Economic Implications of Phenologically Timed Irrigation in Corn Production

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ECONOMIC IMPLICATIONS OF PHENOLOGICALLY TIMED IRRIGATION IN CORN PRODUCTION

by

Dawuda Tsalhatu Gowon

A dissertation submitted in partial fulfilment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Economics

Approved:

UTAH STATE UNIVERSITY
Logan, Utah
1979
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Dawuda T. Gowon
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ABSTRACT

Economic Implications of Phenologically Timed Irrigation in Corn Production

by

Dawuda Tsalhatu Gowon, Doctor of Philosophy

Utah State University, 1979

Corn production data was fitted into a Translog production function. Analysis of the resultant equation was based on what impact irrigation keyed to the crop's phenology would have on yield. A crop product cost function was developed to determine if there is profit (loss) in adapting water application to corn by phenological time period. Reasons for not adapting phenology as a key variable in irrigation include institutional constraints. Without modifying these institutional constraints, adopting the proposed technology may prove prohibitive.

(97 pages)
CHAPTER I
INTRODUCTION

Using methodology developed for predicting crop yield, economists can investigate ways to optimise crop-production through water control. Optimal yields require optimal water application to crops at the crucial time during a phenological stage.

In this dissertation, we are concerned with what the economist can say about the physical relationship between water input and corn (grain or dry matter) output. The economist is concerned because resources required for optimum agriculture production are scarce in certain places. Scarcity demands efficient use of the scarce resources. Using economic theory, a production function will be developed. The production function's usefulness will be demonstrated using empirical evidence and the practicality of the production as regards management decision will be demonstrated.

Econometric investigation is a first step toward a first approximation to a full understanding of the relations called a production function. The economic content of relationships to be estimated is very important for determining estimates and parameter identification.

A large proportion of land surface is desert. Areas that have enough rainfall have the rainfall come not necessarily at times most suitable for crops. The major resource input (input factor) under consideration in this dissertation is water. Water used as irrigation
water is not a "free resource." Irrigation water has a price or at least it can be assigned an imputed value.

Since there is a cost to water, farmers will be limited by a cost constraint or the price which they have to pay for water. Not only does the cost take the form of price paid for the input, cost takes the form of lower productivity associated with irrigation systems and practices.

Objectives and Assumptions

Statement of the Problem and Research Objectives

Water is becoming increasingly scarce in the arid west. This scarcity is demonstrated by farmers drilling deeper for water and transporting water over longer distances. Scarcity implies that owners of water rights will be getting higher rents. Since water supply is relatively fixed, the fixed quantity in the face of increasing demand would raise the price of water. The increase in demand would come through an increase in area under irrigation. The increase could be through product price increase as well as complementary factor price decline. Enlarging present farms and developing virgin land for now are responsible for the increase in demand for water. When there is an increase in demand for water, relative price of water will increase. This calls for investigating more efficient methods of water allocation. Water planners are asked for accurate predictive estimates of how crop production varies with the quantity of water supplied so misallocation of water can be reduced or eliminated.
Furthermore, the question of the best time and method of allocating a predetermined water quantity can be asked. Lack of answers to these questions are a handicap, because production functions clearly showing expected relations between crop yields and water supply at various stages of growth are not available. And if they are not available, no economic analysis can be made of such functions. Also, the state of the art was a barrier to developing water production function. But today, methodology and equipment have been developed to collect reliable data so that accurate water production function can be defined. Even with developed water production function, farmers and policy makers are at a loss when it comes to interpreting the results. Policy implications are not clearly set and thus part of the purpose of the research study remain unresolved.

Specific Objectives

1. To define a production response function for corn where the only variable input is water using phenological timing as the key decision rule.

2. To evaluate and compare the defined production response function with current practices would be the major point of comparison of current practices with the defined production response function.

