Selecting Optimum Conversion Practices in the Pinyon-Juniper Type

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SELECTING OPTIMUM CONVERSION PRACTICES
IN THE PINYON-JUNIPER TYPE
by
Richard J. Marasco

A thesis submitted in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE
in
Agricultural Economics

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1966
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Appreciation is also extended to my parents and wife for the support and encouragement they gave during the study, and to my friends and associates for their help.

Richard Marasco
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INTRODUCTION OF THE PROBLEM AND JUSTIFICATION FOR RESEARCH

A profit criterion which will make possible the selection of optimum conversion practices in the Pinyon-Juniper woodlands can be made operational if: (a) it is possible to predict eradication costs and resulting total costs (eradication costs, seed costs, and seed application costs), and (b) it is possible to determine forage production resulting from initial eradication, as well as when it reaches absolute minimum allowable limit due to tree re-growth. Knowledge of the above relationships makes it possible to determine the optimum practice.

In recent years considerable investment has been made to increase forage resources in the Western United States. A substantial portion of these have taken the form of efforts to convert Pinyon-Juniper woodlands into grazing areas. This is accomplished by introducing various species of forage following eradication of Pinyon-Juniper trees.

Federal land management agencies have controlled approximately 16 percent of the 75 million acres of Pinyon-Juniper woodlands in the Southwest since 1951. Control efforts in Arizona and New Mexico have been undertaken mainly by Bureau of Indian Affairs and Forest Service. Projects in Utah, Colorado, and Nevada have generally been undertaken by the Bureau of Land Management.

Public land management agencies in Arizona and New Mexico direct their efforts toward controlling invasion stands. In such instances dependence is placed on reduction of competition to cause a release of native understory resulting in greater grazing potential of the site. The problems faced by land management agencies in Utah, Colorado, and
Nevada are quite different. Sites in these states are characterized by trees that are mature and heavily stemmed with dense crown canopies. The resulting competition leaves little or no native understory, making grazing impossible. Therefore, tree removal must be followed by reseeding with appropriate species of grass for the area.

Over the past few years the rate of Pinyon-Juniper control has been decreasing. This decline can be attributed to three factors: (a) limited numbers of accessible invasion stands remaining in Arizona and New Mexico, (b) failure to achieve establishment of grass seedlings after control and seeding costs have been incurred, and (c) failure to achieve an unquestionable benefit-cost relationship that shows benefits in excess of costs.

Costs of Pinyon-Juniper eradication in the past have been characterized by a wide range of values. For example, over a sample of 170 observations, out-of-pocket costs ranged from $.45 to $12.35 per acre. This disparity is partially a function of the eradication techniques employed and partially a function of terrain and tree site conditions.

The large number of eradication and seeding techniques that are available and resulting combinations of costs and benefits make the need for a method of selecting an optimum conversion practice imperative. The procedure for determining an optimum conversion practice is set forth in a four-part analysis. The first establishes the relationship between the dependent variable (removal cost) and the independent variables (density, soil, slope, tree heights, and number of acres). The second part determines the minimum total cost associated with the technologically fixed maximum grass establishment for each technique potentially applicable to a particular site. In the third attention is
turned to the effects of tree re-growth and grass production, and the
time interval until re-eradication will be necessary. The final section
brings all portions of the model together and contains an example.

The data employed in this study were taken from office records
kept by the different Bureaus and partially from data collected by the
Bureau of Land Management's ecological sampling team. The data collected
for office records will be published in a report entitled, "Management
Alternatives for Pinyon-Juniper Woodlands - Part B" which will be
published in the fall of 1966. The data collected by the sampling
team are summarized and published in a report entitled, "Management
Alternatives for Pinyon-Juniper Woodlands - Part A."
OBJECTIVES OF STUDY

The primary objective of this study was the determination of a criterion which would make possible the selection of optimum conversion practices in the Pinyon-Juniper type. In order to accomplish this objective it was necessary to determine: (a) the factors influencing eradication costs, (b) the minimum total cost associated with the technologically fixed maximum grass establishment for each technique potentially applicable to a particular site, and (c) the effects of tree re-growth on grass production, as well as the time interval until re-eradication will be necessary.

With the determination of the previous objective serving as a foundation, establishment of a criterion which will serve as the basis for choosing the control method for a particular site is now possible. It is necessary first to calculate the present value of net benefits accruing at various points in time for each technique. The point in time where present value of net benefits is a maximum is, therefore, the optimum time for re-control. Having found the maximum present value of net benefits for each technique, selection of the optimum conversion practice is, therefore, defined as the technique yielding the maximum of the maximum present values of net benefits.
CONCEPTUAL SOLUTION

Determination of a relationship between cost per acre for the eradication and site characteristics such as density and height of the Pinyon-Juniper trees, soil, slope, and number of acres will serve as the primary foundation for determining the optimal conversion practice to be used on a particular site. This makes it possible to predict costs per acre for several alternative control techniques.

Given the relationship between costs per acre for the eradication and site characteristics for each technique, the minimum total cost associated with the maximum grass establishment for each technique in question must be determined. It is important to note that the minimum total cost is considered as that cost incurred to obtain the maximum level of grass establishment per technique.

Establishment will be maximized subject to the constraints of kill, cover, seed rate, and weather index. The levels of kill, cover, and seed rate that maximizes grass establishment will then be substituted into the total cost function, which will be defined as a function of kill, cover, and seed rate. This maximum value of grass establishment will also enable the amount of forage production associated with the particular level of establishment to be determined.

But due to encroachment of residual trees, the levels of forage production may fall over time. Therefore, it is necessary to develop a relationship that will lead to the calculation of the point in time where grass production reaches the minimum allowable limit. With the levels of kill for each technique already determined for a particular
site, it is possible to calculate the number of trees not affected by eradication treatment. Determination of the relationships between production and crown canopy makes possible the calculation of the absolute minimum allowable grass production and the corresponding percent of crown canopy. This relationship leads to the calculation of the average crown diameter which in turn makes it possible to find the relationship between average tree height and average tree diameter.

The resulting relationship between average tree height and average tree diameter leads to the determination of a relationship between tree height and average age of the trees not affected by original eradication. By taking this age and subtracting it from the age of the trees at the time of original chaining yields the replacement interval. Determination of the preceding variables makes it possible to arrive at the present value of net benefits for each technique. Having arrived at an array of present values, selection of the point in time where present value is a maximum is possible. The technique yielding the maximum of the maximum present values of net benefits is, therefore, defined as the optimum conversion practice for the particular site in question.
PART I

DETERMINATION OF THE PREDICTIVE COST MODELS
DEVELOPMENT OF VARIABLES

Hypothesis

In order to determine the relationship between cost per acre and site characteristics, the primary hypothesis to be tested was that changes in tree densities, soil texture, roughness of terrain, and number of acres result in increased eradication costs.

Empirical Procedure

Statistical analysis was used to determine the variables affecting cost of eradication. The particular tools employed here were multiple regression and analysis of variance.

Multiple regression analysis

Multiple regression is a technique which determines the effect of several independent variables upon a single dependent variable. The general model used was as follows:

\[ Y_i = b_0 + \sum_{i=1}^{n} b_i X_i + e_i \]

where:

- \( Y_i \) = the dependent variable.
- \( X_i \) = independent variables.
- \( b_0 \) = \( Y \) intercept.
- \( b_i \) = the regression coefficients.
- \( e_i \) = the stochastic variable.
Analysis of variance

Analysis of variance is a method which makes possible the analysis of the effects of the partial regression coefficients and the regression model. It is also a tabular form of presenting the statistical analysis.

The two hypotheses that were of particular interest are: (a) do the partial regression coefficients differ significantly from zero, and (b) does the model explain a significant amount of the variation in the dependent variable. To test these hypotheses that $B_1 = B_2 = \ldots = B_N = 0$ the following "F" statistic (Johnston, 1963) was employed:

$$F = \frac{R^2/(K-1)}{1-R^2/(N-K)}$$  \hspace{1cm} (1)

where:

$R^2$ = coefficient of determination.

$K$ = number of independent variables.

$N$ = number of observations.

The equation (Anderson and Bancroft, 1952) employed to test the model for significance was the following:

$$F = \frac{MS_{sv}}{MS_E}$$

where:

$MS_{sv}$ = mean squares associated with the source of variation under question.

$MS_E$ = mean squares associated with the residual term.

Selection and Development of Variables

An _a priori_ selection of variables thought to be important in explaining the variation in eradication cost are: size of the project, density of the trees, soil type, and terrain. In the case of bulldozing
the height of the trees was also considered as an independent variable.

Costs per acre for control on projects ranging from 0-1,000 acres were much higher than for projects ranging from 1,000-4,000. It appears that, once a certain size of project is reached, total costs per acre tend to remain constant. The higher costs on small projects can be partially explained by the fact that the average total cost is higher than it would be on a large project, because in the large project case the fixed cost is being spread over a larger number of acres, and the width of the swath in the chaining and double chaining cases might be smaller due to the irregular shape of the smaller projects.

Density of the stand of trees, soil type, and roughness of terrain all affect cost in the same manner. The increase in density of the stand acting along with changes in soil type and roughness of terrain increases the time required to eradicate an acre, causing an increase in the average variable cost of the contractor. Fuel, oil, grease, repairs for the tractor, and operator cost are also variable costs, but due to the scope of this particular study concern was placed primarily on bid costs.

In the bulldozing case the time required to push tall trees was greater than that required to push small trees. Cotner and Jameson (1959, p. 7) regressed tree height in feet against time required in hours and came up with a correlation coefficient of 0.640. Therefore, this increase in time required to push tall trees results in a higher cost per acre for the control.
There are many different methods that may be used to eradicate Pinyon-Juniper woodlands. In this analysis emphasis was given only to arriving at a predictive model for each of the three major techniques: (a) single chaining, (b) double chaining, and (c) bulldozing.

For purposes of tightening up the model and increasing its predictability, it was necessary to divide the study area into two parts in the single chaining case. This division was made on the basis of cost and site characteristics. Through the use of this technique it was possible to divide the control projects into two groups: (a) Arizona and Nevada projects, and (b) Colorado, New Mexico, and Utah projects.

