibilities which have the characteristic that output cannot be maintained at a given level when one input is reduced, unless we increase some other input. The choice among these remaining input combinations is an economic decision in the sense that the decision requires knowledge of input prices.

Briefly stated, economic decisions require knowledge of input prices and best engineering technology. Engineering decisions are concerned with best engineering technology and require technical knowledge of physical processes.

The production function as we have defined it will, in general, be expected to exhibit the following characteristics:

1. If either input is held constant while the other is increased (decreased), output will increase (decrease). Mathematically, this is equivalent to stating that $\partial f/\partial x_1 > 0$ and $\partial f/\partial x_2 > 0$. The partial derivatives $\partial f/\partial x_1$ and $\partial f/\partial x_2$ are called the marginal productivities respectively of $x_1$ and $x_2$ in the production of $y$. In other words, the marginal productivity of input Number 1 is the rate at which output changes with respect to the changes in the quantity of
input Number 1 used, the quantity of input Number 2 being held con-
stant.

2. If output is held constant, a decrease (increase) in one
input will require an increase (decrease) in the other input. Mathe-
matically, $\frac{\partial x_2}{\partial x_1} < 0$, and the partial derivative $\frac{\partial x_2}{\partial x_1}$ is the
marginal rate of substitution between $x_1$ and $x_2$.

3. If $y$ is held constant, the marginal rate at which $x_1$
substitutes for $x_2$ increases as $x_1$ increases. Mathematically,

$$\frac{\partial}{\partial x_1} \left( \frac{\partial x_2}{\partial x_1} \right) = \frac{\partial^2 x_2}{\partial x_1^2} > 0$$

and we say that the production function is convex to the origin in the
$x_1$, $x_2$ plane.

These characteristics of the typical production function can
be summarized with an iso-product map. An iso-product contour (con-
stant output curve) is a curve connecting all those combinations of
$x_1$ and $x_2$ that are required to produce a specified quantity of out-
put. An iso-product map is simply a family of such curves, each curve
corresponding to a different level of output.

The statistical investigation into laws of production by C. W.
Cobb and P. H. Douglas are among the most celebrated in the history of
economics. They proposed the general function

$$y = An^\alpha k^\beta u,$$

$y$ = output,

$n$ = labor input,

$k$ = capital input,

$u$ = random disturbance,
as a fairly universal law of production and estimated it in numerous samples of manufacturing industries throughout the world. This exponential type of production function has no more claim to general validity as a description of technology than other mathematical functions. However, it does have many interesting properties that make it a very convenient choice.

The Cobb-Douglas function has constant elasticities of output variation with respect to labor or capital input.

\[ \alpha = \text{elasticity with respect to labor input}. \]
\[ \beta = \text{elasticity with respect to capital input}. \]

The relationship is nonlinear. For constant levels of capital, the output-labor input relation is shown as a series of curved lines in the following figure:

![Figure 3. Output-labor input relationship.](image)

If either input is zero \((n = 0 \text{ or } k = 0)\), output is zero. Thus, both inputs are necessary to the production process. The curvature is such (each elasticity assumed to be less than unity) that marginal productivity falls as input grows. There is no asymptotic level of output (or ceiling) beyond which production cannot grow, but the rate of increase decreases at high levels of input.
Although the function is nonlinear, it can be transformed with ease into a linear function by converting all variables to logarithms. In logarithms, the associated linear function is

$$\log y = \log A + \alpha \log n + \beta \log k + \log u,$$

or

$$y' = A' + \alpha' n' + \beta' k' + u'. $$

In terms of the primed variables we have a linear function. Scale changes in the basic units of measurement have no essential effect on any of the terms in this logarithmic formulation except the constant $A'$. Therefore, this function is convenient in international or inter-industry comparisons. Since $\alpha$ and $\beta$ are elasticity coefficients, they are pure numbers and can easily be compared among different samples using varied units of measurement.

In a sense, one is able to capture the flavor of essential nonlinearities of the production process and yet benefit from the simplifications of calculation from linear relationships by transforming to logarithms. The logarithmic function is linear in the parameters, which is an essential point to the statistician. Other functions may give a similar type of curvature and keep linearity in parameters. A parabolic function would be an example.

$$x = a_1 + a_1 n + a_2 k + a_3 n^2 + a_4 k^2 + a_5 nk + u.$$ 

However, this type of equation uses many more parameters than does the Cobb-Douglas form. The latter is economical in the use of degrees of freedom, or parameters, and yet gives us nonlinearity.