Procedure

The four measured variables (yield and evapotranspiration for the three stages of growth) will be regressed to fit a translog production function. The equation will have three components corresponding to stage of growth. Careful analysis of their elasticity of
production and marginal physical product would show whether or not
the trend is towards phenological timing as the key decision rule.
The regression results would define the sought after production
function.

If there is a need, some farmers in the Logan area will be
interviewed. Also, Agriculture and Irrigation Experiment Station
will be asked to describe what the current practices are as ob-
served on the field by agents. The result of the first objective
and those of the second objective will be compared and contrasted.

Assumptions
1. Perfect competition exists in both factor and product markets.
   Optimal profit is the goal and this goal can be achieved only
   by employing optimal quantities of inputs (water) at the deter-
   mined phenological stage. Then and only then can optimal quantity
   of output be produced.
2. Corn varieties used in the three locations under consideration
   have the same production capability given their respective geo-
   graphic area, location of farm and soil type.
3. There is no appreciable differences in soil fertility and type
   and quantity of fertilizer applied.
4. Water quality is the same.
Agronomic

Robins and Domingo [21] have reported that soil moisture depletion of one to two days during tasseling resulted in as much as a 22 percent yield reduction, while six to eight days stress reduced yield by 50 percent. They concluded that "yield reductions due to absence of available water after the fertilization period appeared to be related to the maturity of the grain when the available moisture was removed."

Denmead and Shaw [6] found grain yields were reduced by all moisture treatments. Plants subjected to water stress at tasseling were the most affected. The reductions in grain yield were 25 percent when the stress was imposed at vegetative stage, 50 percent by stress at tasseling and 21 percent by stress at ear stages. They also found a tendency for stress imposed in one stage to harden the plant against damage (further yield reduction) from stress at a later stage.

Charles V. Moore [15] showed that it is possible to impute a value to the irrigation cycle. He further developed a model to determine an optimum water price and changing commodity price during growing season. Arlo W. Biere, et. al. [1] demonstrated the sensitivity of a model to the time of water application. They concluded that the higher the available soil moisture around silking
the higher the yield because corn is most sensitive to soil moisture stress at that time. Stewart, et. al. [22] tried to identify the most important stage. They ran separate regressions for four experimental corn-growing sites in four different states (Logan, Utah; Fort Collins, Colorado; Yuma, Arizona; and Davis, California). They found that stress at the pollination stages produced the most drastic effects on grain yield.

Soil Science

Hanks [10] assumed that evaporation from the soil decreases with the square root of time after wetting as well as the stage of growth. Rawitz [19] found growth rate of plants to be affected by decreasing soil water potential and increasing soil resistance. Briggs and Shantz [2] made a comprehensive study of water requirement of plants. de Wit [5] concluded that increase in transpiration tend to increase yield.

Cobb-Douglas Function

One of the easiest functions to manipulate is the Cobb-Douglas function and it was chosen for this dissertation because growth functions are power functions and the Hanks equation used fits the Cobb-Douglas function. Related or similar functions were investigated in the literature and tried. With respect to functions used, the generalized production function has been proposed because it includes special cases of the Cobb-Douglas, the transcendental, and the Cobb-Douglas with variable returns to scale. It still stands that the Cobb-Douglas function is the most direct and easier of the two to manipulate.
For the special case treated by Alain de Janvry [4], the function was \( Y = AX_1^{\alpha_1} + B_1X_2^\gamma_1X_1 \). The Cobb-Douglas case is when \( \beta_1 = \gamma_1 = 0 \) and here we find the degree of substitutability between inputs is affected by the value of the parameters \( \alpha_1, \alpha_2, \alpha_3 \), and by the levels of \( x_1, x_2 \) and their elasticity of substitution. Similarly, the Cobb-Douglas function estimated in this dissertation which has one as its elasticity of substitution has some degree of substitutability between inputs affected by the exponents and the ET ratios.