In the double chaining case it was not necessary to make this type of division. The sites used in this analysis were mainly located in the Utah-Colorado area, although there were a couple of sites located in the Arizona area. The sites located in Arizona were selected on the basis of how well their site characteristics coincided with those in the Utah-Colorado area. Bulldozing on the other hand was taken from sites only in the Arizona area, and as a result, no division was necessary.

In the single and double chaining cases it was not possible to obtain information as to the number of trees per acre for the particular site in question. As a result, percent of the site occupied by Pinyon-Juniper was used. In the bulldozing case information pertaining to the number of trees per acre on the sites in question was available and in turn this figure was used.
In order to carry out the analysis further it was also necessary to code the types of soil and types of terrain. Coding of the soil was accomplished by summarizing all the soil types of an area and the costs associated with them. They were then assigned a numerical number (1, 2, 3,...) according to their average cost with the lowest averaged cost receiving the lowest numerical value (one), and the highest averaged cost receiving the highest numerical value. This was carried out for all techniques, and in the single chaining case it was necessary to derive two separate codings: one for the Arizona-Nevada area, and one for the Colorado, New Mexico, and Utah area.

Terrain was coded in a similar fashion. Sites characterized by level slopes were coded as ones and those characterized by rolling to rough terrain as fours. Appendix B contains tables that present the numerical code for each particular type of soil and slope encountered in the study area.

Tree heights being one of the independent variables characteristic of the bulldozing case alone was an additional variable used in the determination of this predictive model. Although tree height datum was not directly available for the particular sites in question, a method was devised whereby a figure for tree height could be determined.

This method consisted of determining where the site was located. After this was accomplished it was necessary to determine whether or not there was any information concerning tree height on related sites located in that area. When a site was located, the tree height data that was characteristic of the site were then assumed to be similar to that of the site in question.
Single Chaining

Arizona and Nevada area

Determination of a predictive eradication cost model for the Arizona and Nevada area was based on 18 observations. In the regression model the variables were defined as follows:

\[ \hat{Y} = \text{control cost per acre}. \]

\[ X_1 = \text{acres}. \]

\[ X_3 = \text{soil type}. \]

\[ X_4 = \text{terrain or slope}. \]

\[ X_5 = \text{density of Pinyon-Juniper}. \]

The resulting regression equation was:

\[ \hat{Y} = 0.2715 + 0.00009X_1 - 0.000000024X_1^2 + 0.0413X_3 + 0.672X_4 + 3.119X_5. \] (2)

The analysis of variance and significance tests are presented in Table 1.

Table 1. Analysis of variance for single chaining model, Arizona-Nevada area

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Mean squares</th>
<th>Regression coefficients</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>17</td>
<td>.81728E-00a</td>
<td>271498E-00</td>
<td>13.34*</td>
</tr>
<tr>
<td>B(0)</td>
<td>--</td>
<td>--</td>
<td>934625E-04</td>
<td>.</td>
</tr>
<tr>
<td>B(1)</td>
<td>1</td>
<td>.49366E-01</td>
<td>241559E-07</td>
<td>.</td>
</tr>
<tr>
<td>B(2)</td>
<td>1</td>
<td>.23097E-00</td>
<td>413086E-01</td>
<td>.</td>
</tr>
<tr>
<td>B(3)</td>
<td>1</td>
<td>.43422E-01</td>
<td>672003E-00</td>
<td>.</td>
</tr>
<tr>
<td>B(4)</td>
<td>1</td>
<td>.33835E+01</td>
<td>311944E+01</td>
<td>.</td>
</tr>
<tr>
<td>B(5)</td>
<td>1</td>
<td>.68101E-00</td>
<td>311944E+01</td>
<td>.</td>
</tr>
<tr>
<td>Model</td>
<td>5</td>
<td>.23550E+01</td>
<td>311944E+01</td>
<td>.</td>
</tr>
<tr>
<td>Residual</td>
<td>12</td>
<td>.17656E-00</td>
<td>311944E+01</td>
<td>.</td>
</tr>
</tbody>
</table>

Coefficients of determination = .85.

*Significant at 5 percent probability level.

a .8172E-00 is equivalent to .81728.
Calculating the "F" statistic for mean squares due to model yields a value of 13.34. Comparing this calculated value with the tabular value of "F" with (5,12) degrees of freedom at the 5 percent probability level indicates that the calculated value exceeds the tabular value. Therefore, the hypothesis that the model does not significantly explain the variability in the dependent variable (cost) was rejected. As a result, the alternative hypothesis that the model does significantly explain the variation in the dependent variable was accepted.

Testing the null hypothesis that $B_1 = B_2 = ... = B_5 = 0$ was accomplished by substituting the value of $R^2$, $K$, and $N$ into equation (1). Comparing the "F" calculated value resulting with the tabular value of "F" with $(K-1)$ and $(N-K)$ degrees of freedom at the $\alpha = .05$ level resulted in the rejection of the null hypothesis and acceptance of the alternative hypothesis that $B_1 \neq B_2 \neq ... \neq B_5 \neq 0$.

**Colorado, New Mexico, and Utah area**

Determination of a predictive eradication cost model for Colorado, New Mexico, and Utah was based on 23 observations. The variables in this model were defined in the same manner as that used for the Arizona and Nevada area.

The resulting regression equation was:

$$\hat{Y} = .3673 + .000568X_1 - .0000002X_1^2 + .3322X_3 + .4937X_4 + .4705X_5. \quad (3)$$

The analysis of variance and significance tests are presented in Table 2.

Calculating the "F" statistic for mean squares due to model yields a value of 5.13. Comparing this calculated value with the tabular value of "F" with (5,17) degrees of freedom at the 5 percent probability level indicates that the calculated value exceeds the tabular value. Therefore, the hypothesis that the model does not significantly explain the
variability in the dependent variable was rejected. As a result, the alternative hypothesis that the model does significantly explain the variation in the dependent variable was accepted.

Testing the null hypothesis that $B_1 = B_2 = \ldots = B_5 = 0$ was accomplished by substituting the value of $R^2$, $K$, and $N$ into equation (1). Comparing the "F" calculated value resulting with the tabular value of "F" with $(K-1)$ and $(N-K)$ degrees of freedom at the $\alpha = .05$ level resulted in the rejection of the null hypothesis and acceptance of the alternative hypothesis that $B_1 \neq B_2 \neq \ldots \neq B_5 \neq 0$.

Table 2. Analysis of variance for single chaining in the Colorado, New Mexico, and Utah area

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Mean square</th>
<th>Regression coefficient</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>22</td>
<td>.8010E-00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B(0)</td>
<td>--</td>
<td>--</td>
<td>.36727E-00</td>
<td></td>
</tr>
<tr>
<td>B(1)</td>
<td>1</td>
<td>.1053E-00</td>
<td>.56774E-03</td>
<td></td>
</tr>
<tr>
<td>B(2)</td>
<td>1</td>
<td>.9643E-01</td>
<td>-.20014E-06</td>
<td></td>
</tr>
<tr>
<td>B(3)</td>
<td>1</td>
<td>.3128E+01</td>
<td>.33217E-00</td>
<td></td>
</tr>
<tr>
<td>B(4)</td>
<td>1</td>
<td>.3256E+01</td>
<td>.49373E-00</td>
<td></td>
</tr>
<tr>
<td>B(5)</td>
<td>1</td>
<td>.5338E-01</td>
<td>.47051E-00</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>5</td>
<td>.2121E+01</td>
<td></td>
<td>5.13*</td>
</tr>
<tr>
<td>Residual</td>
<td>17</td>
<td>.4131E-00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Coefficient of determination = .602.
*Denotes significance at 5 percent probability level.

Double Chaining

Determination of a predictive model in the double chaining case was accomplished through the use of the same variable employed in the single chaining cases. The resulting regression equation was:
\[ \hat{y} = 1.412 - 0.000613x_1 + 0.00000069x_1^2 + 0.086x_3 + 0.893x_4 + 2.86x_5. \]  
(4)

The analysis of variance and significance test are presented in Table 3. This analysis was based on 21 observations in the Utah, Colorado, and Arizona areas.

Table 3. Analysis of variance for double chaining

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Mean square</th>
<th>Regression coefficient</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>21</td>
<td>.98499E-00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B(0)</td>
<td>-</td>
<td>--</td>
<td>.14116E+01</td>
<td></td>
</tr>
<tr>
<td>B(1)</td>
<td>1</td>
<td>.13786E-00</td>
<td>-.61270E-03</td>
<td></td>
</tr>
<tr>
<td>B(2)</td>
<td>1</td>
<td>.12562E-00</td>
<td>.69303E-07</td>
<td></td>
</tr>
<tr>
<td>B(3)</td>
<td>1</td>
<td>.55012E-00</td>
<td>.86000E-01</td>
<td></td>
</tr>
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<td>B(4)</td>
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<td>.25116E+01</td>
<td>.89309E-00</td>
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</tr>
<tr>
<td>B(5)</td>
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<td>.11441E+01</td>
<td>.2858E+01</td>
<td></td>
</tr>
<tr>
<td>Model</td>
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<td>.29491E+01</td>
<td></td>
<td>7.945*</td>
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<tr>
<td>Residual</td>
<td>16</td>
<td>.37122E-00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Coefficient of determination = .713.
*Denotes significance at the 5 percent probability level.

Calculating the "F" statistics for the mean squares due to the model yields a value of 7.945. Comparing this value with the tabular value of "F" with (5, 16) degrees of freedom at the \( \chi = .05 \) level leads to the rejection of the null hypothesis that the model does not significantly explain the variation in the dependent variable. This leads in turn to the acceptance of the alternative hypothesis that the model does significantly explain the variation in the dependent variable.

Testing the null hypothesis that \( B_1 = \ldots = B_5 = 0 \) was accomplished through the use of equation (1). Comparing the tabular value of "F" with the calculated value of "F" resulted in the rejection of the null
hypothesis and acceptance of the alternative hypothesis that
$B_1 \neq B_2 \neq \ldots \neq B_5 \neq 0$.

