THE WELFARE IMPACTS OF ENVIRONMENTAL REGULATION IN
AN OPEN ECONOMY

by

Nicole Glineur

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Nicole Glineur
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The major theoretical and practical economic issues on pollution have apparently been sorted out. However, the bulk of the literature in environmental economics shares a shortcoming: the disregard of the welfare implications and consequences of pollution control.

Traditionally, pollution is viewed as a joint product. In this study, the issue of trade and environmental regulations is cast as a problem of input regulation, and the subsequent welfare implications of input regulation are then derived. The purpose of the present research is to emphasize the welfare consequences of pollution control in the context of international or interregional trade. The Batra and Casas, Yohe, and McGuire models lay out the theory of the use of the environment in an open economy, deriving the effects of pollution on factor rewards. Using some generalizations of the models, the welfare
impacts of changing regulations which govern the use of the environment are derived.

It is seen that for both a small and large country (region) with identical individuals, there is an optimal level of pollution. In the case where the economy is made up of two different groups of individuals, workers and capitalists (capital owners) in a small country (region), the workers lose while capitalists can either gain or lose. In the large country (region), it cannot be unambiguously determined whether workers and capitalists will be better or worse off than before the regulation changes.

(71 pages)
CHAPTER I

INTRODUCTION

Issue Statement

Today, the major theoretical and practical economic issues on pollution have been sorted out. However, the bulk of the literature in environmental economics shares a shortcoming: the disregard of the welfare implications and consequences of pollution control.

Traditionally, pollution is viewed as a joint product. In this study, the issue of trade and environmental regulation is cast as a problem of input regulation, and the subsequent welfare implications of input regulation are then derived. Environmental regulation has been incorporated into the theory of production and trade but the question of the welfare impacts of pollution control has received only modest attention. Not only is the state of knowledge in the area rudimentary, providing only a limited basis for generalization, it is also devoid of empirical direction.

The purpose of the present research is to emphasize the welfare consequences of pollution control in the context of international or interregional trade, which heretofore, has been neglected. Economists such as Batra and Casas, Grubel, McGuire, Weitzman, and Yohe have established a precedence for such an approach. The Batra and Casas, Yohe, and McGuire models lay out the theory of the use of the environment in an open economy. They have derived the effects of
pollution on factor rewards, however, the welfare impacts of environmental regulation have been left aside. Consequently, it is the welfare ramifications of regulation of the use of the environment as an input that is addressed in this study.

**Objectives**

The overall aim of this study is to analyze, in an open economy and within a general framework, the impact of the exogenously imposed changes in the level of pollution control on the welfare of labor and capital. The specific objectives include:

1. A description of the static general equilibrium Heckscher-Ohlin model, which reveals the effects of environmental controls on factor rewards in an open economy.

2. An analysis of the welfare impact of a change in the pollution control level for a small country (region), i.e., price-taking region, with identical individuals (identical preferences of laborers and owners of capital).

3. A study of the effects of a change in input regulation on the welfare of labor and capital in the small country (region) context.

4. A description of the implications of the large country (region) assumption with variable output prices.

5. An analysis of the welfare impact of a variation in the pollution control level for a large country consisting of identical individuals.

6. An estimate of the change in input regulation implications for
the welfare of labor and capital in a large country (region) context.

Given a welfare analysis which generalizes the Yohe-Batra-Casas model, we summarily conclude that:

A. For a small country (region):
   1) with identical individuals, there is an optimum level of pollution.
   2) with two different groups of individuals, workers and capitalists, the workers will lose, while capitalists can either gain or lose when regulation of the environmental input is relaxed. Workers reduce their consumption of the goods and at the same time pollution is increased; capitalists gain through a rise in capital returns.

B. For a large country (region):
   1) given identical individuals in the economy, there is an optimum level of pollution.
   2) as the economy is divided into two different groups, it cannot be unambiguously determined whether workers and capitalists will be better or worse off than before the regulation changes. Other production parameters, such as the elasticities of substitution between factors, factor intensities, etc., would have to be known.
CHAPTER II

EXTERNALITIES, USE OF THE ENVIRONMENT, AND TRADE:
A REVIEW

The Presence of Externalities

Traditionally, pollution is viewed as an externality, caused by consumption and/or production of pollution-causing goods. The need to devote a section to externalities, while treating pollution is eloquently expressed by the Dorfmans:

"An externality arises when economic agents do not pay for the entire social cost of their activities". Examples of social costs are land disruption, water and air pollution, climatic changes, health, and occupational hazards.

An externality is generated whenever one of the following four situations exists:

1. The inputs or outputs of one firm enter directly into the production function of another firm or firms. For example, the production function of the farmer is affected when the...
pollutants from a factory smokestack causes damage to crops
grown on his neighboring farm.

2. The inputs of a firm enter directly into the utility function
of some individual(s). An example where inputs or outputs of a
firm affect the well-being of society members is when the
pollutants from a factory are detrimental to the health of the
individual(s) subjected to them.

3. The consumption of a good by one individual affects the utility
function of another individual or individuals. For instance,
the education of a child influences the well-being of others by
contributing to the working of society.

4. The consumption of a good by an individual affects the pro­
duction function of a firm or firms. In example, the output of
the farmer is affected when the watershed characteristics of
adjacent land are altered by private constructions.

These effects can be marginal, meaning that the production or
preference functions are affected by small changes in the variables
outside a firm's or an individual's control, or inframarginal, in that
they are affected only by large changes in these variables. The
relationships are direct: they work directly through changes in the
production and/or preference functions rather than indirectly through
the price system. 3

From the four above cases, it can be seen that externalities
cause private and social costs and benefits to diverge. Thus, with
the presence of externalities the optimality property of a competitive
equilibrium is not obtained.
Looking at the first case, if one producer does not consider the positive or negative effects of his activities on other producers, his output will be less or greater than the social optimum. Assume he is creating smoke which is lowering another producer's output. The reduction in output of the second firm represents a real cost, borne by society, of the first producer's activity. Since the producer does not bear this cost himself—his private cost of production is less than the social cost—he will thus tend to overproduce.

Externalities distort market equilibrium away from efficiency. How to correct the divergence, induced by externalities, between private and social costs has been the focus of attention of many economists.

Environmental disruption was first analyzed as a static externality, following Pigou's book, The Economics of Welfare, which led to the recognition that the proper system of taxes and subsidies could correct the suboptimality generated by externalities.

Apart from the concern of Pigou in 1932, little was done in environmental pollution until the 1960s. Before Coase's publication on social cost, it was generally thought that subsidization of the generators of external economies was necessary to increase the socially beneficial activity, and that the taxation of generators of external diseconomies was necessary to decrease the socially harmful activity.

Building from the traditional approach of a two-party externality, Coase emphasizes the reciprocal nature of externalities, the duality of the tax subsidy scheme, and the usefulness of private agreement through bargaining to arrive at the optimal state. The
reciprocal externalities argument is illustrated in the case where the factory's smoke is destroying the crops of a nearby farm. It seems equitable to force the factory to compensate for the crop damage or control the smoke, or both. However, while this helps the farmer, it hurts the factory owner. This is a reciprocal damage situation. If property rights rest with the factory owner, permitting him to produce as much smoke as he wishes, it would benefit the farmer to bribe the factory owner to reduce smoke output up to the point where an additional dollar spent on the bribe just equals the benefit accruing to him from the reduction in smoke damage. In the case where property rights rest with the farmer, the duality assumption becomes a consideration. If the farmer's consent is required before the factory is authorized to discharge any smoke, it would pay the factory owner to bribe the farmer to allow him to produce smoke up to the point where the marginal costs equal the marginal benefits. As the costs and benefits are symmetrical in the two cases, the result is identical, abstracting from bargaining costs which are real and will often prevent a solution. Though in both cases income distribution affects and is affected by various bargaining positions and strategies, the effect on the efficiency of output is the same.

Since the victims will bribe the polluters to reduce pollution beyond the optimal point induced by the tax, Buchanan and Stubblebine showed that a pollution tax can lead to too little pollution. Fisher correctly points out that the significance of this result is weakened by the prohibitive transaction costs of the typical many party pollution case. Also crucial is Mishan's point that property rights
affect the use of resources. Because of the consumer's income constraint, there may be a difference between the amount an individual is willing to pay for clean air, for example, and the amount he or she will require in compensation for the loss of this good. This difficulty is likely to arise when the damaged party is a consumer.

While Coase, Buchanan and Stubblebine adequately examine the above facets of the problem, they do not consider the public goods aspect of many externality problems that deprive their solution of policy relevance.

The Coase theorem fails as a challenge to pollution-control policy involving some form of public intervention. It does offer an insight into the virtues of the market in dealing with certain kinds of externalities, but generally not those associated with pollution or other environment disruption.

Environment as an Input

The externality which we are concerned with in this research is pollution. Traditionally, pollution is viewed as a joint output. Here, pollution is the result of the use of the environment as an input in the production process. Yohe argues that the polluter should pay a compensation for the use of the environment, just as labor effort is rewarded with wages.

As mentioned in Chapter I, the Batra and Casas, Yohe, and McGuire models lay out the theory of the use of the environment in an open economy. They have derived the effects of tighter pollution control on factor rewards as follows:

1. As the environment is regulated, assuming constant prices and provided that the demand for the regulated product is elastic,
the factor used intensively in the nonregulated industry will gain in terms of both goods.

2. As prices vary, the returns to capital and labor move in opposite directions, depending on the factor intensity of the nonpolluting sector.