Another estimation form is the Cobb-Douglas function with variable returns to scale for different production techniques. Ulveling and Fletcher [23] showed that a modified Cobb-Douglas production function can give partial production elasticities and returns to scale. Not only does such a production function allow for pooling of information and therefore preserve degree of freedom, it tests systematically for productivity difference among production techniques.

In a research note, Yujiro Hayami [12] found that "there is no evidence against the use of the Cobb-Douglas production function for the cross country analysis of agricultural production." Such a conclusion Hayami found to be consistent with previous work on cross regional analysis done by Griliches. The work was done in the U.S., Canada and also in Japan.

**Econometrics**

Researchers tend to use multicollinearity to point at a weakness in the use of Cobb-Douglas function, but John P. Doll [7] wrote "Modern econometric theory suggests that the rational underlying this statement is readily available in recent literature and will not
be repeated here. Interestingly enough, very little attention has been directed towards analyzing the impact of the assumptions of the economic analysis upon multicollinearity." Dan Yaron [25] showed that while production functions with fixed intraseasonal distribution are estimable by regression methods, difficulties are involved in the regression approach in the estimation of dated production function.

Zarembka [26] suggests that transformation of variables is a powerful procedure in econometrics to handle the general problem of choice of functional variables, particularly when the functional form is not suggested by theory. Ramsey and Zarembka [18] estimated a production function that is not a constant returns to scale. Constant elasticity of substitution production function was used as a transformation problem. They found some of their result to be outside the one percent confidence limit.

Zellner, Kmenta, and Drèze [27] showed that whatever the functional form specified, production functions are always free of simultaneous equation bias when directly estimated from cross section data on firms. Zellner and Revankar [28] introduced a production function with the generalization referring to assumptions made about the elasticity of substitution and returns to scale.

\textbf{Economics}

Heady and Dillon [13] made a comprehensive study of Agricultural Production functions. They noted that production functions are a derived relationship between dependent and independent variables that is capable of showing how a change in one of the independent variables will affect the other variables. The farmer or farm manager must be
able to understand and react to changing forces that affect input variables. He needs to understand new production techniques if he wishes to apply them. Above all, a basic understanding of the economic principles underlying his agricultural production is required to form the proper choice and decision about his production process. In this context, production functions increasingly becoming a prediction tool are being used by farmers in decisions and economic predictions.

Production functions do not have all the answers to possible economic problems and can therefore not claim absolute dominance, neither can production functions be a substitute to traditional methods of economic analysis. Walter [24] determined cost functions from production function and showed that for profit maximization, cost functions are essential. Marc Nerlove [16] wrote "If there are increasing returns to scale and a growing demand, firms may find it profitable to add more capacity than they expect to use in the immediate future."

Irrigation

Rhoades and Nelson [20] showed that field corn growing under irrigation does tend to show exaggerated effect on plant growth due to brief period of high moisture stress. Norero, Keller, and Ashcroft [17] found that when evapotranspiration value approach maximum, frequent irrigation is necessary in order to maximize production.

Literature review helped in keeping the writers' perspective on yield and input factors. The literature further provided a range of production functions and when those production functions may be used.
Methods of hypothesis testing; how to set production functions; result presentation and interpretation were influenced by what is in the literature.
CHAPTER III
EXPERIMENTAL DESIGN AND DATA ACQUISITION

Line source continuous variable design developed by Hanks and associates (1974) at Logan was adopted. In this design all irrigation after establishment of the crop is from a single sprinkler line parallel to the rows through the center of the plots. The closely spaced (6.1m) sprinkler heads are a type which throws a triangular water pattern such that the maximum application occurs at the sprinkler line, tapering evenly away as one moves outward in either direction. Finally at a distance of approximately 15m, no irrigation water was applied at all. Therefore water application is inherently a variable design. "The approach taken overall in this study was to establish a wide array of measured irrigation regimes, to determine the associated evapotranspiration regimes which occurred, and to measure the resultant dry matter and grain yields from each." Stewart et al. [22].