The parameters of the Cobb-Douglas function, in addition to being elasticities, possess other attributes important in economic
analysis. The sum of the exponents shows the degree of "returns to scale" in production.

\[ \alpha + \beta < 1 \quad \text{decreasing returns to scale}, \]
\[ \alpha + \beta = 1 \quad \text{constant returns to scale}, \]
\[ \alpha + \beta > 1 \quad \text{increasing returns to scale}. \]

Suppose that each input is increased by \( r \) percent.

- \( n \) increased to \( n(1 + r/100) \),
- \( k \) increased to \( k(1 + r/100) \).

The output is increased by less than \( r \) percent, by \( r \) percent, or by more than \( r \) percent, according to whether there are decreasing, constant, or increasing "returns to scale." This is easily seen by substituting into the function

\[ y = An^\alpha k^\beta u, \]
\[ y(1 + r/100)^{\alpha+\beta} = A(n(1 + r/100))^\alpha \cdot (k(1 + r/100))^\beta u. \]

It is an important economic question whether the statistics of an investigation show \( \alpha + \beta \) to be less than, equal to, or greater than unity. The sum of these coefficients shows the degree of "homogeneity" of the function. If \( \alpha + \beta \) is equal to unity, we say that the production function is homogeneous of the first degree.

Marginal productivity of any factor is the slope of the function graphed in the output-factor input dimensions when all other inputs are held constant. It was noted above that the marginal productivity changed as we moved along the curve at different levels of factor input. We noted, however, that the Cobb-Douglas function took on a linear form when expressed in logarithmic instead of arithmetic units. We can, therefore, write
(change in logarithm of output)/(change in logarithm of labor input) = $\alpha$ when the capital input is held constant. The change in the natural logarithm of some variable is the same thing as the percentage change. We can, therefore, also write

\[
(\text{percentage change in output})/(\text{percentage change in labor input}) = \alpha \text{ when the capital input is held constant.}
\]

A ratio of percentage changes is simply a ratio of absolute changes multiplied by the inverse ratio of levels of the two variables. The limiting value of absolute changes for the infinitesimal increments is, however, the concept of marginal productivity. We can, therefore, write

\[
(\text{percentage change in output})/(\text{percentage change in labor output}) = \alpha
\]

\[
((\text{labor input})/(\text{output})) (\text{marginal productivity of labor}) = \alpha.
\]

This brings us to the important property of the function: marginal and average products are proportional, where the factor of proportionality is the associated exponent.

Marginal productivity of labor = $\alpha$ (output)/(labor input)

= $\alpha$ (average productivity of labor).

Similarly, we find

marginal productivity of capital = $\beta$ (average productivity of capital).

The Cobb-Douglas production function had its beginning in 1928 when Senator Paul Douglas, a member of University of Chicago's Economics Department, sought to derive a production function for the United States economy. This work was a pioneering effort in this field. Douglas, together with Charles W. Cobb, attempted to determine the
influence of capital and labor on production in the United States for the years 1899-1922. Capital, labor, and production were all measured in terms of index numbers with the method of least square employed to obtain estimates of the parameters of the function. The following function is a log linear homogeneous Cobb-Douglas production function:  

\[ P = 1.01 L^{3/4} C^{1/4} \]

Since that time production functions have been derived for Australia, \(^2\) India, \(^3\) and other countries.

A form of the Cobb-Douglas production function has also been used to derive manufacturing relationships for different industries. Vernon L. Smith completed a study on the trucking industry in which a Cobb-Douglas function was used to explain the relationship between the inputs and outputs. \(^4\)

Once the input-output relationships for Utah's structural steel fabrication industry, as described by a production, are known, it will be possible to analyze the impact on related industries for given changes in structural steel fabrication.

---


In this thesis a modified Cobb-Douglas function of the form
\[
Y = a X_1^k X_2^j ... X_n^l
\]
will be used, where the \( X_i \) \((i = 1, ..., n)\) represents inputs and \( Y \), the output of fabricated steel.

Sample Data

The data consists of monthly accounting figures from a sample of firms described in the preceding chapter. Variables on which observations were made are: Raw materials purchased from roller mills, other raw materials purchased, wages paid, salaries paid, depreciation, and sales of manufactured goods.