The two-good/three-factor Batra-Casas model is the starting point of this research, followed by the Yohe and McGuire studies, on the backward incidence of tighter pollution controls onto the factors of production. Hence, the focus is on Yohe's work when prices vary with output.

The Batra-Casas model is based on the Heckscher-Ohlin international trade theory. Following Batra and Casas, McGuire, and Yohe, the impacts of pollution control are analyzed in a two-sector (one polluting, one not) static general equilibrium model. Each sector is producing one good using three factors of production: capital, land, and environment. The Batra and Casas model, along with the Yohe and McGuire analysis, assume pollution occurs because of the use of inputs, one being the environment, to produce the primary good. They do not assume joint production of the good and the pollution as is common in pollution-control studies. Since the focus here is on the welfare implications of input regulation, changes in regulations are represented as changes in the use of one of the factors of production in an open economy.

The Heckscher-Ohlin Trade Theory and the Regulation of the Use of the Environment

The overall aim of this study is to analyze, in an open economy
and within a general equilibrium framework, the impact of the exogenously imposed changes in the level of pollution control on the welfare of labor and capital. The analytical framework of the traditional Heckscher-Ohlin model of international trade yields some useful insights about the factor endowments explanation of the basis for trade, and the effects of trade on income distribution. It gives the most comprehensive explanation of a country’s economy. The exchange of goods is the essence of the international trade problem, and, in the Heckscher-Ohlin model, factor endowments guide the allocation of resources according to comparative advantage.

Environmental regulation has been incorporated into the theory of production and trade. International trade theory helps to determine the incidence of pollution control on the factors of production. The Heckscher-Ohlin model shows that the direction of the incidence depends upon the relative factor intensity of the nonpolluting sector. In the framework of the Heckscher-Ohlin model, environmental regulation is characterized as a control over utilization of the environment. The incorporation of the regulated factor environment in the Heckscher-Ohlin model enables us to derive some results. For example, by applying certain properties of the model, we can deduce some consequences of substituting labor and capital for environment in order to reduce pollution.

**Generalization of the Heckscher-Ohlin Model to Three Factors of Production**

By incorporating environment as an input in the production
process, we are using a third factor of production. The simple "two by two" Heckscher-Ohlin model does not capture all the factors of production.

The validity of presumption originated from a Heckscher-Ohlin type of model is questioned. Authors such as Grubel and Ahmad contend that the extension of the Heckscher-Ohlin model into cases involving more than two factors of production is difficult and results in ambiguous answers.

Ahmad raises the following points. First, the generalizations of Heckscher-Ohlin model have only been proven for the two-factor cases. Firm results are not possible when the number of factors is increased. Second, if environment is included as an additional factor to capital and labor in the production process, it is difficult to determine the market price. "Environment, like any factor of production, has a cost, but because of externalities this cost is not properly reflected in the private costs of production."12 Third, "Heckscher-Ohlin theory points only to a tendency for specialization in line with factor endowments."13

Jones and Takayama argue that the challenged restriction of the model to "twos" offers ease of exposition and clear results.

In defense of the generalization of the Heckscher-Ohlin theory to three factors of production, Jones14 offers the following conclusions: if the number of factors of production exceeds the number of outputs, the general Rybcynski result remains valid and a rise in any single commodity price will cause at least one factor to gain in real terms and at least one factor to lose.
He observes that a series of propositions, presented by the Heckscher-Ohlin model, reveal that productive techniques involve a combination of several inputs to produce a single output in each sector: 15

the assumption of no joint production injects an asymmetry into the input-output constellation, and this asymmetry gets reflected in magnified relationships between output prices and factor prices, on the one hand, and factor endowments and industry output at (constant prices) on the other.

Assuming a greater number of inputs creates new possibilities, of which, says Jones, the most important one is the constant factor returns effect of relative price changes. When there is an excess of two factors, the excluded middle can be filled in. Jones concludes that "two-dimensional building blocks not only provide the firm foundations upon which trade theory is constructed but present a standard of comparison against which truly multidimensional results can be appreciated." 16
Footnotes


8. Transaction costs are nonexistent in Coase's two-party setting example.


11. Based on the Batra-Casas model.


13. Ibid., p. 236.


15. Ibid., p. 40.

16. Ibid.
CHAPTER III

THE MODEL

With the two-good/three-factor Batra-Casas model being the starting point of this research, a replica of the model will first be presented. Next, the development of Yohe's and McGuire's studies, on the backward incidence of tighter pollution controls onto the factors of production, are summarized. Hence, the focus is on Yohe's work when prices vary with output. Based on Yohe's results, an attempt to derive the welfare impact of pollution control is made.

The following procedure is adopted. The first analysis will concentrate on the small country case, with pollution entering directly the consumer's utility function. The focus will then be on the large country case, assuming an economy with identical individuals to later relax this assumption and also concentrate on the variable output prices and pollution control levels.

Pollution Control in the Open Economy Case, A La
Yohe-Batra-Casas-McGuire

Replica of the Batra-Casas Model

The model's assumptions are:

1. An economy with two sectors 1 and 2, where 1 is the polluting sector (imposes the externality)
2. Three factors of production: capital (K), labor (L), and environment (E)

3. The two sectors are characterized by strictly quasiconcave, linear homogeneous production functions:

\[ Y_1 = f^1(K_1, L_1, E_1) \]
\[ Y_2 = f^2(K_2, L_2, E_2) \]

with the restriction that a corner solution, in which one of the goods (sector good) can be produced solely by the use of E, cannot be reached.

Full employment economy is then described by:

\[ L_1 + L_2 = a_{L1} Y_1 + a_{L2} Y_2 = L \]
\[ K_1 + K_2 = a_{K1} Y_1 + a_{K2} Y_2 = K \]
\[ E_1 + E_2 = a_{E1} Y_1 + a_{E2} Y_2 = E \]

where \(a_{ij}\) = requirement of input i to produce one unit of output j; L and K are the fixed endowments of labor and capital to the economy; and E is the allowable level (set by the regulator) of the environment that can be used by both sectors of this two-sector economy.

This constraint is binding, so equality holds in the last relationship. The permissible pollution is regulated, i.e., E is regulated (and it is assumed that industries pollute up to that level).

Now assuming constant returns to scale, there is a zero profit condition. Then price equals constant average cost or marginal cost \(P = AC = MC\), so,

\[ P_1 = a_{L1}w + a_{K1}r + a_{E1}q \]
\[ P_2 = a_{L2}w + a_{K2}r + a_{E2}q \]

for \(w = \) wage; \(r = \) capital price; and \(q = \) shadow price for environment.
The $a_{ij}$ are now the factor demands about their respective unit isoquants and take the form,

$$a_{ij} = a_{ij}(w,r,q), \ i = L, K, E \text{ and } j = 1, 2$$ (8)

where $q$ can be viewed as an effluent charge or a quantity standard.

The above equations are the basic equations for the two-good, three-factor model. Equipped with the above structure, the focus is now on the effects of changes in the allowable level of pollution ($E$) on labor and capital.

Differentiating the full employment relationships, the employment relations yield:

$$Y^{*}_{1}L_{1} + Y^{*}_{2}L_{2} = L_{y} - (\lambda_{1}a_{L1}^{*} + \lambda_{2}a_{L2}^{*})$$ (9)

$$Y^{*}_{1}K_{1} + Y^{*}_{2}K_{2} = K_{y} - (\lambda_{1}a_{K1}^{*} + \lambda_{2}a_{K2}^{*})$$ (10)

$$Y^{*}_{1}E_{1} + Y^{*}_{2}E_{2} = E_{y} - (\lambda_{1}a_{E1}^{*} + \lambda_{2}a_{E2}^{*})$$ (11)

where $\lambda_{ij} = \frac{Y_{i}^{*}a_{ij}}{Y_{j}}, \ j = 1, 2; \ i = L, K, E; \ Y_{j}^{*} = \frac{dY_{j}}{Y_{j}}$ (equals percentage change in the variable, the asterik represents the differential of the variable). Hence, $\lambda_{L1} = \frac{Y_{1}^{*}a_{L1}}{L_{y}}$ is the share of the economy's endowment of labor used in production of $Y_{1}$.

Now differentiating equations (6) and (7), the zero profit conditions, $P = AC = MC$, yields to the price relations:

$$P_{1}^{*} = \gamma_{L1}w^{*} + \gamma_{K1}r^{*} + \gamma_{E1}q^{*}$$ (12)

$$P_{2}^{*} = \gamma_{L2}w^{*} + \gamma_{K2}r^{*} + \gamma_{E2}q^{*}$$ (13)

where $wdY_{kj}^{*} + rdY_{k}^{*} + qdY_{E}^{*} = 0$ was used. Then $\gamma_{ij}$ is the value share of factor $i$ in commodity $j$, i.e., $\gamma_{L1} = a_{L1}w/P_{1}$.