The control time schedule was one which was irrigated throughout the three stages of growth. The other three schedules differed from the control treatment since fixed irrigation was discontinued during vegetative stage, second in the pollination stage, third in both vegetative and pollination stages and lastly all plots were irrigated in the maturation stage with a few exceptions which were noted.

Measurements made were of applied water including rainfall, soil water content, total dry matter production, grain yields and
weather components, especially class A pan evaporation. There were twenty rows on each side of the line source, and there were five to six irrigation levels.

This dissertation is based on data collected in 1974 and 1975 at Davis, California; Fort Collins, Colorado; and Logan, Utah. Data pooling is possible because of a uniform approach to measuring crop water requirements and actual evapotranspiration. Stewart et. al. [22] reported that "It is common knowledge that methods now in use for making these estimates are far from perfect and that the use of different methods often produces different results. Accordingly, the Davis (California) research team has developed what are thought to be improved methods of ET estimation, and these were adopted for use at all experimental sites."

Potential evapotranspiration (ET\textsubscript{P}) is closely correlated with pan evaporation and crop growth stage. Accurate measurements of short term ET\textsubscript{P} is required when determining the ratios of ET\textsubscript{P} for crop to E\textsubscript{G}, where E\textsubscript{G} is evapotranspiration for each growth stage. Both measurements depend on the use of sophisticated lysimeter equipment. Such equipment is available in Davis and was utilized in this study. Daily measurements were made of ET\textsubscript{P} and of Class A pan evaporation (E\textsubscript{G}). [11]

For clarity, and to facilitate measurements among growth stages, the data were summed for short periods (mostly five days each) and ET\textsubscript{P}/E\textsubscript{G} ratios were computed for each period. This process gave us the actual evapotranspiration (ET\textsubscript{A}) used in our regression analysis. For ET\textsubscript{A}, each application of water to the soil (including rainfall)
starts a new water period, which requires separate consideration. An
counting was kept of water application as they affected the evaporation
layer. For this, a separate water budget was carried as opposed
to that for the lower soil profile. It should be noted that \( ET_p \)
limits \( ET_A \) in any given water period. When \( ET_A > ET_p \), it is assumed
that drainage down the soil profile was responsible.

In this study we discuss our analysis of the four-hundred and
ninety-three observations collected from Davis, Fort Collins and
Logan considering with emphasis on grain yields of corn. To clearly
show stage of growth effect, a composite test was made. Border pro-
lems were clearly stated and treated.

To get current practice in corn irrigation, literature indicat-
ing irrigation practices were surveyed, and farmers were asked to re-
late current practices.
CHAPTER IV
THE MODEL

Model Specification

Present production functions are developed using a combination of input factors; capital and labour. The product component is mathematically defined as a dependent variable. It is expressed as a function of the independent variables which happen to be the input factors. In this case, one input is used, phenologically timed and observed in growth stages. The environment, the climate and time path are important in this production function.

There are three stages in sequence, which lead to a functional form of \( Y = (\text{ET}_A^V, \text{ET}_A^P, \text{ET}_A^m) \), and it is this final product \( Y \) that has a bearing on the yield needed for regression. Thus \( Y f(\text{ET}_A^V, \text{ET}_A^P, \text{ET}_A^m) \) is to be used in the form of a Translog function. Instead of examining a conventional production function, we will examine a production response function. The conventional production function is:

\[ y = f(K,L) \]  

where \( y \) = output resulting from a combination of input factor labour \( L \), and capital \( K \). Pictorially, it follows the law of variable proportions having the well-known marginal products and average products characteristics.