In order to obtain data which is representative of the industry as it now exists and over a time period for which technology was relatively stable or unchanging, the years 1960, 1961, 1962, and 1963 were chosen. A mailed questionnaire was used to collect the data after permission was granted by an officer of each company. A sample questionnaire is included in Appendix A. The data received is in Appendix C.

Method of Analysis

Before regression was performed on the data it was first corrected for price level change. It is necessary to have the data in real terms since production function theory is based on real terms and the data collected was in money terms. The deflators used for wages and salaries paid are set forth in Table 1. These data were computed from the average hourly earnings for the fabricated metal products industry of the state. A base year of 1957-59 was used.
### Table 1. Wage deflators

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td>102.5</td>
<td>111.6</td>
<td>112.0</td>
<td>115.8</td>
</tr>
<tr>
<td>February</td>
<td>101.2</td>
<td>109.9</td>
<td>112.8</td>
<td>116.2</td>
</tr>
<tr>
<td>March</td>
<td>102.0</td>
<td>112.4</td>
<td>115.8</td>
<td>117.5</td>
</tr>
<tr>
<td>April</td>
<td>103.7</td>
<td>111.2</td>
<td>113.3</td>
<td>118.7</td>
</tr>
<tr>
<td>May</td>
<td>103.3</td>
<td>114.5</td>
<td>115.3</td>
<td>116.2</td>
</tr>
<tr>
<td>June</td>
<td>105.4</td>
<td>115.0</td>
<td>114.9</td>
<td>116.6</td>
</tr>
<tr>
<td>July</td>
<td>102.7</td>
<td>115.8</td>
<td>114.9</td>
<td>116.2</td>
</tr>
<tr>
<td>August</td>
<td>101.2</td>
<td>109.1</td>
<td>114.5</td>
<td>115.8</td>
</tr>
<tr>
<td>September</td>
<td>101.2</td>
<td>109.1</td>
<td>115.3</td>
<td>114.5</td>
</tr>
<tr>
<td>October</td>
<td>101.2</td>
<td>109.1</td>
<td>116.2</td>
<td>115.3</td>
</tr>
<tr>
<td>November</td>
<td>102.0</td>
<td>112.0</td>
<td>114.5</td>
<td>114.5</td>
</tr>
<tr>
<td>December</td>
<td>103.7</td>
<td>114.5</td>
<td>114.5</td>
<td>114.9</td>
</tr>
</tbody>
</table>


The deflators used for sales and raw materials purchased were:

### Table 2. Sales and raw materials deflators

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>101.8</td>
<td>100.1</td>
<td>100.6</td>
<td>98.8</td>
</tr>
<tr>
<td>February</td>
<td>101.4</td>
<td>100.4</td>
<td>100.4</td>
<td>98.6</td>
</tr>
<tr>
<td>March</td>
<td>100.7</td>
<td>100.8</td>
<td>99.8</td>
<td>98.4</td>
</tr>
<tr>
<td>April</td>
<td>100.7</td>
<td>101.1</td>
<td>99.6</td>
<td>98.5</td>
</tr>
<tr>
<td>May</td>
<td>100.6</td>
<td>100.7</td>
<td>99.2</td>
<td>99.3</td>
</tr>
<tr>
<td>June</td>
<td>100.4</td>
<td>100.8</td>
<td>98.6</td>
<td>99.0</td>
</tr>
<tr>
<td>July</td>
<td>100.1</td>
<td>100.6</td>
<td>98.6</td>
<td>99.0</td>
</tr>
<tr>
<td>August</td>
<td>100.4</td>
<td>100.9</td>
<td>99.1</td>
<td>99.0</td>
</tr>
<tr>
<td>September</td>
<td>100.2</td>
<td>101.1</td>
<td>99.0</td>
<td>99.1</td>
</tr>
<tr>
<td>October</td>
<td>99.8</td>
<td>100.9</td>
<td>98.7</td>
<td>99.9</td>
</tr>
<tr>
<td>November</td>
<td>99.5</td>
<td>100.1</td>
<td>98.4</td>
<td>99.9</td>
</tr>
<tr>
<td>December</td>
<td>99.6</td>
<td>100.2</td>
<td>98.7</td>
<td>100.0</td>
</tr>
</tbody>
</table>

These figures are taken from the iron and steel portion of the wholesale price index published in the Survey of Current Business. Base year was 1957-59.

After the data was deflated to represent an index of real rather than money values, it was converted to its log (natural) form.