Now differentiation of the unit isoquant factor demands (derived demand) gives the percentage change in input demands in response to factor price changes. The $a_{ij}$ is homogeneous of degree
zero (in factor prices). The differentiation and the zero homogeneity, and the fact that the factor elasticity, say \( \theta_{im} = \frac{\partial a_{im}}{\partial r} \), gives,

\[
a_{Lj}^{*} = -\gamma_{Kj} \sigma_{jL}(w^{*} - r^{*}) - \gamma_{Ej} \sigma_{jL}(w^{*} - q^{*})
\]

\[
a_{Kj}^{*} = -\gamma_{Lj} \sigma_{jK}(w^{*} - r^{*}) - \gamma_{Ej} \sigma_{jE}(r^{*} - q^{*})
\]

\[
a_{Ej}^{*} = -\gamma_{Lj} \sigma_{jE}(w^{*} - q^{*}) - \gamma_{Kj} \sigma_{jE}(r^{*} - q^{*})
\]

for \( j = 1, 2 \)

then solving for \( Y_{1}^{*} \) and \( Y_{2}^{*} \), and the employment relations (9) and (10) simultaneously result in:

\[
Y_{1}^{*} = \frac{\left\{ \lambda_{K1}[L^{*} - (\lambda_{L1} a_{L1}^{*} + \lambda_{L2} a_{L2}^{*})] - \lambda_{L2}[K^{*} - (\lambda_{K1} a_{K1}^{*} + \lambda_{K2} a_{K2}^{*})] \right\}}{\lambda_{L1} \lambda_{K2} - \lambda_{K1} \lambda_{L2}}
\]

\[
Y_{2}^{*} = \frac{\left\{ \lambda_{L1}[K^{*} - (\lambda_{K1} a_{K1}^{*} + \lambda_{K2} a_{K2}^{*})] - \lambda_{K1}[L^{*} - (\lambda_{L1} a_{L1}^{*} + \lambda_{L2} a_{L2}^{*})] \right\}}{\lambda_{L1} \lambda_{K2} - \lambda_{K1} \lambda_{L2}}
\]

These latter equations can be simplified if we let

\[
D_{i} = -(\lambda_{i1} a_{i1}^{*} + \lambda_{i2} a_{i2}^{*}); \quad i = L, K
\]

and

\[
|\lambda_{im}| = \lambda_{11} \lambda_{m2} - \lambda_{12} \lambda_{m1}; \quad i = L, K, E; m = L, K, E; \text{ and } m = i,
\]

then we get the simplification:

\[
Y_{1}^{*} = \frac{\lambda_{K2}(L^{*} + D_{L}) - \lambda_{L2}(K^{*} + D_{K})}{|\lambda_{LK}|}
\]

\[
Y_{2}^{*} = \frac{-\lambda_{K1}(L^{*} + D_{L}) + \lambda_{L1}(K^{*} + D_{K})}{|\lambda_{LK}|}
\]

Now, substituting these latter two equations, (19) and (20), into the last employment relation (11):

\[
|\lambda_{EK}|(L^{*} + D_{2}) + |\lambda_{LE}|(K^{*} + D_{K}) + |\lambda_{KL}|(E^{*} + D_{E}) = 0
\]

or

\[
|\lambda_{EK}|D_{L} + |\lambda_{LE}|D_{K} + |\lambda_{KL}|D_{E} = R
\]
where $R = |\lambda_{KE}|L^* + |\lambda_{EL}|K^* + |\lambda_{LK}|E^*$. This latter relationship (22) can be expanded after substituting for the $D_i$ and $a_{ij}^*$ terms to get a polynomial in $w^*$, $r^*$, and $q^*$ as

$$R = Aw^* + Br^* + Cq^*$$

(23)

Then, using equations (12), (13), and (23), the model can now be closed and solved for $w^*$, $r^*$, and $q^*$ from the matrix equation set:

$$\begin{bmatrix}
\gamma_{L1} & \gamma_{K1} & \gamma_{E1} \\
\gamma_{L2} & \gamma_{K2} & \gamma_{E2} \\
A & B & C
\end{bmatrix}
\begin{bmatrix}
w^* \\
r^* \\
q^*
\end{bmatrix}
= 
\begin{bmatrix}
P_1^* \\
P_2^* \\
R
\end{bmatrix}
$$

(24)

Yohe assumes $\gamma_{E2} = \lambda_{E2} = \sigma_{LE}^1 = \sigma_{LE}^2 = \sigma_{KE}^2 = 0$, or only one of the sectors, sector 1, uses the environment via pollution. Also, in sector 1, only K (capital) is substitutable with E (environment).

The change in $w$, $r$, and $q$ as allowable pollution levels change

First, factor intensities in each output (1, 2) are needed. It can be represented that sector one is pollution- (or environment-) intensive if

$$\begin{align*}
a_{E1}^{E1} & > a_{E2}^{E2} = 0 \\
a_{K1}^{K1} & > a_{K2}^{K2} \\
a_{E1}^{L1} & > a_{E2}^{L2} = 0 \quad \text{(a Yohe assumption)} \\
a_{L1}^{L2} & > a_{L2}^{L1}
\end{align*}$$

and is capital-intensive in the weak factor intensity sense.

Sector two can be made labor-intensive by

$$\begin{align*}
a_{L2}^{L2} & > a_{L1}^{L1} \\
a_{L2}^{E2} & > a_{E1}^{E1} \quad \text{and} \\
a_{K2}^{L2} & > a_{K1}^{L1}
\end{align*}$$
Let us solve the matrix equation (24) for \( \frac{w^*}{E^*} \) by setting \( K^* = L^* = P_1^* = P_2^* = 0 \). This assumption implies that the polluting country (or region) is a small country, taking world prices as given.

Solving equation (24),

\[
\frac{w^*}{E^*} = -\frac{\gamma_{K2}^* \gamma_{E1}^* |\lambda_{LK}|}{G} < 0
\]

(25)

where \( |\lambda_{LK}| = \frac{a_{L1}^* - a_{L2}^* (\gamma_{Y2} \sigma_{K1} \sigma_{K2})}{a_{K1}^*} \frac{\gamma_{Y1}}{L_K} < 0 \), and \( G \) is the determinant of the left-hand side of the matrix equation system expressed as

\[
G = \gamma_{E1}^* (\lambda_{LY1} \gamma_{Y2} \sigma_{LY1}) + \lambda_{LY2} \gamma_{Y1} \gamma_{LY1} \gamma_{Y2} \sigma_{LY1} \gamma_{Y2} \sigma_{LY2} \\
- \left( \frac{\gamma_{LY1} - \gamma_{LY2}^2 \gamma_{LY1} \gamma_{Y2} \sigma_{LY1}}{\gamma_{LY1}} \right)
\]

(26)

The assumptions made are that the production function is strictly quasiconcave and \( \sigma_{LE} = 0 \) (no labor-environment substitution) then,

\[ \sigma_{LY} \text{ and } \sigma_{KL} \geq 0, \]

which implies that \( G < 0 \) (in the polluting industry).

The above result (25) says that as regulation (control) is relaxed on pollution, the payment to labor will fall. Solving for capital, similarly, as

\[
\frac{r^*}{E^*} = \frac{\gamma_{L2}^* \gamma_{E1}^* |\lambda_{LK}|}{G} > 0
\]

(26)

So, as pollution is increased, payment to capital increases.

Finally,

\[
\frac{q^*}{E^*} = \frac{(\gamma_{L1} \gamma_{Y2} \gamma_{Y1} - \gamma_{L2} \gamma_{Y1})}{G} \frac{\lambda_{LK}}{G} < 0
\]

(27)

where \( (\gamma_{L1} \gamma_{Y2} \gamma_{Y1} - \gamma_{L2} \gamma_{Y1}) - \frac{a_{L1}^* - a_{L2}^* (\gamma_{K1} \gamma_{K2})}{a_{K1}^* a_{K2}^*} \frac{\gamma_{Y1}}{P_1 P_2} < 0 \)

The shadow price on the use of the environment falls as the pollution level is increased.
From there it can be shown that
\[
\frac{\gamma_1^*}{E^*} > 0 \quad \text{and} \quad \frac{\gamma_2^*}{E^*} < 0
\]  
which are the Rybczynski theorem results of trade theory applied to this pollution control case.

As the allowable pollution level varies, it will affect national (regional) income: \( I = wL + rK + qE \)
\[
I^* = \gamma_L(w^* + L^*) + \gamma_K(r^* + K^*) + \gamma_E(q^* + E^*) .
\]
It can be shown that \( \gamma_Lw^* + \gamma_Kr^* + \gamma_Eq^* = 0 \), so
\[
I^* = \gamma_L L^* + \gamma_K K^* \quad \text{for} \quad L^* = K^* = 0 
\]
A decrease in pollution level lowers national income if prices are constant. Thus, under that condition, an increase in the pollution level will raise national income.

**Derivation of the results of varying prices on factor rewards**

We then proceed, following Yohe and others to derive the results under varying prices. The following is based on Yohe's paper on "The Backward Incidence of Pollution Control." To derive the results of varying prices on factor rewards, equation (24) needs to be solved. Yohe concentrated on the effect of variable commodity prices on factor rewards as environment is altered. The rate of changes in \( Y_1 \) and \( Y_2 \) are:
\[
\gamma_1^* = \eta_1 p_1^* \\
\gamma_2^* = \eta_2 p_2^*
\]
where the coefficient \( \eta_1 \) is the price elasticity of demand for the product of the ith sector where \( i = 1, 2 \). Observing that
I = P_1 Y_1 + P_2 Y_2, differentiation of I leads to:

\[ I^* = Y_1^* Y_1 + \gamma_1 P_1^* + Y_2^* Y_2 + \gamma_2 P_2^* \]

or

\[ I^* = Y_1^* Y_1(1 + \frac{1}{\eta_1}) + Y_2^* Y_2(1 + \frac{1}{\eta_2}) \]  

(31)

Using equations (25) and (30)

\[ -w^* \left( \frac{G}{\gamma_{LK}/\gamma_{K2}} \right) = \gamma E_1^* = \frac{\gamma E_{E}^*}{\gamma_1} = \frac{Y_1^* Y_1(1 + \frac{1}{\eta_1}) + Y_2^* Y_2(1 + \frac{1}{\eta_2})}{\gamma_1} \]

Rearranging

\[ w^* = -(\frac{\lambda_{LK} \gamma_{K2}}{\gamma_1 G}) Y_1^* Y_1 - Y_2^* Y_2 - (\frac{\lambda_{LK} \gamma_{K2}}{\gamma_1 G}) (\frac{Y_1^* Y_1}{\eta_1} + \frac{Y_2^* Y_2}{\eta_2}) \]

or

\[ w^* = -(\frac{\lambda_{LK} \gamma_{K2}}{\gamma_1 G}) Y_{E^*} - (\frac{\lambda_{LK} \gamma_{K2}}{\gamma_1 G}) Y_1^* P_1^* + Y_2^* P_2^* \]

(32)

A  

B

The first term of equation (32), A, repeats (25): a percentage change in w from a percentage change in E when output prices are constant.