Bulldozing

In order to arrive at a predictive model in this case it was necessary to add an additional independent variable (tree height). The variables used were defined as follows:

$\hat{Y} =$ control cost per acre.

$X_1 =$ acres.

$X_3 =$ soil type.

$X_4 =$ slope.

$X_5 =$ density of Pinyon-Juniper.

$X_6 =$ tree height.

$X_7 =$ tree height squared $X_6^2$.

The resulting regression equation took the following form:

$$\hat{Y} = -0.027 - 0.0000857X_1 + 0.0000002X_1^2 + 0.007X_3 +$$
$$+ 0.035X_4 + 0.000086X_5 + 0.1997X_6 + 0.00016X_6^2.$$  \hspace{1cm} (5)

The analysis of variance and significance tests are present in Table 4. This analysis was based on 24 observations.

Calculating the "F" statistic for the mean square due to the model yields a value of 17.75. Comparing this value with the tabular value of "F" with $(7,16)$ degrees of freedom at the $\chi = 0.05$ level leads to the rejection of the null hypothesis that the model does not significantly explain the variation in the dependent variable. This in turn leads to the acceptance of the alternative hypothesis that the model does significantly explain the variation in the dependent variable. Testing the null hypothesis that $B_1 = \ldots = B_5 = 0$ with equation (1) results in
the rejection of the null hypothesis and acceptance of the alternative hypothesis that $B_1 \neq B_2 \neq \ldots \neq B_5 \neq 0$.

Table 4. Analysis of variance for bulldozing

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Mean square</th>
<th>Regression coefficient</th>
<th>F</th>
</tr>
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<td>-.27421E-01</td>
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<td>.70449E-02</td>
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</tr>
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<td>B(4)</td>
<td>1</td>
<td>.14841E-01</td>
<td>.35213E-01</td>
<td></td>
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<tr>
<td>B(5)</td>
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<td>.12532E-02</td>
<td>.85586E-04</td>
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<td>B(6)</td>
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<td>.19972E+00</td>
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</tr>
<tr>
<td>B(7)</td>
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<td>.73006E-03</td>
<td>.16345E-03</td>
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<td>114.00*</td>
<td></td>
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<tr>
<td>Residual</td>
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<td>.58453E-02</td>
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<td></td>
</tr>
</tbody>
</table>

Coefficient of determination = .98.

*Denotes significance at the 5 percent probability level.
SUMMARY

Multiple regression procedures were employed to determine the relationship between eradication costs per acre and site characteristics. These resulting predictive models will in turn make it possible for range managers to calculate eradication costs for any site in question, given its site characteristics.

With the preceding models, it is possible to predict eradication costs for a particular site. It now becomes necessary to determine the minimum total cost associated with the mechanically fixed maximum grass establishment.
PART II

DETERMINATION OF THE MINIMUM TOTAL COST ASSOCIATED WITH

THE MECHANICALLY FIXED MAXIMUM GRASS ESTABLISHMENT
DEVELOPMENT OF CONSTRAINTS AND PROCEDURE

Empirical Procedure

Determination of the minimum total cost associated with the maximum grass established was accomplished through the use of linear programming procedures. Programming problems are concerned with the efficient use or allocation of limited resources to meet desired objectives. One of the main characteristics of this type of problem is that there are an infinite number of solutions that satisfy the basic conditions of the problem. The solution that satisfies both the conditions and the objectives set out by the program is termed the optimum solution.

Solutions to linear programming problems can be obtained in a variety of ways. The method employed in this study was the simplex method. This method is an iterative procedure that consists of moving from an extreme point to an adjacent extreme point leaving a lower value of the objective function. These moves are continued until an optimal extreme point has been reached.

Selection and Development of Constraints

Predictive grass establishment models for various areas occupied by Pinyon-Juniper woodland have been developed by Glover (1966). These particular models are expressed as functions containing the following variables: kill, cover, seed rate, and weather. Although in some cases the functions were expressed as a combination of the above variables. For the purpose of this section emphasis was placed on kill, cover, and seed rate.
Given the particular establishment model that was determined by Glover (1966) for the particular area in question, it was possible to arrive at a maximum grass establishment in the following manner. Establishing equation (6) as a hypothetical grass establishment model characteristic of a particular area subject to kill, cover, seed rate, and weather constraints given by equations (7) through (13) made it possible to employ straightforward linear programming to determine the optimal solution:

\[ Z_{\text{max}} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4, \quad \ldots \quad (6) \]

subject to:

\[ \beta_1 x_1 = b_1, \quad \ldots \quad \ldots \quad \ldots \quad (7) \]
\[ \beta_1 x_1 + \beta_2 x_2 \leq b_2, \quad \ldots \quad \ldots \quad \ldots \quad (8) \]
\[ \beta_2 x_2 \leq b_3, \quad \ldots \quad \ldots \quad \ldots \quad (9) \]
\[ \beta_3 x_3 + \beta_4 x_4 \leq b_4, \quad \ldots \quad \ldots \quad \ldots \quad (10) \]
\[ \beta_4 x_4 = b_5, \quad \ldots \quad \ldots \quad \ldots \quad (11) \]
\[ \beta_2 x_2 + \beta_4 x_4 \leq b_6, \quad \ldots \quad \ldots \quad \ldots \quad (12) \]
\[ \beta_3 x_3 \leq b_7, \quad \ldots \quad \ldots \quad \ldots \quad (13) \]

where:

\[ x_1 = \text{kill.} \]
\[ x_2 = \text{seed rate.} \]
\[ x_3 = \text{cover.} \]
\[ x_4 = \text{weather index.} \]

Derivation of the constraining condition was accomplished in the following manner. Constraint (7), which is the kill constraint, was developed by utilizing the regression equations Rivers (1966b, p. 128) developed for determining the expected percent kill for a given technique. These predictive models took the following form:
\[
\log \hat{Y}_{D.C.} = \log_{10} (16.5) + .4035 \log (X), \quad (14)
\]
\[
\log \hat{Y}_{S.C.} = \log_{10} (6.18) + .6113 \log (X), \quad (15)
\]

where:
\[
\hat{Y}_{S.C.} = \text{expected percent kill for single chaining.}
\]
\[
\hat{Y}_{D.C.} = \text{expected percent kill for double chaining.}
\]
\[
X = \text{the percent of the trees greater than or equal to 7 inches in diameter in the stand.}
\]

Solving the above equations for a particular \(X_i\) yields corresponding expected value of kill, namely \(\hat{Y}_i\).

Due to the lack of a predictive model for kill in the bulldozing case, it was necessary to determine the mean kill associated with it. Therefore, it was necessary to assume that the expected value of kill in all cases for bulldozing was the mean kill associated with it. This assumption was based on the fact that for any given density the desired kill can be obtained if the associated cost is incurred.

Making the substitution of the expected value of kill into constraint (7) yields:
\[
\beta_1 X_i = C_1,
\]

where:
\[
C_1 = \text{constant for the particular site in question.}
\]

The "b" values in the remaining constraints were determined in the following manner. It was determined that as the percent of kill increased, the seed rate would decrease reaching an asymptote at 7 pounds per acre. Setting kill equal to 100 percent and multiplying it by its coefficient and adding it to seven multiplied by its coefficient, yields the value of \(b_2\).
In the case of \( b_4 \), it was observed that when \( x_4 \) was at its maximum, \( x_3 \) was equal to zero. Substituting the levels of these variables into (10) yields the numerical value of \( b_4 \).

The value for \( b_6 \) was determined in a similar fashion. The value of \( x_2 \) was set equal to 7 pounds per acre when \( x_4 \) was at its average value for the area. Solving for \( b_6 \) with these two particular values yields its numerical value.

In the case of \( b_5 \), the weather index constraint, it was necessary to determine the index that is most likely to occur at the time eradication is planned. This value was determined by methods set forth by Glover (1966).

The values of \( b_3 \) and \( b_7 \) were defined as the maximum allowable value that \( x_2 \) and \( x_3 \) could take on multiplied by their respective coefficients. In the case of \( b_3 \), the value of \( x_2 \) was relaxed enabling it to range up to whatever level considered feasible. In turn \( b_7 \) was set at 3 inches because this was selected as the maximum depth a seed drill could achieve.

In areas where establishment was expressed as a function of a combination of the original variables, it was necessary to eliminate the irrelevant constraints. Also in cases where kill fell between the limits of 70 to 100 and where expected weather index fell between the mean weather index and its maximum value for the particular area, it was necessary to eliminate constraints (7), (8), (9), and (12). In the situation where the value of kill fell within the 70-100 limit and the expected weather index value fell outside of its defined range, it was necessary to eliminate only constraint (8). When the expected index value fell within the defined range and kill fell outside its ranges, it was necessary to eliminate constraint (12).
In situations where the value of kill and expected weather index fell within the defined ranges, it was observed that 7 pounds of seed yields high level of establishment. Therefore, whenever this situation arose seed rate was automatically plugged in at 7 pounds per acre.

**Selection and Development of Variables for Minimum Total Cost Function**

Determination of a minimum total cost associated with the maximum mechanically set grass establishment was accomplished by defining minimum total cost as a function of out-of-pocket cost (cost of eradication, cost of seed, and cost of seed application). The cost of eradication was calculated by substituting the site characteristic into the predictive cost models for the particular site in question. This figure was defined as the cost associated with the average kill for the technique in question.

Calculating the expected value of kill \( E(K_j) \) from equations (14) and (15) yields expected values of kill for single chaining and double chaining. \( CK \) (cost of eradication) for these two cases was then calculated by setting up a proportion between the expected value of kill to \( CK \) and the mean kill for the technique to the predicted cost of eradication.

In the bulldozing case it was necessary to set up a proportion between the mean tree density (107.9) for this technique to the cost resulting from the cost model and the density associated with the site under question to \( CK \). Solving for \( CK \) yields the cost of eradication for this technique. The above adjustment in \( CK \) is necessary because the predicted cost generated by the predicted cost models was assumed to be the cost associated with the mean kill for a given technique.
Cost of the seed was determined by taking the rate of seed that maximizes grass establishment and multiplying it by the price per pound of the appropriate species of grass for the area. Cost of seed application was determined in the following manner. If the level of $X_3$ was less than or equal to one, the application technique used was aerial seeding. The cost that was associated with this technique was determined by finding the mean cost for all the projects studied that were aerial seeded.