The Mathematical Model

The first hypothesis tested was

\[ Y = aX_1^i X_2^j X_3^k X_4^l X_5^m \]

where \( X_1 \) represents raw materials from roller mills, \( X_2 \) other raw materials, \( X_3 \) wages, \( X_4 \) salaries, and \( X_5 \) the amount of capital used (depreciation). A multiple regression analysis yields the following results:

\[ Y = 39X_1^{0.054} X_2^{0.292} X_3^{0.535} X_4^{-0.343} X_5^{0.300} \]

with an \( R^2 = .69 \). Although the model fits the data rather well, the negative coefficient for \( X_4 \) implies a negative marginal productivity for salaried personnel. This can in part be explained by the interrelationships between the variables. If output were to rise the \( X_1, X_2, X_3, \) and \( X_5 \) variables would likely rise directly, but \( X_4 \) or salaries would likely remain somewhat constant. So there would not be as direct a relationship between output and salaries as between output and the other variables. This could cause the coefficient to be negative.

Wages and salaries were then combined into one variable. Since
depreciation was made available only on a yearly basis, and since
the total amount was less than two percent of total sales, it was
deleted from the model.

Therefore, the second hypothesis was:

\[ Y = a x_1 x_2 x_3 \]

where \( x_1 \) represents raw materials from the roller mills, \( x_2 \) was
other raw materials, and \( x_3 \) was wages and salaries. The resulting
parameters were:

\[ Y = 70.6 x_1^{0.028} x_2^{28} x_3^{39} \]

with an \( R^2 = .64 \). Although the \( x_1 \) variable (materials from the
roller mills) was the largest of the inputs (and logically an impor­
tant one), it was not significant at either the .01 or .05 level.6
This might well be explained by the warehousing function carried on
by some of the companies. When they receive a contract, they try to
order as much of the structural steel pre-cut from the mill as pos­
sible. This reduces the expense of cutting at their individual plants.
Furthermore, with a warehousing operation, they maintain some inventory
for the occasional buyer. A certain amount of structural steel is
purchased each month. Some will be sold through the warehouse and the
remainder will be used for the contracts. There is no way of separating
what was used for contracts and what was used for the warehousing

---

5The other data were made available by months.

6The analysis of variance for these models can be found in
Appendix B.
operation. So this could account for the non-significance of the $X_1$ variable.

The roller mill products and the other raw materials inputs were then combined to form another hypothesis.

Thus, the third hypothesis tested was:

$$Y = aX_1 X_2^\frac{1}{2}$$

where $X_1$ represents all raw materials and $X_2$ wages and salaries. The results were:

$$Y = 55.7 X_1^{.408} X_2^{.287}$$

with an $R^2 = .60$.

The analysis of variance for the $Y = aX_1 X_2^\frac{1}{2}$ model is as follows:

<table>
<thead>
<tr>
<th>Source</th>
<th>d.f.</th>
<th>Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>$X_1$</td>
<td>1</td>
<td>2.6959</td>
</tr>
<tr>
<td>$X_2$</td>
<td>1</td>
<td>1.6610</td>
</tr>
<tr>
<td>Model</td>
<td>2</td>
<td>7.1658</td>
</tr>
<tr>
<td>Error</td>
<td>81</td>
<td>.1173</td>
</tr>
</tbody>
</table>

7 The interpretation of $X_1$: The average of the percentage of $X_1$ that consists of roller mill products was 58.7 percent with a standard deviation of 16.7 percent. The percentages were found to be normally distributed. Confidence intervals were then constructed around this average to give some guide for interpretation of the function. The confidence interval is:

$$P(62.26 > X > 55.14) = .95$$

Therefore, of the $X_1$ variable, 58.7 percent is structural steel and the remainder other raw material inputs such as paint, rivets, etc.
The tests for significance are:

\[ F_{X_1} = 22.98 \]
\[ F_{X_2} = 14.16 \]
\[ F_{\text{mod}} = 61.09 \]

Therefore the two variables plus the model are significant at both the .01 and .05 level. This means that the probability is less than .01 that these results are due to random variation.