A production function will be developed using (2) in a slightly modified form. The modification is to have water as the only variable input whose optimal quantities would vary according to stages of growth.
of corn—the subject of investigation. Labour and capital will be held constant. The function then is expressed as:

\[ y = f(K, L, \text{ET}_{A_v}, \text{ET}_{A_p}, \text{ET}_{A_m}) \] (3)

where \( \text{ET}_{A_v}, \text{ET}_{A_p}, \text{ET}_{A_m} \) are actual evapotranspiration.

\( \text{ET}_{A_v}, \text{ET}_{A_p}, \text{and} \text{ET}_{A_m} \) are measured in centimeters for the phenologically timed period called vegetative, pollination and maturation stages respectively.

**Hanks' Model**

Research to evaluate the influence of irrigation management on corn production where water and salinity limited production was carried out in Arizona, California, Colorado, and Utah: Hanks, et. al [11], Stewart, et. al. [22]. We use Hanks' model modified as a Translog equation. A Translog production function is a generalized production function. It expresses the logarithm of output as a Taylor series approximation of a generalized production function in terms of the logarithms of input about any arbitrary point. Translog form takes into account interaction between inputs. We use Hanks' model because it provides for a direct strong relationship between evapotranspiration and yield regardless of growth stage. Additionally, it can predict transpiration, while others take measured data. Another advantage is that the Hanks' model is readily transferable, all it requires to predict yields are basic soil, climate, crop, and irrigation data. Furthermore, the Hanks' model shows that yield is related to transpiration. According to Hanks, the yield--transpiration relationship is important but the factors are difficult to separate.
The Hanks water budget model shows yield as a function of evapotranspiration. It is represented in equation form as:

\[ \frac{Y}{Y_p} = C \left( \frac{E_{T_A}}{E_{T_P}} \right)^{\lambda_1} \left( \frac{E_{T_A}}{E_{T_P}} \right)^{\lambda_2} \left( \frac{E_{T_A}}{E_{T_P}} \right)^{\lambda_3} \]  

where:

- \( Y \): Tons per hectare of harvested grain or dry matter.
- \( Y_p \): Potential yield is the highest measured value of \( Y \).
- \( C \): Parameter of production to be later defined as regression constant.
- \( E_{T_A} \): Measured evapotranspiration. It is the amount of applied water depleted by plants, taking into account losses from drainage and runoff. \( E_{T_A} \) is measured in centimeters.
- \( E_{T_P} \): Potential evapotranspiration: The highest measured value of \( E_{T_A} \).

The subscripts \( v, p, m \) represent a phenological time period. Where: \( v \) = vegetative stage, \( p \) = pollination stage, \( m \) = maturation stage.

Vegetative stage is defined as extending from planting to first tassel. This varies with location, but for Logan, Utah it averaged sixty-three days based on a two-year (1974-75) experiment. Pollination stage includes from first tassel to blister kernel. For the two-year experiment in Logan, this averaged twenty-six days. Maturation stage, from blister kernel to physiological maturity, averaged forty-three and a half days for the 1974, 1975 Logan experiment.

The exponents \( \lambda_1, \lambda_2, \lambda_3 \), in the Hanks' model represent elasticity of production defined here as the relative importance of water for the three different stages. The \( \lambda_1 \) values represent the elasticity
of production of the crop to an increase in actual evapotranspiration \((\text{ET}_A)\) during its vegetative stage. Similarly, \(\lambda_2\) and \(\lambda_3\) represent the elasticity of production of the crop to an increase in actual evapotranspiration \((\text{ET}_A)\) during pollination and maturation stages, respectively.

Yield is measured either as grain or as dry matter. Grain yield is the actual amount of corn kernels harvested, weighed dry in tons per hectare. Dry matter yield is the actual weight of everything on the corn plant from a few inches up from the roots where the stalk was cut. It was weighed dry in tons per hectare.