The second term, B, shows the impact of changing output prices (percentage change in w from a percentage change in E with varying output prices), if \( P_1^* = P_2^* = 0 \) (32) becomes (25). Thus, the new price effect is

\[ S_w = -(\frac{\lambda_{L} \lambda_{K2} - \lambda_{L} \lambda_{K1}}{\gamma_1 G}) Y_1^* P_1^* + Y_2^* P_2^* \].

If \( P_1^* \neq P_2^* \neq 0 \), depending on the price elasticity of demand for \( Y_1 \) and \( Y_2 \), the price effect, \( S_w \), will either raise or diminish the change in real wage. Yohe concludes that elastic (inelastic) demand for \( Y_2 \) and inelastic (elastic) demand for \( Y_1 \) leads to a(n) reduction (increase) of the incidence effect on real wages.
Yohe, then, proceeds to show the parallel effects for $r^*$ and $q^*$

$$r^* \sim \left( \frac{\lambda_{LK} \gamma_{L2}}{\gamma_{1G}} \right) \gamma_{E1} E^* + S_r$$  \hspace{1cm} (33)

where

$$S_r = \left( \frac{\lambda_{LK} \gamma_{L2}}{\gamma_{1G}} \right) \gamma_{1} P^*_1 + \gamma_{2} P^*_2$$

$$q^* \sim \left( \frac{(\gamma_{L1} \gamma_{K2} - \gamma_{L2} \gamma_{K1}) \lambda_{LK}}{\gamma_{1} \gamma_{E1}} \right) \gamma_{E1} E^* + S_q$$  \hspace{1cm} (34)

where

$$S_q = \left( \frac{(\gamma_{L1} \gamma_{K2} - \gamma_{L2} \gamma_{K1}) \lambda_{LK}}{\gamma_{1} \gamma_{E1}} \right) \gamma_{1} P^*_1 + \gamma_{2} P^*_2$$

Welfare Impacts on Pollution Control

Using Yohe's results on the effects of pollution control on factor rewards when both pollution levels and prices are changing, we attempt to develop the welfare implications of the model.

The focus will first be on the small country case, assuming initially identical individuals, then separating the individuals within the economy into two different groups: workers and capital owners. The second part of the analysis will treat the large country case, following the same procedure as for the small country, with the distinction that in the later case output prices can vary.

The small country case

In this case, it is assumed that the country is too small to influence the world product prices. The small country cannot affect the terms of trade; once the country is trading, pre-existing world prices are not disturbed, the small country is a price taker.
Economy with identical individuals

To analyze the welfare impacts of pollution control, it is necessary to look at the utility function of the consumers. Two assumptions are made:

1. The economy is composed of identical individuals; in other words, each individual has the same utility function.

2. Pollution enters directly the consumer’s utility function.

Each consumer’s behavior is characterized by the following utility function, which, given identical individuals, is the welfare function of society.

\[ U = u(y_1, y_2, E) \] (35)

where \( y_1 \) and \( y_2 \) are, respectively, the quantity consumed of goods \( Y_1 \) and \( Y_2 \), and \( E \) is the pollution from sector one.

For a given level of \( E \), the consumers, in this case society, maximize their utility function subject to price and national income constraints. The Lagrangian for this maximization problems is:

\[ \Lambda U = U(y_1, y_2, E) + \lambda(I - py_1 - y_2) \] (36)

where \( p = \frac{1}{p_2} \) is the constant terms of trade (small country assumption); \( \lambda \) is the Lagrange multiplier associated with the income constraint; \( I = wL + rK + qE \) (as defined above); and \( I(p, E) = pY_1 + Y_2 \).

The Lagrange function yields the following results:

\[ y_1 = y_1(p, E, I) \]
\[ y_2 = y_2(p, E, I) \]
\[ \lambda = \lambda(p, E, I) \]

It can be seen that the solution to the constrained optimization
problem represents the demand functions of society. The optimal consumption depends on price, the use of environment, and income.

Substituting the Lagrangian solutions into the utility functions yields the maximum utility level which is represented by the indirect utility function:

\[ V = V[y_1(p, E, I(p, E)), y_2(p, E, I(p, E)), E] \] (37)

or

\[ V = V[p, E, I(p, E)]. \]

The next step is to maximize utility with respect to \( E \):

\[ \frac{dV}{dE} = V_E + V \frac{3I}{3E} = 0 \] (38)

The envelope theorem is applied to examine the effect of a change in environmental regulation on the maximized utility level. The optimal level of \( E \) satisfies the following simplification:

\[ \lambda \frac{3I}{3E} = -U_E \] (39)

where \( \lambda \) is the marginal utility of income and \( \frac{3I}{3E} \) is positive following equation (30).

**Interpretation of equation (39).** This result leads to the welfare-maximizing rule. Environment should be used to the point where marginal profit equals marginal cost. In this case, the optimum level of environment use occurs at the point where the marginal utility of income generated from its use is equal to the marginal disutility from its use. Thus, given identical individuals, the small country, unable to affect its terms of trade, has an optimal level of pollution control.

The above implies the use of more environment, leaving society
worse off, although society might be unaware of the existence, benefit, or harm of additional external diseconomies.

**Effects of change in input regulation on welfare of workers.**

Workers have the following utility function:

\[ U^L = U^L(y^L_1, y^L_2, E) \]  

(40)

where \( y^L_1 \) and \( y^L_2 \) are, respectively, the workers consumption of goods \( Y_1 \) and \( Y_2 \) subject to their budget constraint: \( wL = py^L_1 + y^L_2 \). To maximize the workers' utility function, the following Lagrangian can be formed:

\[ \Lambda U^L = U^L(y^L_1, y^L_2, E) + \lambda (wL - py^L_1 - y^L_2) \]  

(41)

The solution of the Lagrange function are the workers' demand functions:

\[
\begin{align*}
y_1 &= y_1(p, w, E) \\
y_2 &= y_2(p, w, E) \\
\lambda &= \lambda(p, w, E)
\end{align*}
\]

To get the indirect utility function, the above solutions are substituted into the utility function:

\[ v^L = v^L[y, [p, w(E), E], y_2[p, w(E), E]] \]  

(42)

simplifying

\[ v^L = v^L[p, w(E), E] \]

Changing the level of \( E \) yields to the following:

\[
\frac{d v^L}{d E} = 3v^L \frac{3}{6w} + 3v^L \frac{3}{6E} 
\]

(43)

Using the envelope property:
where $U_E$ is the marginal utility of additional pollution control, and
\[ \lambda = \frac{\partial V}{\partial w} \] is the Lagrange multiplier equal to the marginal contribution to the maximum utility level made by an increase in income or the marginal utility of income.

**Interpretation of equation (44).** The wage effect is negative. When sector one is environment- (pollution-) intensive and capital-intensive in the weak factor intensity sense and when sector two is made labor-intensive, it has been seen in equation (25) that \( \frac{\partial w}{\partial E} < 0 \). $U_E$, representing the disutility to workers from an increase in the use of environment, is also negative, so that $\frac{dV^L}{dE}$ cannot be positive.

Hence, workers are losing when environmental regulation is relaxed. A relaxation in pollution control increases $E$ to $E'$, decreases $w$, and causes the budget constraint to shift to the left. The consequences of this inward shift are a restriction in workers' consumption of $Y_1$ and $Y_2$, while they are consuming more pollution. Since the small country's terms of trade are unaffected, the slope of the budget constraint remains constant. As a result of the above, workers move to a lower indifference curve and are, thus, worse off than before the regulation change. This is shown graphically in Figure 1, where labor's budget constraint is $wL = py^L_1 + y^L_2$.

It has been seen that a relaxation in pollution control decreases wages in both sectors. This is illustrated by the inward shift of the budget line: $AB$ is the prechange budget constraint, and $CD$ is the afterchange budget constraint. The intercept of the budget
Fig. 1. The welfare effect of a change in the level of pollution control for labor.
line on the $Y_1$ axis is $\frac{wL}{p}(y_2 = 0)$; the intercept of the budget line on the $Y_2$ axis is $wL (y_1 = 0)$. $u$ is the workers original utility curve; $u'$ is their afterchange utility curve, following a decrease in real income. $e$ is the workers original consumption point, at which $y_1$ and $y_2$ are used up; and $e'$ is the workers afterchange consumption point, at which $y_1'$ and $y_2'$ are being consumed.

Figure 1 illustrates that, when a country’s terms of trade are constant and when there is a relaxation in environmental regulation, the workers level of satisfaction (which depends upon their consumption and is shown by their utility curve) diminishes; they consume less of the goods and more pollution. Thus, the workers lose.