Expressed in log form, the above function becomes:

\[ \log_e Y = 4.019 + .408 \log_e X_1 + .287 \log_e X_2. \]

This function, when used to make forecasts, yields valid results only in the median range of the data. This can be explained by the inventory changes which cannot be neted away from the input data. That is, some months when output was low, purchases of raw materials and labor were higher as a result of new contracts which were to extend over several months. When sales were high—sales which included output produced earlier—purchases of raw materials and labor inputs were low. As a result of varying lengths of contracts, there was no relationship between the high and low sales months. This made it impossible to make an appropriate adjustment on the data. Actually, the function fits the data, but the unadjusted data is not realistic since it includes inventory changes. In the next chapter a function will be derived from generated data which eliminates these changes.
In the last chapter a production function was derived from questionnaire data which contained inventory changes. In this chapter a function will be derived from data adjusted to eliminate the effects of these changes.

**Limits of Substitution**

From a theoretical point of view, the Cobb-Douglas production function has unlimited substitution between variables. This is inherent in the algebraic properties of the model. With unlimited substitution between variables the function will be of little or no value in forecasting the effect of increases or decreases in demand for the final product on the inputs.

In order to establish the degree of substitution between inputs for the current problem, ratios of total sales to labor inputs and total sales to raw material inputs were computed.

\[
\text{Ratio}_L = \frac{\text{total monthly sales}}{\text{monthly labor costs}} = L_1, L_2, L_3, \ldots, L_n
\]

\[
\text{Ratio}_M = \frac{\text{total monthly sales}}{\text{monthly raw material costs}} = M_1, M_2, M_3, \ldots, M_n
\]

The ratios were graphed and the percent between \( \pm \) one standard deviation and \( \pm \) two standard deviation was computed. The ratios were found to be normally distributed. The labor ratios have a mean of
3.98 and a standard deviation of 1.99; i.e.,

\[ \bar{L} = \frac{1}{n} \sum_{i=1}^{n} L_i = 3.98 \]

\[ S_L = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (L_i - \bar{L})^2} = 1.99 \]

Given this information, a 95 percent confidence interval yields the following:

\[ P(3.54 < 3.98 < 4.40) = .95. \]

The raw material ratios have a mean of 1.64 and a standard deviation of .83; i.e.,

\[ \bar{M} = \frac{1}{n} \sum_{i=1}^{n} M_i = 1.64 \]

\[ S_M = \sqrt{\frac{1}{n-1} \sum\{M_i - \bar{M}\}^2} = .83 \]

The corresponding 95 percent confidence interval is:

\[ P(1.46 < 1.64 < 1.82) = .95. \]

These confidence intervals indicate that there is very little substitution between the inputs for a given output. This was suspected because of the nature of the industry where a given amount of steel and labor is necessary for production. It is apparent that the range of substitution of labor is greater than the range of substitution of raw materials. This results from contracts or jobs which require a certain amount of structural steel with the labor requirement flexible to vary depending on the amount of handling, etc. These limits may be
somewhat overstated because of the heterogeneity of the contracts.

Given an increase in demand for final output, all other things equal, the raw material needs can be forecast from its ratio, or the labor needs from its ratio. These forecasts are expected to be accurate since the confidence intervals for each of the ratios—total sales to labor costs, total sales to raw material costs—is very small.

In order to determine (a) if there is any change in the degree of substitution between inputs as output increases, and (b) the points at which further substitution is impossible (points at which the isoproduct contours become vertical on one side and horizontal on the other), the data were classified according to the following groups and subjected to an analysis of variance.

Table 3. Mean and standard deviation of ratios by groups

<table>
<thead>
<tr>
<th>Sales in dollars</th>
<th>Number of observations</th>
<th>Labor ratios Mean</th>
<th>Labor ratios Standard deviation</th>
<th>Raw materials ratios Mean</th>
<th>Raw materials ratios Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$35,000 - 50,000$</td>
<td>14</td>
<td>3.90</td>
<td>3.07</td>
<td>1.67</td>
<td>.92</td>
</tr>
<tr>
<td>$51,000 - 65,000$</td>
<td>11</td>
<td>3.81</td>
<td>2.22</td>
<td>1.32</td>
<td>.65</td>
</tr>
<tr>
<td>$70,000 - 95,000$</td>
<td>19</td>
<td>4.07</td>
<td>2.53</td>
<td>1.49</td>
<td>.97</td>
</tr>
<tr>
<td>$96,000 - 115,000$</td>
<td>12</td>
<td>4.16</td>
<td>1.79</td>
<td>1.87</td>
<td>1.07</td>
</tr>
<tr>
<td>$125,000 - 175,000$</td>
<td>15</td>
<td>3.93</td>
<td>1.87</td>
<td>1.86</td>
<td>.66</td>
</tr>
<tr>
<td>$180,000 - 340,000$</td>
<td>10</td>
<td>3.93</td>
<td>1.08</td>
<td>1.61</td>
<td>.41</td>
</tr>
</tbody>
</table>