**Difficult Issues Associated With Model Specification**

A functional form showing important variables that will be used to develop a production function is:

\[
\begin{align*}
Y_{\text{actual}} &= f[K, L, (\text{ET}_A)_v, (\text{ET}_A)_p, (\text{ET}_A)_m] \\
Y_{\text{potential}} &= f[K, L, (\text{ET}_p)_v, (\text{ET}_p)_p, (\text{ET}_p)_m] \\
(\text{ET}_p)_v &> (\text{ET}_A)_v; (\text{ET}_p)_p > (\text{ET}_A)_p; (\text{ET}_p)_m > (\text{ET}_A)_m \ldots \ldots (5)
\end{align*}
\]

This specification means that actual yield is a function of capital, labour, and actual evapotranspiration \((\text{ET}_A)\). \(\text{ET}_A\) is an index of moisture estimate needed at a given phenological stage. Potential evapotranspiration \((\text{ET}_p)\) is the highest measured \(\text{ET}_A\). If the relationship \(\text{ET}_p > \text{ET}_A\) holds and assuming \(L\) and \(K\) are held constant, then yield can be written in Cobb-Douglas form as:

\[
Y = C_A (\text{ET}_A)_v^{\lambda_1} (\text{ET}_A)_p^{\lambda_2} (\text{ET}_A)_m^{\lambda_3} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)
\]

where \(C_A\) is actual production parameter. To minimize the impact of combining data from three different locations, a ratio of actual to potential observations \((\text{ET}_A/\text{ET}_p)\) is required. Actual observation to
represent data collected from the field and maximum values represent potential observation. Such a ratio also helps minimize climatic effect from one year to the next, it minimizes disease effect etc. So for the potential counterpart of equation (6) we will have:

\[ Y_p = C_p \left( \frac{ET_p}{V} \right)^{\alpha_1} \left( \frac{ET_p}{P} \right)^{\alpha_2} \left( \frac{ET_p}{m} \right)^{\alpha_3} \] .......................... (7)

where \( C_p \) is a potential production parameter.

Forming the ratio:

\[ \frac{Y}{Y_p} = \frac{C_A \left( \frac{ET_A}{V} \right)^{\lambda_1} \left( \frac{ET_A}{P} \right)^{\lambda_2} \left( \frac{ET_A}{m} \right)^{\lambda_3}}{C_p \left( \frac{ET_p}{V} \right)^{\alpha_1} \left( \frac{ET_p}{P} \right)^{\alpha_2} \left( \frac{ET_p}{m} \right)^{\alpha_3}} \] .......................... (8)

Simplification and assuming \( \alpha_i = \lambda_i V_i \) (where \( V_i \) stands for overall \( i \)) will yield the model equation:

\[ Y = C \left( \frac{ET_A}{ET_p} \right)^{\lambda_1} \left( \frac{ET_A}{ET_p} \right)^{\lambda_2} \left( \frac{ET_A}{ET_p} \right)^{\lambda_3} \] .......................... (9)

When equation (4) is rewritten in a more general form within the Translog production function specification, we will get:

\[ \ln \left( \frac{Y}{Y_p} \right) = 1n\lambda_0 + \lambda_1 \ln \left( \frac{ET_A}{ET_p} \right) + \lambda_2 \ln \left( \frac{ET_A}{ET_p} \right) + \lambda_3 \ln \left( \frac{ET_A}{ET_p} \right) + \lambda_4 \left( \ln \left( \frac{ET_A}{ET_p} \right) \right)^2 \]

\[ + \lambda_5 \ln \left( \frac{ET_A}{ET_p} \right)^2 + \lambda_6 \ln \left( \frac{ET_A}{ET_p} \right)^3 + \lambda_7 \ln \left( \frac{ET_A}{ET_p} \right) \ln \left( \frac{ET_A}{ET_p} \right) \]

\[ + \lambda_8 \ln \left( \frac{ET_A}{ET_p} \right) \ln \left( \frac{ET_A}{ET_p} \right) \ln \left( \frac{ET_A}{ET_p} \right) + \lambda_9 \ln \left( \frac{ET_A}{ET_p} \right) \ln \left( \frac{ET_A}{ET_p} \right) \ln \left( \frac{ET_A}{ET_p} \right) \] .......................... (10)

And equation (10) will be the one used in this study.