Effects of change in input regulation on the welfare of capitalists. The utility function of the capital owners is:

$$u^K = u^K(y^K_1, y^K_2, E)$$  \hspace{1cm} (45)

where $y^K_1, y^K_2$ are, respectively, the capitalists consumption of goods $Y_1$ and $Y_2$ subject to their income constraint:

$$r^K = p y^K_1 + y^K_2.$$  

To maximize the capitalists utility function, a Lagrangian is formed:

$$\Lambda u^K = u^K(y^K_1, y^K_2, E) + \lambda (r^K - p y_1^K - y^K_2)$$  \hspace{1cm} (46)

The solutions of the Lagrangian are the capitalists demand functions:

$$y_1 = y_1(p, r, E)$$
$$y_2 = y_2(p, r, E)$$
$$\lambda = \lambda(p, w, E).$$

Their indirect utility function is

$$v^K = v^K[y_1[p, r(E), 22 E], y_2[p, r(E), E]]$$  \hspace{1cm} (47)

or
\( v^K = v^k[p, r(E), E] \).

Differentiating with respect to \( E \):

\[
\frac{dV^K}{dE} = \frac{\partial v^K}{\partial r} \cdot \frac{\partial r}{\partial E} + \frac{\partial v^K}{\partial E}
\] (48)

Using the envelope property:

\[
\frac{dV^K}{dE} = \lambda r \frac{\partial r}{\partial E} + U_E
\] (49)

**Interpretation of equation (49).** The revenue effect is positive: when sector one is environment-intensive and weakly capital-intensive and when sector two is labor-intensive, equation (26) suggests that \( \frac{\partial r}{\partial E} > 0 \). As pollution is increased, payments to capital increase. \( U_E \) is negative so that \( \frac{dV^K}{dE} \) can be either positive or negative. Hence, capitalists are either losing or gaining when environmental regulation is modified. A relaxation in pollution control increases \( E \) to \( E' \), increases \( r \), and causes the income constraint to shift to the right. The consequences of this outward shift, for the capital owners, is an increase in the consumption of \( Y_1 \) and \( Y_2 \), while at the same time they also are consuming more pollution. The capitalists move to a higher indifference curve which is shown graphically in Figure 2, where the capitalists' income constraint is \( r_k = p y_1^K + y_2^K \). The outward shift of the budget line illustrates the increase in capital reward. \( AB \) is the prechange income constraint, and \( CD \) is the afterchange income constraint. The intercept of the budget line on the \( Y_1 \) axis is \( \frac{r_k}{p} (y_2 = 0) \); and the intercept of the budget line on the \( Y_2 \) axis is \( r_k (y_1 = 0) \). \( u \) is the capitalists' original utility curve; and \( u' \) is their afterchange utility curve, following an
Fig. 2. The welfare effect of a change in the level of pollution control for capital.
increase in real income. \( e \) is the capitalists' original consumption point, at which \( y_1 \) and \( y_2 \) are used up; and \( e' \) is the capitalists' afterchange consumption point, at which \( y_1 \) and \( y_2 \) are consumed.

Figure 2 illustrates that, when a country's terms of trade are constant and when there is a relaxation in environmental regulation, the capitalists' welfare either improves if the income effect outweighs the pollution effect or worsens if the pollution effect is greater than the income effect.

The large country case

In the second case, the country can affect its terms of trade. A large country can influence world prices. It is then necessary to summarize the large country terms of trade effects prior to concentrating on the welfare aspect of the problem.

Implications of the large country assumption

This section derives the likely terms of trade effects of a relaxation in environmental regulation. We first assume that sector two is the importing sector, \( Y_2 \) being the imported good, to later look at the situation where sector one is the importing sector. Having dealt with the two sectors separately, we will finally treat the world as a whole.

sector two is the import-competing sector. Sector two is assumed to be the home country while sector one is the foreign country. Goods move from sector one to sector two. So far, it has been assumed that a sector's level of satisfaction or real income depends on the bundle of commodities consumed. Thus, the level of
satisfaction depends on the quantity consumed of \( Y_2 \), shown by the utility function.

The demand for imports or the excess demand for \( Y_2 \) is:

\[
M_2(p, E) = y_2[p, I(p, E)] - y_2(p, E)
\]

(50)

where \( M_2 \) is the home excess demand for \( Y_2 \).

Changing the level of \( E \):

\[
\frac{\partial M_2}{\partial E} = \frac{\partial y_2}{\partial E} \cdot \frac{\partial I}{\partial E} - \frac{\partial y_2}{\partial E}
\]

or

\[
\frac{\partial M_2}{\partial E} = m(p \frac{\partial y_1}{\partial E} + \frac{\partial y_2}{\partial E}) - \frac{\partial y_2}{\partial E}
\]

where \( m = \frac{\partial y_2}{\partial I} \) denotes the home marginal propensity to import \( ^{23} \); and if

\[
I = pY_1 + Y_2, \quad \frac{\partial I}{\partial E} = p \frac{\partial Y_1}{\partial E} + \frac{\partial Y_2}{\partial E}
\]

Rearranging,

\[
\frac{\partial M_2}{\partial E} = p \cdot m(\frac{\partial Y_1}{\partial E}) - (1 - m) \frac{\partial Y_2}{\partial E}
\]

(51)

**Interpretation of equation (51).** Referring to the Rybczynski results established in equation (28):

\[
\frac{\partial Y_1}{\partial E} > 0 \quad \text{and} \quad \frac{\partial Y_2}{\partial E} < 0, \quad \frac{\partial M_2}{\partial E} \text{ is positive,}
\]

which indicates that, given constant terms of trade, home imports increase as \( E \) increases. The above results are shown in Figure 3. OH and OF are, respectively, the home and foreign offer curves. \( ^{24} \) We recall that good \( Y_1 \) is sector one's import and sector two's export good and that the opposite holds for good \( Y_2 \).
Fig. 3. Effects of an increase in the use of the input, environment (E), on the terms of trade, when sector two is the import-competing sector.
The initial free trade equilibrium is shown by point e, with the equilibrium terms of trade equal to p, the slope of Op. At the world relative price of $Y_1$, the home sector chooses to demand quantity $eA$ of $Y_2$ above its local production. In order to obtain this through imports, it has to export $OA$ units of $Y_1$, which has the equivalent value.

An increase in $E$ causes the home offer curve to shift from $OH$ to $OH'$, which engenders a lowering of the terms of trade from $Op$ to $Op'$. The new equilibrium point is $e'$, where $e'B$ of $Y_2$ is imported in exchange of $OB$ exports of $Y_1$. The relative price of $Y_2$ increases to the level shown by $Op'$, so that, under the assumed conditions, sector two can expect a deterioration in its terms of trade. This implies an increase of the social cost of pollution control from what it would be if the terms of trade remained unchanged.

Sector one is the import-competing sector. In this case, $Y_1$ is the home import good, and $Y_2$ is the home export good. The home country's demand for imports or the excess demand for $Y_1$ is:

$$M_1(p, E) = y_1[p, I(p, E)] - Y_1(P, E)$$

(52)

where $M_1$ is the home demand for $Y_1$. Differentiating with respect to $E$:

$$\frac{\partial M_1}{\partial E} = \frac{\partial y_1}{\partial I} \cdot \frac{\partial I}{\partial E} - \frac{\partial y_1}{\partial E}$$

or

$$\frac{\partial M_1}{\partial E} = m(p\frac{\partial y_1}{\partial E} + \frac{\partial y_2}{\partial E}) - \frac{\partial y_1}{\partial E}$$
where \( m = \frac{3Y}{3I} \). Reordering:

\[
\frac{\partial M}{\partial E} = p \cdot \frac{\partial Y}{\partial E} - (1 - m) \frac{\partial Y}{\partial E} \tag{53}
\]

**Interpretation of equation (53).** Again, recalling the preceding results, \( \frac{\partial Y}{\partial E} < 0 \) and \( \frac{\partial Y}{\partial E} > 0 \); so that \( \frac{\partial M}{\partial E} < 0 \), which reveals a decrease in the home sector's imports. Let us see what happens to the home exports:

\[
X_2(p,E) = Y_2(p,E) - y_2[p,I(p,E)] \tag{54}
\]

where \( X_2 \) is the home exports of \( Y_2 \).

Changing the level of \( E \):

\[
\frac{\partial X_2}{\partial E} = \frac{\partial Y_2}{\partial E} - \frac{\partial Y}{\partial E} \cdot \frac{\partial I}{\partial E} \]

or

\[
\frac{\partial X_2}{\partial E} = \frac{\partial Y_2}{\partial E} - m \left( \frac{\partial Y_1}{\partial E} + \frac{\partial Y_2}{\partial E} \right) \]

where \( m = \frac{\partial Y_2}{\partial I} \). Reordering,

\[
\frac{\partial X_2}{\partial E} = -pm\left(\frac{\partial Y_1}{\partial E}\right) + (1 - m)\left(\frac{\partial Y_2}{\partial E}\right) \tag{55}
\]

**Interpretation of equation (55).** \( \frac{\partial X_2}{\partial E} \) is positive. Thus, given constant terms of trade, an increase in \( E \) will reduce the home exports (\( Y_2 \) in this case). The above results are shown in Figure 4.