The analysis of variance was used to determine if there were any significant differences between group means. The following results were obtained:
Analysis of variance (labor):

<table>
<thead>
<tr>
<th>Source</th>
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<th>s.s.</th>
<th>m.s.</th>
<th>F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groups</td>
<td>5</td>
<td>1.013</td>
<td>.2025</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>75</td>
<td>149.51</td>
<td>1.9974</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>80</td>
<td>150.52</td>
<td></td>
<td>.1013</td>
</tr>
</tbody>
</table>

Analysis of variance (raw materials):

<table>
<thead>
<tr>
<th>Source</th>
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<th>s.s.</th>
<th>m.s.</th>
<th>F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>5</td>
<td>3.0391</td>
<td>.6080</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>75</td>
<td>60.61</td>
<td>.8080</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>80</td>
<td>63.65</td>
<td></td>
<td>.7525</td>
</tr>
</tbody>
</table>

\[F_{5,75} = 2.35\]

We see from the analysis of variance that for both labor and the raw material inputs there is no significant difference between the group means at different levels of production. Thus we will assume that at each level of output the 95 percent confidence intervals on the labor and raw material input ratios represent the bounds beyond which further substitution is impossible.

**Production Function**

As indicated in the preceding chapter, the production function derived from monthly questionnaire data was, as a result of inventory problems, valid only in the mid-range of the outputs. This problem of overlapping of inventories could in part be solved by aggregating the data into quarterly sets, but there would still be some overlapping of inventories. Another possibility for solving the problem would be to
use only data for those months where there was little or no overlap, but this would raise a question of validity and perhaps give a false picture. It was thus decided to generate the labor and raw material inputs for different levels of output by means of the input ratios described above. That is, for a given level of output, divide output by the mean of the L ratios to obtain the labor input requirement, and similarly divide output by the mean of the M ratios to obtain the raw materials requirement; i.e., given an output level of $100,000 the input requirements would be $100,000/3.98 for labor and $100,000/1.64 for raw materials. Since there were no significant differences between the means of the input ratios for different levels of output and since the confidence intervals were small, it is expected that the inputs generated for the various output levels will be reliable. On the basis of the generated data, the following parameters were obtained for the function

\[ Y = 2.2 X_1^{6.66} X_2^{3.34} \]

where \( X_1 \) represents raw materials and \( X_2 \) labor. This equation has constant returns to scale and will be used to make forecasts for the structural steel fabricating industry.

The marginal physical products for the two inputs are as follows:

\[ \text{MPP}_{X_1} = 1.45 X_1^{-0.34} X_2^{0.34} \]

\[ \text{MPP}_{X_2} = 0.748 X_1^{0.66} X_2^{-0.66} \]

Using the limits of substitution as defined by the above
confidence intervals, contours for the function can be generated for
different levels of output. Figure 4 represents a set of such contours
derived by using the labor ratios to find the labor requirements for
given output levels, and the function to find the raw material require-
ments. The points at which the curves become vertical or horizontal
represent the limits of substitution between inputs.

Estimating

Virtually all of the output of the structural steel fabricating
industry is sold to the construction sector of the state. Therefore,
factors that affect the construction industry will also affect the
fabrication of structural steel. The returned questionnaires indicate
that the majority of the sales were to general building contractors
in the state and the rest went to the general construction sector.¹

One factor that could have a significant influence on the
structural steel fabricating industry is the $57 million bonding bill
that was passed by the Utah State Legislature for the construction of
additional college buildings. Of this $57 million, $2 million will be
used for land purchases and the remainder for building construction.²

The two main types of building construction are the steel
super structure type and the concrete super structure type. The contract
for the steel super structure type is about 15 percent structural steel

¹ General building contractors are primarily engaged in the con-
struction of dwellings, office buildings, stores, etc. The general
construction sector includes contractors who build highways, bridges,
dams, etc.