If the level of $X_3$ was greater than one or equal to three, then the application technique used was drilling. The cost associated with this particular technique was also the mean cost for all the projects studied that were seeded by drilling methods. It is important to note that $X_3$ was limited to values ranging from zero to three.

Given the above values it was possible to define the minimum total out-of-pocket costs as follows:

\[ MTC = CK + CS + CA, \]  

(16)

where:

- \( MTC \) = minimum total cost.
- \( CK \) = cost of eradication.
- \( CS \) = cost of seed.
- \( CA \) = cost for applying seed.

Derivation of an Optimal Solution for Several Hypothetical Situations

The following hypothetical situation illustrates how to set up the programming problem when: (a) the establishment model is expressed as a function of all the variables given in equation (6), (b) the establishment model is characterized by having kill and expected weather indexes
falling into the ranges previously defined, and (c) when the establishment model is expressed as a function of a combination of the independent variables.

Case I is an illustration of the programming problem when the establishment model is expressed as a function of all the variables given in equation (6). Cases II and III illustrate establishment models that: (a) have the level of kill and expected weather index values falling within the ranges previously defined, and (b) when it has in its make-up a combination of the independent variables. It must be kept in mind that all the above cases are strictly hypothetical in nature; that is, all the coefficients in the establishment model employed were arbitrarily chosen.

Case I

Case I is an example of a hypothetical situation where the establishment model is a function of all the variables contained in equation (6) with the values of \( X_1 \) and \( X_4 \) falling outside their defined ranges. Defining the following variables as:

\[
X_1 = .60 = \text{kill.}
\]
\[
X_2^{\text{max}} = 10.0 = \text{maximum seed rate.}
\]
\[
X_4 = 128.0 = \text{weather index.}
\]
\[
X_3^{\text{max}} = 3.0 = \text{maximum depth of cover.}
\]
\[
X_4^{\text{m-a}} = 372.0 = \text{maximum and average weather index.}
\]

\[
Y_E = -.07 + .231X_1 + .026X_2 + .16X_3 + .00117X_4,
\]

and setting up the following restrictions makes it possible to employ linear programming procedures to determine the optimal solution.
\[
.231x_1 = .1386.
.231x_1 + .026x_2 = .4130.
.026x_2 = .2600.
.16x_3 + .00117x_4 = .4352.
.00117x_4 = .1498.
.026x_2 + .00117x_4 = .6172.
.16x_3 = .48.
\]

\[
Z_{\text{max}} = -.07 + .231x_1 + .026x_2 + .16x_3 + .00117x_4. \quad (17)
\]

\[Z_{\text{max}} = .764 \text{ and the level of the variables yielding this value are:}
X_1 = .60, \quad X_2 = 10.0, \quad X_3 = 1.8, \quad X_4 = 128.0. \quad \text{It was necessary now to convert } Z_{\text{max}} \text{ from index terms (Glover, pp. 11-14) into probability terms. This was accomplished by referring to Table 1b in Anderson and Bancroft (1952, p. 382) which yields the area under the normal curve. In the above case } Z_{\text{max}} \text{ in probability terms is equal to .77; that is given the previous values of } X_1, X_2, X_3, \text{ and } X_4, \text{ the range manager has a 77 percent change of obtaining grass establishment.}
\]

It is now possible to calculate the minimum total out-of-pocket costs associated with mechanically set grass establishment. If the expected cost of eradication is defined as $3.50 and the mean kill of the technique in question associated with it is .74, the resulting value of CK is equal to $2.84.

The seed cost (CS) may be calculated by multiplying \(x_2\) by the cost per pound of the particular species of grass to be used. For this particular example the cost of grass per pound was set at $.25 per pound. Multiplying \(x_2 = 10\) by $.25 yields CS = $2.50.

In the case of the cost of seed application it was found that \(x_3\) was greater than one which implies drilling of the seed. Therefore, the
mean cost for drilling was substituted as CA. Substituting CK, CS, and CA into equation (16) yields:

\[ \text{MTC} = 2.84 + 2.50 + 1.50. \]
\[ \text{MTC} = 6.84. \]

**Case II**

Case II is an example of how to set up the programming problem when \( x_1 \) and \( x_4 \) fall outside of the defined ranges. Defining:

\[
\begin{align*}
X_1 & = .81. \\
X_2 & = 10.0. \\
X_3 & = 3.0. \\
X_4 & = 128.0. \\
X_4 & = 372. \\
X_4 & = 186. \\
Y &= -.07 + .231X_4 + .026X_2 + .16X_3 + .00117X_4,
\end{align*}
\]

and setting up the following constraints:

\[
\begin{align*}
.231X_1 & = .1894. \\
.026X_2 & = .2600. \\
.16X_3 + .00117X_4 & = .4352. \\
.00117X_4 & = .1498. \\
.026X_2 + .00117X_4 & = .3996. \\
.16X_3 & = .4800. \\
Z_{max} & = -.07 + .231X_1 + .026X_2 + .16X_3 + .00117X_4.
\end{align*}
\]

\( Z_{max} = .8770 \) and the level of the variables yielding this value are:

\( X_1 = .81, X_2 = 9.6, X_3 = 1.8, X_4 = 128.0. \) Converting \( Z_{max} \) into probability terms yields \( Z_{max} = .79. \) The calculation of the minimum total cost associated with the mechanically set maximum grass establishment now proceeds in the same fashion as that for Case I.
In cases where $x_4$ falls within the limits previously defined and $x_1$ falls outside its defined limits, the problem takes the following form:

\[ 0.231x_1 = 0.1894. \]
\[ 0.231x_1 + 0.026x_2 \leq 0.4130. \]
\[ 0.026x_2 \leq 0.2600. \]
\[ 0.16x_3 + 0.00117x_4 \leq 0.4352. \]
\[ 0.00117x_4 = 0.1498. \]
\[ 0.16x_3 \leq 0.4800. \]

$$Z_{\text{max}} = -0.07 + 0.231x_1 + 0.026x_2 + 0.16x_3 + 0.00117x_4.$$  

Determination of the optimal solution now proceeds in the manner as defined for the previous examples.

If $x_4$ and $x_1$ both fall within the defined limits, the constraints and the objective function reduce to the following form:

\[ 0.16x_3 + 0.00117x_4 \leq 0.4352. \]
\[ 0.00117x_4 = 0.1498. \]
\[ 0.16x_3 \leq 0.48. \]

$$Z_{\text{max}} = 0.16x_3 + 0.00117x_4. \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

After determining the optimal solution for $Z_{\text{max}}$ in equation (18), substitution of the particular levels of $x_3$ and $x_4$ along with $x_1 = 0.81$ and $x_2 = 7.0$ must be made back into equation (17). This value of $Z$ must then be converted into probability terms. The calculation of the minimum total cost associated with the mechanically set maximum grass establishment now proceeds in the usual manner.

**Case III**

Case III is an illustration of how to set up the programming problem when the establishment model does not contain all of the variables.
presented in equation (6). If the particular establishment model under question was a function of kill and weather alone, the problem was set up in the following way:

\[ .231X_1 = .1386. \]
\[ .16X_3 + .00117X_4 \leq .4352. \]
\[ .00117X_4 = .1498. \]
\[ .16X_3 \leq .48. \]
\[ Z_{\text{max}} = -.07 + .16X_3 + .00117X_4. \]

\( Z_{\text{max}} \) was determined by using the simplex method.

The previous method is applicable to all cases where the establishment model is made up of a combination of the original independent variable. The only difference is the fact that different constraints are eliminated.

In cases such as the above where seed rate falls out of the establishment model, it is necessary to use the seed rate that was generally used in the area to calculate MTC. This variable dropped out of the establishment model because the seed rate used remained constant over all the projects. After the CS has been determined, calculation of MTC can proceed in the same manner.
PART III

DETERMINATION OF THE RE-CONTROL DATE
DEVELOPMENT OF VARIABLES AND PROCEDURES

One of the characteristics of the Pinyon-Juniper type is that even if control plots are completely cleared of trees, there is a tendency for re-growth, this re-growth presents the need for future maintenance or re-eradication. Because of the time it takes the trees to develop, it was necessary to consider one cycle, that is, the initial control plus re-control.

Determination of the mechanically set date for eradication may be determined in two ways. In order to set the stage for these two methods, it was necessary to test the following hypotheses:

(a) that there exists a direct relationship between average crown diameter and total tree height,
(b) that there exists a relationship between total tree height and average age of the tree,
(c) that there exists a relationship between time and crown canopy,
(d) that there exists a relationship between crown canopy and production.

Empirical Procedure

Of primary concern in the statistical analysis was the determination of the relationships between the various independent and dependent variables. Three statistical tools were used to determine and test these relationships. The first was a simple regression analysis. The analysis of variance was the second. This made possible the partitioning...
of the sum of squares due each source of variation. The third involved the "F" test to determine significance of each of the models.

**Selection and Development of Variables**

Development of the following relationships was accomplished by summarizing data collected by the Bureau of Land Management's ecological sampling team in two field seasons. Rivers (1966a, 1966b) developed relationships between the following independent and dependent variables. His relationships were subdivided into species and site class. The relationships developed in this part of the analysis are general relationships, which were developed solely for purposes of clarity. In order to develop a relationship between average crown diameter and total average tree height, it was necessary to summarize the data collected.

The tree height data were taken in terms of total average tree heights and the age data were the average age of the trees at the 1 foot level. The value for tree age at the 1 foot level was obtained through the method presented by Rivers (1966b, pp. 135-143).

Developing a relationship between time and crown canopy, where time is the independent variable, was accomplished in the following manner. Given the number of trees per acre not affected by the initial control, it was possible through the age-density-percent crown canopy relationship developed by Rivers (1966b, pp. 125-162) to determine the relationship between time and crown canopy.