Redefining \( \frac{ET_A}{ET_p} \) as \( W_v \), \( \frac{ET_A}{ET_p} \) as \( W_p \) and \( \frac{ET_A}{ET_p} \) as \( W_m \), and \( \frac{Y}{Y_p} \) as \( Y \),

then equation (10) can be rewritten as:

\[ Y = 1n\lambda_0 + \lambda_1 \ln W_v + \lambda_2 \ln W_p + \lambda_3 \ln W_m + \lambda_4 \left( \ln W_v \right)^2 + \lambda_5 \left( \ln W_p \right)^2 + \lambda_6 \left( \ln W_m \right)^2 + \]
and equation (11) can be rewritten as
\[ Y = \lambda_0 W_v^{\lambda_1} W_p^{\lambda_2} W_m^{\lambda_3} \log W_v \log W_p \log W_m \] (12)

which reduces to
\[ Y = \lambda_0 W_v^{\lambda_1 + \lambda_4} \log W_v \log W_p^{\lambda_2 + \lambda_5} \log W_m^{\lambda_3 + \lambda_6} \] (13)
CHAPTER IV
ECONOMIC ANALYSIS

Shape of Function

Given empirical evidence, with no water, there will be no grain product. Introduction of water implies $\text{ET}_A > 0$ and thus some grain product, even if only as measurable dry matter. An increase in water supply implies a cumulative $\text{ET}_A$ and that the evapotranspiration is increasing. The evapotranspiration rate goes up because, as the plant develops, so does its transpirational capacity. This would increase the tonnage of dry matter yield. Similarly, as the transpiration capacity of corn increases so would grain yield. Production is increased as corn kernels increase in size and fill up the corn cob. If we keep $(\text{ET}_A)_v, (\text{ET}_A)_m$ at a level where the crop is not stressed and let any increase in total $\text{ET}_A$ come only during $(\text{ET}_A)_p$, dry matter and grain yields will both increase.

The question of actual evapotranspiration equaling respective stage potential evapotranspiration $[(\text{ET}_A)_v, (\text{ET}_A)_m = (\text{ET}_A)_v, (\text{ET}_A)_m]$ can be problematic. Taking the ratio $\frac{Y}{V}$ we find $Y = V_p$ because $Y = (1)^{\lambda_1} \cdot (1)^{\lambda_2} \cdot (1)^{\lambda_3}$ (assuming $\text{CA} = 1$). Thus $Y = V_p = 1$. Hence, with an increase in production factor inputs, actual yield is supposed to approach potential output, $V_p$. Where $\lambda_i = 0 \forall i$, potential yield is not obtainable because the elasticity or factor share of each stage is zero. Where then is the economic problem?
Once the actual yield measured equals maximum yield, the function will no longer exhibit increasing returns to scale. If one chooses to increase a factor of production when actual yield \( Y = Y_p = 1 \), zero returns to scale should be expected, and beyond \( Y = Y_p \) negative returns to scale should be expected.

As long as \( Y_p > Y \), there is a fraction, and fractions of ET imply that better management (defined as stage-oriented water application) could make actual yield approach potential Yield \( (Y_p) \). Normally when \( \sum_{i=1}^{3} \lambda_i > 1 \), it is a case of increasing returns to scale. How \( Y_p \) is approached would dictate the rate of increase in \( Y \).

**Economic Basis of Analysis**

One of our identified problems is explaining satisfactorily to farmers when they should adopt a technical advancement. The technological advancement being a new production function which shows how water should be optimally allocated based on some form of phenological timing.

Farmers currently are operating at an efficiency and a cost. They have at the margin equated their Marginal Cost (MC) and marginal benefit and are operating "efficiently" given their present condition. If farmers are operating efficiently given their present condition why offer a technological advancement? Why should they change? How can the change be made, and what is the vehicle of change?