² Salt Lake Tribune (Salt Lake City, Utah), February 3, 1965.
Figure 4. Iso-product curves
and for the concrete super structure is about 7 percent structural steel. The majority of these materials will come from Utah's structural steel fabricating industry.\textsuperscript{3} If one assumes that the bonding program will create an increase in the demand for structural steel of $6 million, the additional requirements for labor and raw materials can be calculated from the production function. The amount of labor can be computed directly from the ratios described in the forepart of this chapter. This is found to be $1,527,600 of additional labor. The average wage for structural steel fabrication work is $2.68 an hour.\textsuperscript{4} Therefore, 569,776 more manhours will be required. Given an increased demand of $6 million for final output and 569,776 more manhours of labor, $3,658,536 worth of additional raw materials will be required. Approximately 58.7 percent or $2,147,561 of these raw materials will be structural steel. If other things were to remain equal and the construction created by the bonding were to make an increase of $6 million, then an extra $1.5 million will have to be spent on labor and $3.5 million more will have to be spent on raw materials by the structural steel fabricating industry.

Another factor which will have an important effect on structural steel fabrication is the highway construction plan for the future. From the study by Wilber Smith, a consultant engineer, $776,000,000 should be spent on highway construction in the next 18

\textsuperscript{3}Schaub and Haycock, Architects, Interview, May 6, 1965, Logan, Utah.

years. About 3.5 percent of this total are structures which are purchased from the structural steel fabricating industry. This represents a demand for $27 million in output from the industry. Given this increased demand, $6,852,900 labor or 2,557,052 manhours will be required. The amount of raw materials will be $16,463,415, of which $9,665,025 will be structural steel.

The trend in construction is down slightly from the last several years. The authorized construction for the first four months of 1963 was $65,207,000. In the same period of 1964, it was $55,571,000, and in the first four months of 1965, it was $44,202,000. It is expected, however, that this is only a temporary lag and that construction activity will continue to follow an upward trend. As Utah's population grows, the demand for new construction will increase as more homes, business offices, manufacturing facilities, etc., are needed. With increased construction, there will be an increase in the demand for structural steel.

With the function derived in this thesis, it is possible to determine the relative importance of the various inputs. The function will make it possible to analyze the factors that will be necessary for future growth and what the impact will be on related industries of given changes in structural steel fabrication. One must remember when making forecasts that there will also be a multiplier and accelerator

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effect. Also, in making the predictions in this thesis, pure competition was assumed in the factor market. The function which best represents the structural steel fabricating industry in Utah is:

\[ y = 2.2 x_1^{.66} x_2^{.34} \]

Within the limits described in the preceding section, this function may be used for forecasting purposes.
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(24) The newspaper clipping collection of Leonard J. Arrington, Department of Economics, Utah State University.


APPENDICES
Gentlemen:

On July 15 Mr. Don Thomas of our research staff called on you to explain the study we are conducting on the structural steel fabricating industry. At that time you indicated that you would complete a questionnaire for us. We would appreciate as much information as is convenient for you to give us. If you need any help in filling out this questionnaire, please feel free to contact Mr. Thomas. If you keep your records quarterly, make your entries for the appropriate blanks disregarding the notation for months.

We would like to have total purchases of raw materials broken down into purchases from roller mills and all other purchases. Also, the breakdown of labor payments into total wages paid (hourly personnel) and total salaries (monthly personnel) if it is at all possible would be appreciated.

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<td>Salaries</td>
</tr>
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Total amount of depreciation in 1963

Type of depreciation used
### 1962

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Total amount of depreciation in 1962

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Total amount of depreciation in 1961
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</table>

Total amount of depreciation in 1960 __________________________________________

What percent of your sales were to general building contractors in Utah?

What percent of your sales were to general construction (roads, etc.) in Utah?

What percent of your sales were to subcontractors for building in Utah?

What percent of your sales were to others in Utah?

What percent of your purchases were from Geneva Steel Mills?

What percent of your purchases were from other sources in Utah?

What percent of your purchases were from other steel mills (non-Utah)?
Appendix B

Analysis of variance

\[ Y = 39 x_1^{0.054} x_2^{2.92} x_3^{5.35} x_4^{-0.343} x_5^{3.00} \]

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\[ R^2 = 0.69 \]

Analysis of variance

\[ Y = 70.6 x_1^{0.026} x_2^{2.28} x_3^{3.9} \]

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\[ R^2 = 0.64 \]
Table 4. Continued

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<th>X₃</th>
<th>X₄</th>
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</table>

X₁ = raw materials from roller mills.

X₂ = other raw materials.

X₃ = wages paid.
\( X_4 \) = salaries paid.
\( X_5 \) = capital from yearly depreciation figures.
\( Y \) = total monthly sales.