The relationship between production and crown canopy was determined by summarizing the data collected. A reciprocal transformation was then carried out on the data to make possible the derivation of the desired relationship.
ANALYSIS AND RESULTS

In this section the analysis and results are presented for each hypothesis tested. The order of presentation is as follows: (a) tree height vs. average crown diameter, (b) tree height vs. age, (c) time vs. crown canopy, and (d) production vs. crown canopy.

Tree height vs. average crown diameter

The 64 observations involved in this analysis were the averages of all tree heights and average crown diameters on the 64 plots. The resulting regression line took the following form:

\[ Y = 4.56 + 1.32X, \quad (19) \]

where:

- \( Y \) = total average tree height for the site.
- \( X \) = average crown diameter.

\[ X = 2 \frac{\sqrt{X_i}}{N}, \quad (20) \]

where:

- \( X_i = \frac{Ch}{NS'} \)
- \( Ch \) = percent crown canopy.
- \( NS' \) = number of trees following eradication.

The resulting analysis of variance and test of significance are presented in Table 5.
Table 5. Analysis of variance for average crown diameter and tree height

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Mean squares</th>
<th>Regression equation</th>
<th>F</th>
</tr>
</thead>
<tbody>
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<td>Total</td>
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</tr>
<tr>
<td>B(0)</td>
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<td>-</td>
<td>0.45556E+01</td>
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</tr>
<tr>
<td>B(1)</td>
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<td>0.686604E+04</td>
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<td></td>
</tr>
<tr>
<td>Model</td>
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<td>0.686605E+04</td>
<td>0.132104E+01</td>
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</tr>
<tr>
<td>Residual</td>
<td>62</td>
<td>0.698313E+01</td>
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<td></td>
</tr>
</tbody>
</table>

Coefficient of determination = 0.941.
*Denotes significance at .05 level.

Tree height vs. age

Age data employed in this analysis was the age at the 1 foot level. Tree height was measured in total tree height. The analysis was based on 64 observations which were average tree heights and ages for the 64 plots. The resulting regression equation took the following form:

\[ \hat{Y} = 3.31X^{1.31}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21) \]

where:

\[ \hat{Y} = \text{age at the 1 foot level of NS'}. \]

\[ X = \text{tree height}. \]

The analysis of variance and test of significance are presented in Table 6.
Table 6. Analysis of variance of age vs. tree height

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Mean squares</th>
<th>Regression equation</th>
<th>F</th>
</tr>
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<td>Residual</td>
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<td>.539203E-01</td>
<td></td>
<td>370.57*</td>
</tr>
</tbody>
</table>

Coefficient of determination = .857.
*Denotes significance of the .05 probability level.

Production vs. crown canopy

Development of a relationship between forage production and crown canopy was accomplished by dividing the study area up into three provinces: (a) Escalante-Sevier, (b) La Sal, and (c) Coronado. The boundaries for these provinces were based on difference in geology, topography, soil parent material, climate, and vegetation. Figure 1 is an illustration of where these boundaries fell (Isaacson, p. 29).

The analysis was carried out by summarizing the data collected by the Bureau of Land Management (BLM) sampling team. The regression equations for each of the provinces are given in Table 7.

Use of the reciprocal transformation in analyzing the production vs. crown canopy data made it possible to determine at what level production approaches a horizontal asymptote. This characteristic will be of value later on in the study, and as a result, discussion will be delayed until that point.
Figure 1. Boundaries of the ecological provinces
Table 7. Regression equation for production vs. crown canopy

<table>
<thead>
<tr>
<th>Province</th>
<th>Number of observations</th>
<th>Regression equation</th>
<th>Mean squares</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Escalante</td>
<td>18</td>
<td>$\hat{Y} = 6.402 + \frac{7.208}{bX}$</td>
<td>307.98</td>
<td>.7125</td>
</tr>
<tr>
<td>La Sal</td>
<td>20</td>
<td>$\hat{Y} = 5.788 + \frac{17.954}{X}$</td>
<td>764.36</td>
<td>.9331</td>
</tr>
<tr>
<td>Coronado</td>
<td>18</td>
<td>$\hat{Y} = 45.22 + \frac{3.8019}{X}$</td>
<td>317.48</td>
<td>.7288</td>
</tr>
</tbody>
</table>

$\hat{Y}$ is equal to expected forage production.

$X$ is equal to re-growth time.

The analysis of variance and tests for significance of the models of all the provinces above led to rejection of the null hypothesis that the models didn't explain the variation in the dependent variable. Therefore, the alternative hypothesis that the models do significantly explain the variation in the dependent variable was accepted.

Relation of time and crown canopy

The analysis of crown canopy and re-growth time was determined by deriving a relationship between crown canopy and time for each range of NS' within each site class. Rivers (1966a, P. 85) based site classification on the tree height at the 10 inch diameter class read from his height/diameter curves. The following table uses his site class breakdown for Pinyon.

The regression equation for each of these classifications are presented in Table 8.
Table 8. Regression equation for time vs. crown canopy

<table>
<thead>
<tr>
<th>Site class (Pinyon)</th>
<th>Interval of NS'</th>
<th>Regression equation</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24'-31'</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-100</td>
<td>$\hat{Y} = 0.0007X^{1.60}$ (22)</td>
<td>.8649</td>
<td></td>
</tr>
<tr>
<td>100-200</td>
<td>$\hat{Y} = 0.0014X^{1.56}$ (23)</td>
<td>.8405</td>
<td></td>
</tr>
<tr>
<td>200-up</td>
<td>$\hat{Y} = 0.00159X$ (24)</td>
<td>.8056</td>
<td></td>
</tr>
<tr>
<td>18'-23'</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-100</td>
<td>$\hat{Y} = 0.0009X^{1.485}$ (25)</td>
<td>.8987</td>
<td></td>
</tr>
<tr>
<td>100-200</td>
<td>$\hat{Y} = 0.0009X^{1.649}$ (26)</td>
<td>.9752</td>
<td></td>
</tr>
<tr>
<td>200-300</td>
<td>$\hat{Y} = 0.0014X^{1.61}$ (27)</td>
<td>.9413</td>
<td></td>
</tr>
<tr>
<td>300-up</td>
<td>$\hat{Y} = 0.00137X$ (28)</td>
<td>.9839</td>
<td></td>
</tr>
<tr>
<td>11'-17'</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-100</td>
<td>$\hat{Y} = 0.00065X^{1.102}$ (29)</td>
<td>.8279</td>
<td></td>
</tr>
<tr>
<td>100-up</td>
<td>$\hat{Y} = 0.00047X^{1.24}$ (30)</td>
<td>.9583</td>
<td></td>
</tr>
</tbody>
</table>

$a\hat{Y}$ is equal to percent crown canopy.

$bX$ is equal to re-growth time.

The analysis of variance for each site class and interval indicated that in all cases the "F" test led to the rejection of the hypothesis that the model is not significant. Therefore, this led to the acceptance of the alternative hypothesis.
SUMMARY

The previous relationships made it possible to set up two methods for determining the re-control date. The re-control date for the purposes of this study was defined as the time that re-eradication should take place. Method I employs the general relationships developed in the previous section and Rivers (1966a, 1966b) between tree height, age, and crown diameter. Method II is the more straightforward method for determining the re-control date, employing only the time-crown canopy and forage production-crown canopy relationships.

**Method I**

This method of determining the mechanically fixed date of re-control is based upon the following relationships: first, the relationship between percent of kill and percent of the stand greater than or equal to 7 inches in diameter; second, the relationship between average crown diameter and tree height; third, the relationship between tree height and average tree age; and fourth, the relationship between production and crown canopy.

Given a certain percent of the stand equal to or greater than 7 inches in diameter makes it possible to determine the expected value of kill for each of the techniques. Substituting the expected values of kill into the following relationship:

\[ NS - E(K_n)(NS) = NS', \quad \cdots \cdot \cdot \cdot \quad (34) \]

where:

- \( NS \) = number of trees before eradication.
E(K_j) = expected value of kill for the j^{th} treatment (j = 1 \ldots N) yields the number of trees following eradication.

Determination of the crown canopy where production approaches a horizontal asymptote in equations (31), (32), and (33) was accomplished by putting confidence limits on \( \alpha \), where \( \alpha \) is defined as follows:

\[
\text{Limit } f(x) = \alpha, \quad x \to \infty,
\]

where:

\[
f(x) = \alpha + \frac{\beta}{x}.
\]

\( f(x) \) = the amount of forage production.

\( X \) = percent of crown canopy.

Upon establishing confidence limits on \( \alpha \), determination of the minimum allowable level of production and crown canopy was made possible.

Substituting the average crown diameter of the trees into equation (19) yields the total tree height. The resulting value in turn was substituted into equation (21) yielding the age at the 1 foot level when NS' reaches the level of crown canopy (Ch) where production of forage reaches the minimum allowable limit.

Defining \( T_r \) as the number of years until the mechanically set date for re-control for the j^{th} treatment:

\[
T_r = f(z_2) - f(z_2)', \quad \ldots \ldots \ldots \ldots \ldots (35)
\]

where:

\[
f(z_2) = \text{age when trees reach Ch}.
\]

\[
f(z_2)' = \text{age of the trees less than 7 inches in diameter at the same time of original eradication}.
\]
Method II

This method employs the following relationships: (a) the relationship between time and crown canopy, and (b) production and crown canopy. Given a certain percent of the stand greater than or equal to 7 inches in diameter it was possible to determine the expected value of kill for each of the techniques. Substituting this value into equation (34) yields the number of trees following eradication. Entering this value plus the site class characteristic of the particular site into Table 8 yields the equation that represents the appropriate time-crown canopy relationship.

By referring to Table 7 the equation that represents the appropriate production-crown canopy relationship for the area in question may be obtained. Substituting the correct relationship between time and crown canopy into the relationship between production and crown canopy yields a production as a function of time given by equation (35):

\[ y = \alpha + \frac{B}{\alpha x + B}, \quad \ldots \quad (36) \]

Defining the production function as \( \frac{d}{dp} \) and taking the limit \( \frac{d}{xp} \rightarrow \infty \) yields the horizontal asymptote \( \alpha \). Establishing confidence limits on \( \alpha \) and selecting the upper limit as the minimum level of production yields the age that the NS will have to reach before production reaches its minimum allowable limit.
PART IV

PRESENTATION OF COMPLETE MODEL AND EXAMPLE
PRESENTATION OF COMPLETE MODEL

Introduction

The complete model for determining optimum conversion practices in the Pinyon-Juniper type consists of a combination of the following submodels: (a) the chaining models, (b) maximum grass establishment and associated minimum total cost model, and (c) the re-control model.