The initial free trade equilibrium takes place at point \( e \), where \( p \), the terms of trade, equals the price of \( Y_2 \) in terms of \( Y_1 \). Sector one demands \( eA \) of \( Y_1 \) and exports \( OA \) of \( Y_2 \). An increase in \( E \) causes the home offer curve to shift from \( OH \) to \( OH' \). This shift to
Fig. 4. Terms of trade effects of an increase in the use of the input, environment (E), when sector one is the import-competing sector.
the left initiates an increase in the terms of trade from $O_p$ to $O_p'$. At the new equilibrium point $e'$, $OB$ of $Y_2$ is exported in exchange of $e'B$ of $Y_1$. Hence, under the assumed conditions, sector one can expect an improvement in its terms of trade.

The large country's terms of trade effects on the world. If the home budget is:

$$py^H_1 + y^H_2 = p^H_1 + y^H_2$$

where $H$ is home, and the foreign budget is:

$$py^F_1 + y^F_2 = p^F_1 + y^F_2$$

Then the world budget is:

$$P(M_1 + M^F_1) - (M_2 + M^F_2) = 0 ,$$

where $M^F_1$ is the foreign country's excess demand for $Y_1$, and $M^F_2$ is the foreign country's excess demand for $Y_2$.

Sector two is cleared by Walras law, hence, the equilibrium condition is:

$$M_1 + M^F_1 = 0 \quad \text{then } M_1 = -M^F_1$$

and the home budget constraint $PM_1 + M_2 = 0$. Replacing $M_1$ by $-M^F_1$, we obtain the balanced trade condition:

$$PM^F_1 = M_2$$

Assuming the home sector imports $Y_2$ and the foreign country does not pollute, equation (58) becomes:

$$PM^F_1(p) = M_2(p, E)$$

Following total differentiation,

$$\frac{\partial M^F_1}{\partial p} dp + M^F_1 dp = \frac{\partial M_2}{\partial p} dp + \frac{\partial M_2}{\partial E} dE$$
Rearranging
\[
\frac{\partial M_2}{\partial p} \frac{1}{M_1} + \frac{\partial M_2}{\partial p} \frac{M_1}{M_1^2} dp = \frac{\partial M_2}{\partial E} \cdot dE
\]
or
\[
[M_1^2 (1 - \varepsilon_F) - \frac{M_2}{\varepsilon_H} \cdot \varepsilon_F] dp = \frac{\partial M_2}{\partial E} \cdot dE
\]

where \(\varepsilon_H\) and \(\varepsilon_F\) are, respectively, the elasticity of home and foreign demand for imports.

From equation (58), \(PM_1^F = M_2\). Hence,
\[
[M_1^F (1 - \varepsilon_F - \varepsilon_H)] dp = \frac{\partial M_2}{\partial E} \cdot dE
\]

where
\[
\frac{dp}{dE} = \frac{-\frac{\partial M_2}{\partial E}}{M_1^F (\varepsilon_H + \varepsilon_F - 1)}
\]

Assuming the Marshall-Lerner condition for stability\(^{27}\) holds, if the foreign sector imports \(Y_1\), \(M_1^F > 0\), and if the home sector imports \(Y_2\), \(M_1^H > 0\), \(^{28}\) then, \(\frac{dp}{dE} < 0\).

Generally, the preceding analysis can be interpreted as yielding the following results. If there is an increase in \(E\), providing the Marshall-Lerner stability condition is satisfied, there will be an improvement in the terms of trade when the polluting sector is the importing sector. When the polluting sector is the exporting sector, the terms of trade will deteriorate. With these results in mind, we are now ready to analyze the welfare impacts of an increase in \(E\).

Welfare impacts of pollution control in the large country

**Economy with identical individuals.** Society's indirect utility
function\(^{29}\) is:

\[ V = V[p, E, I(p, E)] \]

In the large country case, the maximization of utility with respect to \( E \), yields to:

\[ \frac{dV}{dE} = \frac{\partial V}{\partial p} \frac{dp}{dE} + \frac{\partial V}{\partial E} + \frac{\partial I}{\partial p} \frac{dp}{dE} + \frac{\partial V}{\partial E} \frac{\partial I}{\partial E} = 0 \]

Again, using the envelope theorem gives:

\[ \frac{dV}{dE} = (-\lambda y_1) \frac{dp}{dE} + U_E + (\lambda y_1) \frac{dp}{dE} + \lambda \frac{\partial I}{\partial E} = 0 \]

Reordering,

\[ \lambda \frac{\partial I}{\partial E} = -U_E - \lambda (y_1 - y_1) \frac{dp}{dE} \quad (62) \]

**Interpretation of equation (62)**. Again, the above result leads to the welfare maximizing rule: the environment should be used to the point where marginal income for a country (region) equals marginal cost. If the home sector exports \( Y_1 \) and imports \( Y_2 \), \( \frac{dp}{dE} < 0 \) and \( Y_1 - y_1 > 0 \). The marginal cost for the large country will consist of the marginal disutility from the use of the environment and the cost due to the deterioration in its terms of trade. Again, given identical individuals, the large country has an optimal level of pollution control.

If the home sector exports \( Y_2 \), \( \frac{dp}{dE} < 0 \) and \( Y_1 - y_1 < 0 \).

**Economy with two groups of consumers**:

1. The effects of change in the input regulation on the welfare of workers. Labor's indirect utility function is:\(^{31}\)

\[ V^L = V^L[p(E), w(E), E] \]

where \( p(E) \) indicates that, in this case, we have variable
commodity prices. Changing the level $E$, yields:

$$\frac{dV^L}{dE} = \frac{\partial V^L}{\partial p} \cdot \frac{dp}{dE} + \frac{\partial V^L}{\partial w} \cdot \frac{dw}{dE} + \frac{\partial V^L}{\partial E}$$

Using the envelope theorem gives,

$$\frac{dV^L}{dE} = -\lambda y_1 \frac{dp}{dE} + \lambda \frac{dw}{dE} + \frac{U^L_E}{dE}$$

or

$$\frac{dV^L}{dE} = \lambda (\frac{dw}{dE} - y_1 \frac{dp}{dE}) + \frac{U^L_E}{dE}$$

(63)

**Interpretation of equation (63).** The sign of $\frac{dV^L}{dE}$ is indeterminate. Again, if the home sector exports $Y_1$, $\frac{dp}{dE} < 0$ and $U^L_E < 0$, $\frac{dw}{dE}$ can either be negative or positive. In the large country case, the product prices can vary, hence, the second term of equation (32), showing the impact of changing output prices or the price effect on $w$, will either raise or diminish the change in real wages. If there is an increase in the level of pollution in a large sector, the workers will either gain or lose.

2. Effects of change in input regulation on the welfare of capitalists. Capitalists' indirect utility function is:

$$V^k = V^k[p(E), r(E), E] .$$

Changing the level of $E$ yields

$$\frac{dV^K}{dE} = \frac{\partial V^K}{\partial p} \cdot \frac{dp}{dE} + \frac{\partial V^K}{\partial r} \cdot \frac{dr}{dE} + \frac{\partial V^K}{\partial E}$$

Using the envelope theorem gives

$$\frac{dV^K}{dE} = -\lambda y_1 \frac{dp}{dE} + \lambda \frac{dr}{dE} + \frac{U^K_E}{dE}$$
Rearranging,

\[
\frac{dV^K}{dE} = \lambda(K_{dE} - Y_{1dE}) + u^K_E \tag{64}
\]

Interpretation of equation (64). If the home sector exports \(Y_1, \frac{dp}{dE} < 0\), and \(U^K_E < 0\), \(
\frac{dr}{dE}
\) can either be negative or positive. The welfare impact of a change in the pollution level on capital cannot be clearly defined, the capitalists will either gain or lose.

In equations (63) and (64),

\[
\frac{dw}{w} \quad \text{and} \quad \frac{dr}{r} \quad \frac{dr}{dE}
\]

are, respectively, unsigned. Let us reiterate the example of equation (63):

\[
\frac{dV^L}{dE} = \lambda(L_{dE} - Y_{1dE}) + U^L_E,
\]

then by assumption \(\frac{dw}{dE}\) is unsigned. Equation (32) shows why parameters, such as the value shares, elasticities of substitution etc. have to be known:

\[
w^* = \frac{dw}{w} = \frac{\lambda_{LK}Y_{K2}}{\gamma_1G}(E^*E_E) - \frac{\lambda_{LK}Y_{K2}}{\gamma_2G}(\gamma_1P_1^* - \gamma_2P_2^*) \tag{32}
\]

where * is the percentage change and where G is the matrix determinant of equation (24) and \(G < 0\). \(Y\)'s are value shares, and \(\lambda\)'s are the shares of each region's input in the production of good \(Y_1\) or \(Y_2\). The term

\[\left|\lambda_{LK}\right| = \lambda_{LY_1}\lambda_{XY_2} - \lambda_{LY_2}\lambda_{XY_1} < 0\]

by the assumptions acquired from the Yohe, Batra-Casas model.

The first term of equation (32) is a repetition of equation
(25) showing the effect given constant prices: as regulation is relaxed on pollution, the payment to labor will fall.

The second term of the equation is the effect of changes in prices. This latter effect either dampens or accelerates the first effect as prices change.
Footnotes


18. \[ dp = 0, \ p = \text{constant}. \]

19. The envelope theorem shows the relationship between the indirect utility function and the Lagrangian. Here the envelope property is concerned with a change in the maximum utility level of consumers caused by a change in pollution control. A change in a parameter generally induces a variation in the optimum levels of choice variables. According to the envelope property, if the parameter change is very small, the induced change in choice variables can be ignored.

20. Capitalists do not provide labor.

21. Wage is a function of the use of the environment. Equation (25) shows that as regulation is relaxed on pollution, the payments to labor will fall.