Once a site has been selected to apply control measures to, it is possible through the use of the above models to determine: (a) the cost of eradication, (b) the maximum technologically fixed grass establishment and its associated minimum total cost, (c) the mechanically fixed date for re-control, and (d) the resulting forage production.

Upon the determination of the previous variables, it is possible to calculate the point in time where present value of net benefits reaches a maximum value for each technique. Once this point in time has been determined, the optimum time for re-control can be defined as the point in time where present value of net benefits reaches a maximum value. The technique yielding the maximum of the maximum present values of net benefits is, therefore, the optimum practice.

If a number of sites are involved, this same technique provides a method whereby selection of the site yielding the maximum present value of net benefits can be determined. For example, if there are three potential sites in question and the optimum technique for each site has been determined, all that remains is the comparison of the maximum present values of the net benefits. The site yielding the maximum of the maximum present value of the net benefits is, therefore, defined as the optimum site.
Complete Model

For convenience in developing the complete model, it was necessary to adopt the following notation:

\[ MTC_{0j} = \text{the initial cost of control for the } j^{th} \text{ treatment.} \]
\[ MTC_{rj} = \text{the cost of re-control of the } j^{th} \text{ treatment.} \]
\[ Z(E) = \text{the establishment model for the area in question.} \]
\[ \beta_{ij} \sum x_{ij} = \text{constraining conditions on } Z(E). \]
\[ \text{PV(NB)}_j = \text{present value of expected maximum net benefits.} \]
\[ \text{PV(NB')}_j = \text{present value of re-control benefits.} \]
\[ t_o = \text{time of initial control.} \]
\[ t_x = \text{optimum time for replacement.} \]
\[ t_r = \text{the mechanically fixed date for re-control.} \]
\[ \text{PV(L)} = \text{present value of the loss in benefits by delaying replacement until time } t_x. \]
\[ V_f = \text{value of forage.} \]

1 In order to arrive at a figure for \( MTC_{rj} \), it was necessary to add one tree a year from the time of initial control up to the time of re-control to NS'. This value in turn is substituted into the predictive cost equations as the figure for density. In the cases where density is expressed as a percent, the conversion factor is \( \cdot 001 = \text{one tree.} \)

2 Forage being expressed in pounds per acre necessitated conversion of pounds to AUM's per acre (AUM = the amount of forage necessary to sustain a 1000 pound cow through one month). This was accomplished by dividing the amount of forage by 1000 and multiplying by the price of an AUM.
For a given set of site characteristics it is possible to generate values of $K_{0j}$ through the use of equations (2) through (5). The kills associated with the $K_{0j}$'s in the double and single chaining case may be calculated by plugging the value for the percent of the stand greater than or equal to 7 inches in diameter into equations (14) and (15). The bulldozing case necessitates setting the expected value of kill equal to its mean value for the observations studied.

By setting up a ratio of mean kill and expected cost of eradication to expected value of kill and adjusted expected cost of eradication, the value of $CK$ for double and single chaining may be determined. In the case of bulldozing it becomes necessary to revert back to the proportion set up between mean density to expected value of cost and the density of the site under question to $CK$. The value for $CK$ is determined now by solving the above proportion for $CK$.

Maximizing $Z(E)$ subject to $\beta_{1j} \sum X_{1j}$ yields the value or level of $X_2$ (seed rate) and $X_3$ (cover depth). Multiplying $X_2$ by the cost of the seed per pound of the species of grass desired and by setting $CA$ equal to the cost per acre implied by the level of $X_3$ yields the values of $CS$ and $CA$ respectively. If the level implies aerial seeding, $CA$ is equal to $.47$, and if it implies drilling, $CA$ is equal to $1.50$. Taking the summation of $CK$, $CS$, and $CA$ yields the minimum total cost associated with the maximum probability of grass establishment.

Glover (1966, p. 49) in his thesis dealing with predicting production of grass on Pinyon-Juniper woodlands, developed a production function for grass which was expressed in terms of percent utilization, weather index, and probability of grass establishment. His resulting equation took the following form:
where:

\( u = \text{percent utilization.} \)

\( w = \text{weather index.} \)

\( E = \text{probability of establishment.} \)

Given the initial expected value of production, it is possible to express production as a function of crown canopy (equations 31, 32, and 33). The general form is given by equation (38):

\[
P_i = f(u, w, E), \quad \ldots \ldots \ldots \ldots \quad (37)
\]

Setting confidence limits on \( \alpha \) in these equations makes it possible to determine the crown canopy associated with the minimum allowable amount of forage production. The minimum allowable amount of forage production is, therefore, defined as the upper limit on \( \alpha \).

The \( \beta \) values in equation (38) represents the \( \beta \) corresponding to the maximum value production in the data used to generate the respective production functions. If the expected initial amount of production generated by equation (37) deviates from the maximum value, it is then necessary to adjust the \( \beta \) value. This adjustment was made by setting up a proportion between the \( \beta \) values given in equations (31), (32), and (33) to the maximum production value in the data for area used to generate the function, and the adjusted \( \beta \) value to the generated initial amount of production. Solving this proportion yields the adjusted value. This adjustment in the \( \beta \) values was necessary because it takes a longer length of time for forage production to reach its minimum allowable limit than it would for low values of \( P_i \).

If the percent of canopy where the upper limit of \( \alpha \) intersects, equation (38) is defined as \( Ch \) and in turn is substituted into the following equation:
the resulting value is the average square feet of crown area of individual unskilled trees at maturity to satisfy Ch. This value of $X_i$ in turn is then substituted into equation (20) which yields the average crown diameter associated with Ch. This leads to the substitutions of the appropriate variables into equations (19) and (21). The mechanically fixed date for re-control is now obtained by substituting the results of equation (21) into equation (35).

Determination of the date for re-control can also be carried out in the following manner. After determining the initial expected value of production from equation (37) and adjusting the $\beta$ values in equation (38), it is possible to express production as a function of time. This is made possible because there exists a relationship between canopy and time which is given by the relationships in Table 9. Upon making the substitution of the time-crown canopy relationship into equation (38) the production function takes the following form:

$$\hat{Y}'_p = \alpha + \frac{\beta}{\alpha'X\beta'}$$

where:

$$\hat{Y}'_p = \text{forage production.}$$

$$X = \text{time.}$$

It is now possible to establish confidence intervals on $\alpha$. Selecting the upper limit as the minimum level of production and equating it to the right hand side of equation (39) yields the age that the NS' will have to reach before production reaches its minimum allowable limit. The re-control date is, therefore, defined as the point in time where the upper limit on $\alpha$ intersects the production function.
With the mechanically fixed date of re-control \( \left( t_r \right) \) determined, it becomes possible to determine the present value of benefits associated with initial and re-control.

![Graph illustrating forage benefits](image)

**Figure 2.** Graph illustrating the forage benefits accruing due to initial control and re-control.

Therefore:

\[
PV(B) = V_f \left[ \int_{t_0}^{t_1} P_i e^{-rt} dt + \frac{1}{(1+r)t_0} \int_{t_0}^{t_x} p \Delta y_p e^{-r(t_x-t_0)} dt \right], \quad (40)
\]

\[
PV(B') = \frac{V_f}{(1+r)t_x} \left[ \int_{t_0}^{t_1} P_i e^{-rt} dt + \frac{1}{(1+r)t_x-t_0} \int_{t_0}^{t_x} y_p e^{-r(t_x-t_0)} dt \right], \quad (41)
\]

\[
PV(L) = \frac{V_t}{(1+r)t_0} \int_{t_0}^{t_x} (P_i - \Delta y_p) e^{-r(t-x-t_0)} dt, \quad (42)
\]

and,

\[
PV(NB)_j = \left[ PV(B) + PV(B') \right] - \left[ MTC_{ij} + \frac{MTC_{ij}}{(1+r)t_x} + PV(L) \right].
\]
The point in time where PV(NB) reaches a maximum is, therefore, the optimum time for re-control to take place for the $j^{th}$ technique. The maximum value of PV(NB) must now be determined for each technique. The technique yielding the maximum of the maximum PV(NB) is, therefore, the optimum conversion practice.

It is important to note at this point that the calculation of PV(NB) was based on the following assumptions: (a) the technique employed to re-control was the same as that employed at time $t_0$, (b) that $P_i$ for re-control is equal to $P_i$ for the initial control, (c) that the loss in benefits encountered between initial control and re-control were the only losses considered relevant, and (d) that total cost is a function of $MTC_{oj}$, $MTC_{rj}$ (described in a footnote on p. 46), and PV(L).
EXAMPLE

The following example is set forth in four steps: (a) estimation of the initial expected cost of eradication, (b) estimation of the minimum total cost associated with mechanically fixed maximum grass establishment, (c) estimation of the mechanically fixed date of re-control, and (d) determination of the optimum date of replacement and the optimum conversion practice.

Step I

Assume the site under question has the following site characteristics: (a) density equal to 325 trees per acre with average height of 15 feet, (b) sandy clay soil, (c) gentle slope, and (d) the number of acres to be eradicated is equal to 920. Given these characteristics the expected costs of single chaining, double chaining, and bulldozing are by $K_{01}$, $K_{02}$, and $K_{03}$ respectively.

\[
K_{01} = 0.3673 + 0.0006X_1 - 0.000002X_1^2 + 0.3322X_2 + 0.4937X_3 + 0.4705X_4
\]

\[
= 0.3673 + 0.0006(920) - 0.000002(920)^2 + 0.3322(4) + 0.4937(2) + 0.4705(.325) \quad \text{[equation (3)]}
\]

$K_{01} = $3.56.

\[
K_{02} = 1.412 - 0.0006X_1 - 0.00000069X_1^2 + 0.086X_2 + 0.893X_3 + 2.88X_4
\]

\[
= 1.412 - 0.0006(920) - 0.00000069(920)^2 + 0.086(5) + 0.893(2) + 2.86(.325),
\]

$K_{02} = $4.17 \quad \text{[equation (4)].}

\[
K_{03} = -0.027 - 0.00085X_1 - 0.0000002X_1^2 + 0.007X_2 + 0.03X_3 + 0.000085X_4 + \ldots + 0.1997X_5 + 0.00016X_5^2
\]

$K_{03} = $3.26 \quad \text{[equation (5)].}$
If it is assumed further that 60 percent of the stand is greater than or equal to 7 inches in diameter, it is possible to arrive at the expected value of kill for each technique. Substituting 60 percent into equation (14) and (15):

\[
\hat{Y}_{D.C.} = 16.5 \times (60)^{0.4035},
\]
\[
\hat{Y}_{D.C.} = 85.
\]
\[
\hat{Y}_{S.C.} = 6.18 \times (60)^{0.6113}.
\]
\[
\hat{Y}_{S.C.} = 75.
\]
\[
\hat{Y}_B = 0.83.
\]

where:

\( \hat{Y}_{D.C.} \) = kill associated with the technique of double chaining.

\( \hat{Y}_{S.C.} \) = kill associated with the technique of single chaining.

\( \hat{Y}_B \) = kill associated with the technique of bulldozing.

The value of CK for each of the three techniques may now be calculated in the following way:

\[
\frac{\hat{Y}_{D.C.}}{CK_{D.C.}} = \frac{\text{mean kill (taken from Table 19)}}{K_{02}}
\]
\[
\frac{0.85}{CK_{D.C.}} = \frac{0.896}{$4.17}.
\]
\[
CK_{D.C.} = $3.96.
\]
\[
\frac{\hat{Y}_{S.C.}}{CK_{S.C.}} = \frac{\text{mean kill (taken from Table 19)}}{K_{01}}
\]
\[
\frac{0.75}{CK_{S.C.}} = \frac{0.74}{$3.56}.
\]
\[
CK_{S.C.} = $3.61.
\]
To facilitate the calculation of minimum total cost associated with the mechanically fixed optimum grass establishment, the hypothetical site was assumed to be located in the Monticello area. Glover's establishment model (Glover, 1966, p. 15) took the following form:

\[ Z(E) = -1.47 + 2.57X_1 + .009X_2 - .01X_1X_2, \]

where:

\[ X_1 = \text{kill} = .85. \]
\[ X_2 = \text{weather index} = 200. \]

It is important to note that in the above model seed rate and depth of cover have dropped out of the establishment model. This can be explained due to the fact that in this area, seed rate and depth remain constant over all the projects. It is now possible to set up the constraints in the following manner:

\[ Z(E)_{\text{max}} = -1.47 + 2.57X_1 + .009X_2 - .01X_1X_2, \]

subject to:

\[ 2.57X_1 = 2.1845 \text{ (for double chaining)}. \]
\[ .009X_2 = 1.800. \]

Due to the fact that the variables \( X_1 \) and \( X_2 \) are defined as .85 and 200 respectively, the programming model reduces down to one of simply substituting these values into \( Z(E) \). The value for \( Z(E) \) in turn is converted into probability terms through the use of Table 1b in Anderson.
and Bancroft (1952, p. 382). The values of $Z(E)_{\text{max}}$ in probability terms for double chaining, single chaining, and bulldozing are .79, .77, and .78 respectively.

CS may be determined by multiplying seven, which is the amount of seed that management in Monticello employs, by the price of the specific species of grass. Therefore, CS = $\$2.80$, where the price of grass per pound is assumed to be $.40$. CA in this case was set equal to $.53$ per acre (the mean price of aerial seeding) because aerial seeding was the technique employed in the Monticello area.

The value of MTC is obtained by summing CK, CS, and CA for all three cases. Therefore, the MTC's for double chaining, single chaining, and bulldozing are given by $\text{MTC}_{02} = \$7.29$, $\text{MTC}_{01} = \$6.94$, $\text{MTC}_{03} = \$13.13$.

**Step III**

The expected value of initial production may be obtained by substituting the appropriate variables into the following equation Glover (1966, p. 50) developed for the Monticello area.

$$P_i = 528.762 - 0.00277X_1 + 450.288X_2 + 6.289X_3,$$

where:

$P_i$ = expected initial production.

$X_1$ = percent of utilization.

$X_2$ = probability of establishment.

$X_3$ = normal weather index.

The values of $X_1$ and $X_3$ were determined by means presented in Glover (1966). The value of $X_2$ used to calculate $P_i$ for each technique was the value resulting from the maximization of $Z(E)$ for each technique.
Therefore, the values of $P_i$ for double chaining, single chaining, and bulldozing are 940, 930, and 935 pounds respectively.

It now becomes necessary to adjust $\beta$ values in equation (32) in the manner previously suggested (p. 48).

\[ \hat{Y}_{PD.C.} = 5.788 + \frac{62.00}{.00007X^{1.60}}, \quad \ldots \quad (44) \]
\[ \hat{Y}_{PS.C.} = 5.788 + \frac{57.9}{.00007X^{1.60}}, \quad \ldots \quad (45) \]
\[ \hat{Y}_{PB} = 5.788 + \frac{60.10}{.00007X^{1.60}}, \quad \ldots \quad (46) \]

where:
\[ .00007X^{1.60} = \text{crown canopy-time relationship obtained from Table 8 by assuming site class equal to 24'-31' and NS' being between 0-100.} \]

At this time it becomes necessary to set confidence limits on $\alpha$. This was accomplished through the use of the following equation (Johnston, 1963).

\[ \alpha = t_{0.05} \frac{\hat{\sigma}_u \sqrt{\sum x^2}}{\sqrt{N}} \]

where:
\[ \hat{\sigma}_u = \text{the square root mean square error given in Table 7 divided by the square root of N.} \]
\[ N = \text{the number of observations given in Table 7}. \]

Then:
\[ 5.788 \pm 10.28, \]

is the confidence limits on $\alpha$ for the La Sal province with 16.068 equal to the upper limit on $\alpha$. Equating equation (44), (45), and (46) to 16.068 yields the mechanically fixed date of re-control. The date of intersection for double chaining is 400 years. The mechanically fixed
dates for re-control in the single chaining and bulldozing cases are 370 and 385 years respectively.

**Step IV**

It is now possible to determine the present value of benefits accruing from initial control and re-control by making the appropriate substitution into equations (40) and (41). The values of these equations were determined by using approximating methods. For purposes of illustration PV(NB) was calculated at $t_x = 73$ for double chaining, $t_x = 70$ for bulldozing, and $t_x = 67$ for single chaining. It is important to note that in a real life situation it would be necessary to iterate $t_x$ and calculate PV(NB) for each $t_x$ to determine the point in time where PV(NB) is at its maximum for each technique. For the purpose of clarity the values of $t_x$ for the various techniques presented above were assumed to be the points in time where PV(NB) is at its maximum.

Assuming that the values of $t_x$ given above to be the optimum times for re-control and $V_f = 1.00$ along with $r = .02$, the maximum PV(NB)'s for double chaining, single chaining, and bulldozing are given by $35.55$, $34.88$, and $28.03$. Therefore, the optimum conversion practice given the above assumptions is double chaining.
CONCLUSION

In recent years considerable investment has been made to increase forage resources in the Western United States. A substantial portion of these have taken the form of efforts to convert Pinyon-Juniper woodlands into grazing areas. This is accomplished by introducing various species of forage following eradication of mature Pinyon-Juniper trees.

Because of the large number of eradication and seeding techniques available and resulting combination of costs and benefits, the need for a method of selecting an optimum treatment for a given site becomes a necessity.

The model developed in this study presents a framework that makes possible selection of optimum conversion practices in the Pinyon-Juniper type. This particular model is concerned with the maximization of economic objectives, that is, arriving at the particular investment that maximizes net benefits in present value terms.

For purposes of confining the study it was necessary to emphasize only the three major eradication techniques of single chaining, double chaining, and bulldozing. The only techniques considered as possible seed application methods were aerial seeding and drilling.

It may be concluded that because of the number of years between initial control and the optimum time for re-control that tree canopy does not create an immediate threat to forage production. Therefore, more emphasis must be placed on effects random factors have on forage production.
Appendix A

Review of Literature

The following publications have been carefully selected for this review on the basis of how applicable their information was to this particular study.

Cotner and Jameson (1959) presented a discussion on the importance of costs in selecting a method for controlling Pinyon-Juniper. This study was based only on control operations in Arizona.

This study was concerned with costs of two methods: (a) burning individual trees, and (b) bulldozing. Upon analysis of the data, it was determined that the size of trees, the number of trees per acre, and labor were the primary variables that determine costs. The authors also pointed out that there might be some secondary variables such as soil and terrain affecting costs.

In conclusion the authors were mainly concerned with the specific variables of trees per acre and tree height. The particular framework for predicting costs holds up only if the type of equipment studied is available plus information concerning the other variables pointed out above.

In 1963, Cotner (1963a) published a bulletin called, "Controlling Pinyon-Juniper." This bulletin reports the scope of Pinyon-Juniper problems in Northern Arizona, the current status of control work, the methods of control used, and a procedure for predicting costs for the leading control methods (bulldozing, burning, and cabling).

Variables found to be of primary importance were tree heights, densities, and average tree sizes.
In conclusion the author was mainly concerned with invasion stands, and as a result his prediction methods are applicable to sites with this particular characteristic.

Cotner (1963b) presents a discussion of a framework which makes it possible to determine where, when, and how resource improvements should be made, with economic decisions considered the most relevant variable. This framework includes: (a) determination of physical benefits relationship, (b) determination of improvement techniques and costs, (c) discounting the physical benefits stream, (d) selecting the right improvement techniques, and (e) determination of economic feasibility.

Hand chopping, individual tree burning, bulldozing, and cabling were the four major control methods discussed in determining the improvement technique to be employed. It was shown that the feasibility of an individual technique varied from site to site, with cabling being the most proficient on large acreages of middle-age trees on relatively level terrains, and hand-chopping being applicable on sites with low densities of small trees.