22. Capital price is a function of the use of environment. Equation (26) shows that as pollution is increased, payment to \( K \) increases.

23. The marginal propensity to import indicates the change in imports associated with a given change in income.

24. The offer curves diagram contrasts the quantity of a commodity \( Y_2 \) one sector (two) wishes to import against the quantity of the other commodity \( Y_1 \) offered in exchange as exports.

25. \[ p = \frac{p_1}{p_2} = \text{price of exports} \cdot \frac{\text{price of imports}}{\text{price of exports}}. \]

26. Walras law states that, at the equilibrium prices, excess demand equals zero in all markets.

27. \( \varepsilon^H + \varepsilon^F > 1 \): in a stable market, the sum of the two countries' (regions) elasticities of demand for imports exceed unity. The Marshall-Lerner stability condition suggests that in order for the market to be stable, offer curves cannot be too inelastic, and that an increase in the relative price of a good reduces world excess demand for that good.

28. Refer to equation (28).
29. Refer to equation (37).

30. Refer to equation (61).

31. To see how we obtained the indirect \( u \) function of workers, refer to equations (40) through (42).
CHAPTER IV

CONCLUSIONS

Some directions regarding input regulation in the context of trade, and in the context of welfare economics, were derived from this analysis. Input regulation takes the form of controlling the utilization of one factor of production, the environment, in a three-factor production function. The input, environment, was incorporated in a Heckscher-Ohlin type of model. In order to reduce environmental deterioration, substitution of the environmental factor for capital was allowed.

The model developed was based on the following assumptions:

1. An economy with two sectors 1 and 2, where 1 is the polluting sector, and is environment-intensive and capital-intensive in the weak sense.

2. Three factors of production: capital, labor, and environment.

3. The two sectors are characterized by strictly quasiconcave linear homogeneous production functions where any one good cannot be produced solely by the environment.

4. Permissible pollution is regulated.

Based on these assumptions, it is concluded that, in an open economy, and within a general equilibrium framework, the impacts of the exogenously imposed changes, a relaxation (increase in the use of the environment) in the level of pollution control on the welfare of labor and capital are:
A. For a small country (region),
1. With identical individuals, assuming pollution enters directly the consumer's utility function, the optimum use level of the input environment, occurs at the expected point where the marginal utility of income generated from its use is equal to the marginal disutility from its use.
2. With two different groups of individuals, workers and capitalists, workers have to reduce their consumption of the goods and at the same time pollution is increased. Therefore, in this context, labor is worse off following an increase in the use of the environment. Capitalists gain since returns to capital increase.

B. For a large country (region),
The large country can affect its terms of trade. Provided the Marshall-Lerner condition is satisfied, if there is an increase in the use of environment, the large country can improve its terms of trade when the polluting sector is the importing sector. If the polluting sector is the exporting sector, the terms of trade will deteriorate.
1. Given identical individuals in the economy, the environment should be used to the point where marginal income for a country (region) equals marginal cost, which consists of the marginal disutility from the use of the environment plus the cost due to the deterioration in its terms of trade.
2. When two different groups are assumed, whether workers and capitalists will be better or worse off than before the regulation changes, cannot be unambiguously determined.
The derivations under this assumption lead to a problem of signing the welfare changes. The parameters in the model that would determine the sign of these changes to individuals include the magnitude of the factor intensities, the elasticities of substitution between factors, the value share of the goods in national income, the signs of the price changes, and the strong and weak definition of factor intensity rankings.

In the large country case, with two groups of consumers, and given variable prices, the results are indeterminate. Empirical estimates of input substitution, etc. are needed to sign the effects under these cases. The change in wages and return to capital in response to changes in the use of the environment are unsigned. The components of these change relationships, as derived in Chapter III, include factor shares, factor intensities, and input substitution elasticities. Wage changes and, similarly, changes in returns to capital are affected by not only factor intensities and value shares but also by the new price effect as prices are allowed to vary. Variable commodity prices may alter the sign, which was negative for the wage change relationship and positive for changes in returns to capital in the constant prices case, depending on the value share, factor intensity, and elasticity of substitution parameters. This price can serve to dampen or accelerate the effect which changes wages and returns to capital under constant prices. It also is possible that the price effect can change in such a way as to reverse the direction of changes in wages and returns to capital compared to the changes under constant prices. The
occurrence of such a reversal depends on the magnitude of the commodity price changes and on the shares of the commodities in national income (sum of regional incomes). As prices vary with output, and, for example, price changes in the polluting sector are positive while price changes in the other sector are negative and greater in magnitude, the wage changes are negative.

For example, during the 1970s, electricity (coal-fired production) prices were leading the CPI and most manufactured goods prices. That direction for price changes would suggest, using the above conclusions, that the negative effect on labor would be accelerated, and the positive effect on capital likewise accelerated.

Regional and Environmental Issues and the Welfare

Implications of the Model

Although the results of differentiation of the model only indicates the theoretical directions of the changes in factor rewards as the use of the environment is altered by changes in environmental regulation, these directions do relate to international and regional development and environmental regulation issues.

It is increasingly being realized that economic activities in one region produce external effects on neighbors. This is one of the physical aspects of the interregional pollution problem. Another one is that through the purchase of products which use the environmental input from a remote region, such as clean air, a state can benefit from free air. Part of the electric power of Los Angeles for example is imported from Utah. With the addition of the Intermountain Power Project still more power is to be produced (using Utah and Wyoming
coal) and transmitted to Los Angeles. By buying electricity generated at such a distance, Los Angeles imports clean air and exports part of its pollution to Utah. It is precisely this interregional externality which caused the concern over the siting of such a large power producing project as Intermountain Power. The plant is currently being constructed in Utah's western desert where the use of air apparently is not as critical to environmental esthetics associated with national parks on the Colorado Plateau where most of Utah's coal is located. Indeed, there is some evidence to suggest that the costs of the power production from the project will be significantly greater than current and projected costs of coal-fired power production at the Huntington, Emery, and other power production near coal mines in Carbon County in eastern Utah (see, for example, Snyder et al. 1983, and Snyder and Keith 1981).32

Utah depends significantly on the development of energy resources such as coal and environmental resources such as clean air and national parks to maintain a viable state economy. It has been shown in this study that changes in environmental regulations can influence factor rewards in a regional economy. Coal is used in combination with capital, labor, and the environment to produce electricity and steel in Utah. Coal-fired electric power production and steel production are relatively environment-intensive industries which exist in Utah; and the results of this study, although theoretical in nature, do give us some insight on the effects of environmental policy changes on returns to factors in the region.
Although full evidence is not given here, Utah cannot be classified in the large region case as far as coal and power production are concerned. Utah coal is low in sulfur (the main pollutant) and high in heat content, but Utah's coal cannot be surface-mined and competes with several substitute coals mined in the West. For example, Utah faces competition from Wyoming surface-mined coal, which is also relatively low in sulfur content (but has a slightly lower heat content). Although, as pointed out by Hachman, the greater the cost of transportation, the less the importance of the difference in mining cost between Utah underground and Wyoming surface coal. If, indeed, Utah could be considered a price-taking region (small region) in this sense, then relaxation of environmental standards pertaining to emissions from mining on electric power production which emits sulfur dioxide would result in a reduction in the welfare of labor while owners of capital could gain or lose. Of course, this inference is only justified if all the conditions implied by the assumptions of the model used hold.

Environmental regulations clearly favor some regions at the expense of others. There is also a commercial side to the regional problem. Interregional trade stems from qualitative and price differences between region. As revealed by Ackerman and Hassler, in the case of coal, the volume, pattern, and direction of trade are bound to be influenced by measures adopted by Congress to combat sources of pollution and, indeed, have been.

The growing concern about the degradation of the environment caused by industrial activities has led the government to adopt
stricter measures to protect and enhance the quality of the environment. The U. S. National Environment Act (NEPA) of 1970 indicated the emergence of a series of legislative actions to protect the environment. In 1978, the U. S. Environmental Protection Agency (EPA) proposed a $40 billion regulation to reduce sulfur emissions from coal-fired plants. This indicated a preference in sulfur removal (or tightening of controls) over the use of low sulfur coal (substitution of the energy input for the use of the environment). In the West, coal is relatively low in sulfur, the most notorious pollutant associated with coal burning. These energy restrictions, motivated by environmental concerns, increase domestic costs. In order to eliminate the residual from the exhaust gases, new coal-fired electricity generating plants are required to use the best control technologies (including scrubbers), which increase the cost of generating electricity. The strip mining regulations for Western coal along with the protection of the market shares of high cost Eastern coal, especially through the requirement for 90 percent sulfur removal from coal even if low-sulfur western coal is burned, also contribute to the removal of some of the competitive advantage of lower cost, lower sulfur content coal.

The electric power industry also faces opposition to locating its facilities in certain areas. While the 1971 Clean Air Act's new source performance standard (NSPS) encouraged the use of low sulfur coal, the 1978 provisions were partly conceived to make high sulfur coal as economically attractive to utilities as low sulfur coal in order to counter the relocation of U.S. coal production to the West.
In spite of the 1978 revisions, Western coal production is still rising. The West has a lower mining cost (surface mining in Wyoming and Montana) which enables the producers to ship coal long distances. Also, the expansion of the western coal-fired electrical capacity could raise local demand over the long run. Under current standards, the major determinants of coal selection remain mining costs and transportation rates, low sulfur content is only a small influence.