In selecting the optimum year for control a comparison of rate of benefit increase, and the rate of cost increase is made. If the two are equal, an optimum has been reached. The optimum technique is, therefore, defined as the technique providing the highest net return over time.

In conclusion the author is mainly concerned with invasion stands of Pinyon-Juniper and determining the optimum time for initial control.

In 1964, Arnold, Jameson, and Reed (1964) published a report that presented information on forage values and use of Pinyon-Juniper woodlands in Arizona. Information includes: (a) a discussion of the problems created by Pinyon-Juniper, (b) various uses and products derived from
these stands, (c) extent, location, and stand characteristics, (d) effects of tree increase on other plants, (e) principle methods used to control Pinyon-Juniper, (f) response of vegetation to tree control, and (g) costs and resulting benefits of control.

In conclusion the study shows that: (a) the costs comparison of the different techniques show that chaining and cabling are the least expensive methods, (b) to obtain the maximum benefits from these methods some follow-up treatment on the slash is required, and (c) in areas of low densities of small trees it was found that individual eradication methods prove to be more efficient.
Appendix B

Miscellaneous Tables

Table 9. Single chaining slope classification for Arizona-Nevada area

<table>
<thead>
<tr>
<th>Code</th>
<th>Average bid cost</th>
<th>General description of slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.58</td>
<td>Level</td>
</tr>
<tr>
<td>2</td>
<td>2.27</td>
<td>Gentle slope</td>
</tr>
<tr>
<td>3</td>
<td>3.24</td>
<td>Rolling</td>
</tr>
<tr>
<td>4</td>
<td>4.27</td>
<td>Rolling with gullies</td>
</tr>
</tbody>
</table>

*The different types of slope were ranked in accordance with the average cost associated with it for the particular area in question.

Table 10. Single chaining soil classification for Arizona-Nevada area

<table>
<thead>
<tr>
<th>Code</th>
<th>Average bid cost</th>
<th>General description of soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.25</td>
<td>Sandy loam</td>
</tr>
<tr>
<td>2</td>
<td>1.52</td>
<td>Limestone silt</td>
</tr>
<tr>
<td>3</td>
<td>1.61</td>
<td>Sandy clay</td>
</tr>
<tr>
<td>4</td>
<td>1.94</td>
<td>Clay silt</td>
</tr>
<tr>
<td>5</td>
<td>3.09</td>
<td>Clay loam</td>
</tr>
<tr>
<td>6</td>
<td>3.94</td>
<td>Sandy silt</td>
</tr>
</tbody>
</table>

*The different types of slope were ranked in accordance with the average cost associated with it for the particular area in question.
Table 11. Single chaining slope classification for Utah, Colorado, and New Mexico area

<table>
<thead>
<tr>
<th>Code</th>
<th>Average bid cost</th>
<th>General description of slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.61</td>
<td>Level</td>
</tr>
<tr>
<td>2</td>
<td>3.15</td>
<td>Gentle</td>
</tr>
<tr>
<td>3</td>
<td>3.91</td>
<td>Gentle rolling</td>
</tr>
<tr>
<td>4</td>
<td>4.53</td>
<td>Rolling</td>
</tr>
</tbody>
</table>

The different types of slope were ranked in accordance with the average cost associated with it for the particular area in question.

Table 12. Single chaining soil classification for Utah, Colorado, and New Mexico area

<table>
<thead>
<tr>
<th>Code</th>
<th>Average bid cost</th>
<th>General description of soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.36</td>
<td>Clay loam</td>
</tr>
<tr>
<td>2</td>
<td>2.59</td>
<td>Sandy</td>
</tr>
<tr>
<td>3</td>
<td>3.38</td>
<td>Sandy loam</td>
</tr>
<tr>
<td>4</td>
<td>3.72</td>
<td>Sandy clay</td>
</tr>
<tr>
<td>5</td>
<td>3.83</td>
<td>Sandy clay loam</td>
</tr>
<tr>
<td>6</td>
<td>3.90</td>
<td>Silty loam</td>
</tr>
</tbody>
</table>

The different types of slope were ranked in accordance with the average cost associated with it for the particular area in question.
Table 13. Slope classification for bulldozing

<table>
<thead>
<tr>
<th>Code</th>
<th>Average bid cost</th>
<th>General description of slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.34</td>
<td>Level</td>
</tr>
<tr>
<td>2</td>
<td>4.54</td>
<td>Gentle</td>
</tr>
<tr>
<td>3</td>
<td>5.89</td>
<td>Medium</td>
</tr>
<tr>
<td>4</td>
<td>10.58</td>
<td>Medium-steep</td>
</tr>
<tr>
<td>5</td>
<td>11.47</td>
<td>Rolling-steep</td>
</tr>
</tbody>
</table>

*The different types of slope were ranked in accordance with the average cost associated with it for the particular area in question.*

Table 14. Soil classification for bulldozing

<table>
<thead>
<tr>
<th>Code</th>
<th>Average bid cost</th>
<th>General description of soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.63</td>
<td>Rocky-sand</td>
</tr>
<tr>
<td>2</td>
<td>3.28</td>
<td>Rocky</td>
</tr>
<tr>
<td>3</td>
<td>4.92</td>
<td>Sandy loam</td>
</tr>
<tr>
<td>4</td>
<td>4.07</td>
<td>Silty clay loam</td>
</tr>
<tr>
<td>5</td>
<td>5.70</td>
<td>Clay loam</td>
</tr>
<tr>
<td>6</td>
<td>7.64</td>
<td>Sandy</td>
</tr>
</tbody>
</table>

*The different types of slope were ranked in accordance with the average cost associated with it for the particular area in question.*
Table 15. Slope classification for double chaining\textsuperscript{a}

<table>
<thead>
<tr>
<th>Code</th>
<th>Average bid cost</th>
<th>General description of slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.38</td>
<td>Level</td>
</tr>
<tr>
<td>2</td>
<td>4.33</td>
<td>Gentle</td>
</tr>
<tr>
<td>3</td>
<td>4.52</td>
<td>Rolling</td>
</tr>
<tr>
<td>4</td>
<td>5.60</td>
<td>Rolling with gullies</td>
</tr>
</tbody>
</table>

\textsuperscript{a}The different types of slope were ranked in accordance with the average cost associated with it for the particular area in question.

Table 16. Soil classification for double chaining\textsuperscript{a}

<table>
<thead>
<tr>
<th>Code</th>
<th>Average bid cost</th>
<th>General description of soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.65</td>
<td>Silty loam</td>
</tr>
<tr>
<td>2</td>
<td>3.91</td>
<td>Medium loam</td>
</tr>
<tr>
<td>3</td>
<td>4.44</td>
<td>Sandy clay</td>
</tr>
<tr>
<td>4</td>
<td>4.73</td>
<td>Sandy loam</td>
</tr>
<tr>
<td>5</td>
<td>4.75</td>
<td>Sandy clay</td>
</tr>
<tr>
<td>6</td>
<td>7.00</td>
<td>Sandy</td>
</tr>
</tbody>
</table>

\textsuperscript{a}The different types of slope were ranked in accordance with the average cost associated with it for the particular area in question.
Table 17. The analysis of variance for mean kills of the three eradication techniques

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sums of squares</th>
<th>Mean squares</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>108</td>
<td>2.6653</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment</td>
<td>2</td>
<td>.3832</td>
<td>.19159</td>
<td>8.899**</td>
</tr>
<tr>
<td>Error</td>
<td>106</td>
<td>2.2821</td>
<td>.02153</td>
<td></td>
</tr>
</tbody>
</table>

**Denotes significance at the .01 probability level.

The hypothesis tested was $H_0: U_1 = U_2 = U_3$ due to the results of the "F" test this hypothesis was rejected. This led to the acceptance of the alternative hypothesis $H_a: U_1 \neq U_2 \neq U_3$.

Table 18. The treatment mean kills for the techniques used in the study

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Number of observations</th>
<th>Treatment mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single chaining</td>
<td>39</td>
<td>.7400</td>
</tr>
<tr>
<td>Dozing</td>
<td>47</td>
<td>.8309</td>
</tr>
<tr>
<td>Double chaining</td>
<td>23</td>
<td>.8965</td>
</tr>
</tbody>
</table>

aData summarized from office reports.
Table 19. Production and crown canopy for three provinces

<table>
<thead>
<tr>
<th>Coronado</th>
<th>La Sal</th>
<th>Escalante</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plot no.</td>
<td>Production (lbs.)</td>
<td>Crown canopy (%)</td>
</tr>
<tr>
<td>383</td>
<td>72</td>
<td>0.273</td>
</tr>
<tr>
<td>317</td>
<td>29</td>
<td>0.373</td>
</tr>
<tr>
<td>311</td>
<td>82</td>
<td>0.177</td>
</tr>
<tr>
<td>315</td>
<td>25</td>
<td>0.333</td>
</tr>
<tr>
<td>313</td>
<td>62</td>
<td>0.207</td>
</tr>
<tr>
<td>314</td>
<td>67</td>
<td>0.267</td>
</tr>
<tr>
<td>360</td>
<td>76</td>
<td>0.263</td>
</tr>
<tr>
<td>362</td>
<td>53</td>
<td>0.303</td>
</tr>
<tr>
<td>366</td>
<td>54</td>
<td>0.237</td>
</tr>
<tr>
<td>398</td>
<td>83</td>
<td>0.180</td>
</tr>
<tr>
<td>399</td>
<td>93</td>
<td>0.173</td>
</tr>
<tr>
<td>403</td>
<td>55</td>
<td>0.270</td>
</tr>
<tr>
<td>26</td>
<td>119</td>
<td>0.067</td>
</tr>
<tr>
<td>382</td>
<td>79</td>
<td>0.223</td>
</tr>
<tr>
<td>288</td>
<td>100</td>
<td>0.073</td>
</tr>
<tr>
<td>364</td>
<td>40</td>
<td>0.270</td>
</tr>
<tr>
<td>384</td>
<td>34</td>
<td>0.360</td>
</tr>
<tr>
<td>18</td>
<td>158</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