The difference in the quality of the polluting resource or in the amount of the pollutants in the resource among regions may influence the location of future plants, marginal changes in plant expansion or contraction and the production techniques. Most of these changes would have both a direct and indirect effect on employment, i.e., a technological change induced by compliance with environmental regulation may alter the capital-labor mix in the production process.

The degree to which the preference classes, workers, and capitalists, ultimately benefit from a relaxation (increase in the use of the environment) in control is not obvious; nor is it evident how changes in consumption and production patterns induced by regulation changes affect various regions, industries, or occupations. The popular myth is that tighter pollution control triggers unemployment.

The substitution of capital and labor for environmental use or pollution in the production process must be analyzed. Looking at the welfare implications, it appears that unambiguous conclusions cannot be derived without knowing the empirical context.

In this analysis it was assumed that only sector one uses the environment via pollution and in that sector, only capital is
substitutable with the environment. Yohe suggests that higher elasticities indicate easier substitution and, therefore, facilitates the cleaning of the production effluent. He also suggests that, since capital is used almost exclusively in the production process, the elasticity between labor and pollution is negligible.

Empirical research has shown that polluting industries operate with a higher capital/labor ratio than other manufacturing industries. Over the last decade, due to increased demand for electricity, construction lags, inflation and, air pollution control costs, the utilities costs for power plant construction have risen sharply, exacerbating the capital-intensive nature of the industry.

Also, historically, energy (coal in this case), and capital have been shown to be complements; consequently, price increases slow down capital formation. Energy and labor are substitutes, suggesting that the use of labor should be rising, which in turn, would cause the average productivity of labor to fall. It is most likely that the elasticity of substitution of capital and labor for environment is less than one. This has been found to be the case in electric power production.\textsuperscript{35}

In recent years, pollution has come to the fore of public discussion, the concerns about the trade-off between economic growth and environmental quality have become central to economic policy. Environment disruption can cause repercussion beyond the boundaries of a region, not necessarily taking the form of physically transmitting pollutants, but influencing the pattern, volume, and terms of trade. The results of this study address some of these issues.
Limits of the Research

The conclusions drawn are not unambiguous predictions, particularly in the large country (region) case. Rather, they took the form of stating a range of likely outcomes of the process. This theory thus precludes a great deal. However, we hope that within the range of likely outcomes, it will provide some valuable information about the welfare impacts of pollution control.

Empirical research is beyond the scope of this study. Our analysis was confined to the theoretical aspects of the issue. Nevertheless, we used some empirical examples to highlight the directions that changes in factor rewards may take given the conclusions of the derived theory. A myriad of assumptions are made in this study. It was not our purpose to elaborate a general comprehensive theory of the welfare impact of pollution. Rather, an attempt was made to show what these effects might be. In order to do so, a simple model is used and a focus on its special properties and implications for altering controls was completed.

We find in Solow's view on assumptions a reflection of our belief that in order to understand a complex real world, one needs to construct a simple imaginary world, each of which includes one, or a few, important aspects of the real world, and to study their workings. "Simplifying assumptions are not an excrescence on model-building; they are its essence." Once the simple models are understood, they can be made more complicated by combining them or by introducing more realistic elements, and eventually all the important aspects of the real world might be understood.
Suggestions for Future Research

The actual empirical work is needed to confirm the directions given by the theory. However, one encounters a data problem, the facts available are from controlled observations.

More work needs to be done to investigate the interregional context. Environmental restrictions may vary among regions, but the producers affected by those divergences are linked by a common commodity price through trade.

Another extension of the model, out of the scope of this study, is to deal with prices and tax regulation and imposition on energy sources in the presence of externalities. This should be a fruitful research effort since there are alternative tax structures which are imposed on energy resources (depending on the state involved), some of the purposes of which are to presumably alter externalities which are generated by the extraction and use of these resources.

General predictions have been laid down about the direction and the extent of changes as the degree of regulation is altered. The empirical work should give deeper insights to the signs of some of those changes. We must conclude that our ability to test the model empirically is most limited. To this incapacity, must be added the fact that other problems have been neglected. However, we do hope that our attempt to derive the welfare implications of pollution control reveals some valuable directions which can be developed empirically in future endeavors, and which can give direction to public policy.
Footnotes


33. Frank Hachman, The Utah Energy Facility Siting Study, Phase I: Great Basin, Chapter 4, Utah Consortium for Energy Research and Education, University of Utah/Utah State University/Brigham Young University, 1981.


BIBLIOGRAPHY


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Appendix 1: Proof of Equation 22

Equation (22) can be rewritten as

\[ \sum_j (R_{jLE}^L - R_{jLK}^E)(y^j)^* + (R_{jLK}^E - R_{jKE}^L)\tau^j + (R_{jKE}^L - R_{jLE}^E)q^j = R \]

where

\[ R_K^j = \left| \lambda_{LK} \right| \left| \lambda_{LE} \right| j y_j \gamma_j \gamma_j = \frac{\lambda_{LK} \gamma_j (\gamma_{K2} - \gamma_{K1})}{\gamma_K} \]

\[ R_E^j = \left| \lambda_{KE} \right| \left| \lambda_{LY} \right| j y_j \gamma_j \gamma_y = \frac{\lambda_{KE} \gamma_j (\gamma_{E2} - \gamma_{E1})}{\gamma_E} \]

\[ R_L^j = \left| \lambda_{KL} \right| \left| \lambda_{LY} \right| j y_j \gamma_j \gamma_y = \frac{\lambda_{KL} \gamma_j (\gamma_{L2} - \gamma_{L1})}{\gamma_L} \]

where \( \gamma_s = \frac{s P_s}{B} \) for \( s = 1, 2; j = 1, 2; s \neq j \); and \( \gamma_i = \) share of factor \( i \) in nation income, i.e. \( \gamma_i = \frac{w_i}{B} \). Hence, equation (22') is reduced to equation (23): \( R = Aw^* + Br^* + cq^* \), where

\[ A = \frac{\left( \gamma_{K2} - \gamma_{K1} \right) \sum_j \left| \lambda_{Ly} \right| \left| \lambda_{yj} \right| \gamma_j s \sigma_j}{\gamma_K} - \frac{\left( \gamma_{E2} - \gamma_{E1} \right) \sum_j \left| \lambda_{Ly} \right| \left| \lambda_{yj} \right| \gamma_j s \sigma_j}{\gamma_E} \]

\[ B = \frac{\left( \gamma_{L2} - \gamma_{L1} \right) \sum_j \left| \lambda_{Ly} \right| \left| \lambda_{yj} \right| \gamma_j s \sigma_j}{\gamma_L} - \frac{\left( \gamma_{E2} - \gamma_{E1} \right) \sum_j \left| \lambda_{Ly} \right| \left| \lambda_{yj} \right| \gamma_j s \sigma_j}{\gamma_E} \]

\[ C = \frac{\left( \gamma_{K2} - \gamma_{K1} \right) \sum_j \left| \lambda_{Ly} \right| \left| \lambda_{yj} \right| \gamma_j s \sigma_j}{\gamma_K} - \frac{\left( \gamma_{L2} - \gamma_{L1} \right) \sum_j \left| \lambda_{Ly} \right| \left| \lambda_{yj} \right| \gamma_j s \sigma_j}{\gamma_L} \]
Appendix 2: Proof of Equation (28)

From equations (25) through (27) we get:

(a) \[ w^* - r^* = \frac{-(\gamma_{k2}\gamma_{e1} + \gamma_{l2}\gamma_{e1})}{G}\lambda_{lk}|E^*| \]

(b) \[ r^* - q^* = \frac{(\gamma_{l2}\gamma_{e1} - \gamma_{l1}\gamma_{k2} + \gamma_{l2}\gamma_{k1})}{G}\lambda_{lk}|E^*| \]

The substitution of equation (a) and (b) into (14) and (15) and then into the \( D_L, D_K \) terms to derive

(c) \[ D_L = \frac{-(\gamma_{k2}\gamma_{e1} + \gamma_{l2}\gamma_{e1})}{G}\lambda_{lk}|E^*| \left(\lambda_{kl}\gamma_{l1}\sigma_{lk}^1 + \lambda_{l2}\gamma_{k2}\sigma_{lk}^2 \right) < 0 \]

(d) \[ D_K = \frac{(\lambda_{k2}\lambda_{e1} + \lambda_{l2}\lambda_{e1})}{G}\lambda_{lk}|E^*| \left(\lambda_{kl}\gamma_{l1}\sigma_{lk}^1 + \lambda_{l2}\gamma_{k2}\sigma_{lk}^2 \right) + \frac{\lambda_{k1}\gamma_{e1}\sigma_{lk}^1}{\lambda_{l1}\lambda_{k1}} \left(\gamma_{l2}\gamma_{e1} + (\gamma_{l2}\gamma_{k1} - \gamma_{l1}\gamma_{k2})\right)\lambda_{lk}|E^*| > 0 \]

with \( L^* = K^* = 0 \), (19) and (20) are reduced to:

(e) \[ Y_1^* = \frac{\lambda_{k2}D_L - \lambda_{l2}D_K}{\lambda_{l1}\lambda_{k2} - \lambda_{k1}\lambda_{l2}} \]

(f) \[ Y_2^* = \frac{-\lambda_{k1}D_L - \lambda_{l1}D_K}{\lambda_{l1}\lambda_{k2} - \lambda_{k1}\lambda_{l2}} \]

The substitution of (c) and (d) into (e) and (f) yields equation (28)

\[ \frac{Y_1^*}{D^*} > 0 \quad \frac{Y_2^*}{E^*} < 0 \]

which are the Rybcynski results.