The change is recommended because allocating water optimally implies operating at the lowest cost. In other words, it would shift the MC curve lower as compared to the original MC curve. (See Figure 1).
Fig. 1. Possible Marginal Cost Curve Movement.
MC_1\) represents marginal cost of firm operating at present and \(MC_2\) represents the new marginal cost curve which is lower than the original one because water has been optimally allocated taking into account the phenological growth pattern of corn crop.

**Marginal Productivity**

The efficiency point is arrived at when the following equation holds:

\[
\frac{\text{MPP}_W}{P_W} = \frac{\text{MPP}_P}{P_P} = \frac{\text{MPP}_m}{P_m} = 1 \quad \text{MC} \quad \text{MR}
\]  

(10)

Of course, we are able to equate with marginal cost and marginal revenue because we have assumed perfect competition in both input (factor) and output markets.

The value of marginal product of water (VMP_w) is defined as price (a constant in a given year) times marginal physical product of water (MPP) at a given stage. MPP represents the marginal contribution of a unit of water to the total product of grain yield. VMP changes because MPP values differ for each stage of growth. For the Cobb-Douglas case, equation (9) will be used to determine MPP for each stage of growth and this requires adjusting equation (9) to:

\[
y = \frac{\text{Y}_P C}{\text{ET}_V^{\lambda_1} \text{ET}_P^{\lambda_2} \text{ET}_M^{\lambda_3}} \quad \text{ET}_A^{\lambda_1}_V \text{ET}_A^{\lambda_2}_P \text{ET}_A^{\lambda_3}_m
\]  

... (11)

Defining \(Y_P C\) as \(\hat{C}\), the equation can be written as

\[
y = \hat{C} \quad \text{ET}_A^{\lambda_1}_V \text{ET}_A^{\lambda_2}_P \text{ET}_A^{\lambda_3}_m
\]  

... (12)

Redefining \(\text{ET}_A\) as \(\bar{W}_V\), \(\text{ET}_A\) as \(\bar{W}_P\) and \(\text{ET}_A\) as \(\bar{W}_m\), then
\[
C = \frac{\frac{Y}{p_v}}{W^\lambda_1 W^\lambda_2 W^\lambda_3}
\]

and equation (7) becomes
\[
Y = c W^\lambda_1 W^\lambda_2 W^\lambda_3 \cdots \cdots \cdots \cdots (13)
\]

Differentiating \( Y \) with respect to evapotranspiration of each growth stage would yield the marginal physical product of that growth stage.

**Marginal Productivity Relationship**

Using non-optimally allocated water as was done in the experiment, there was a tendency for MPP of water to be highest during pollination stage for grain yield.

\[
\text{MP}_{W_p} > \text{MP}_{W_v} > \text{MP}_{W_m} \cdots \cdots \cdots (14)
\]

Equation (12) means that a change in total yield divided by a corresponding change in \( W \) is highest at the pollination stage and the vegetative stage is higher than that of maturation stage. Marginal condition demands that

\[
\frac{\text{MP}_{W_v}}{p_{W_v}} = \frac{\text{MP}_{W_p}}{p_{W_p}} = \frac{\text{MP}_{W_m}}{p_{W_m}} \cdots \cdots \cdots (15)
\]

or alternatively

\[
\frac{\text{VMP}_{W_v}}{p_{W_v}} = \frac{\text{VMP}_{W_p}}{p_{W_p}} = \frac{\text{VMP}_{W_m}}{p_{W_m}} K > 1 \cdots \cdots \cdots (16)
\]

where \( p_{W_v}, p_{W_p}, p_{W_m} \), is price of water at a given stage. Since price of water remains the same during the irrigation season, then \( p_{W_v} = p_{W_p} = p_{W_m} \).
But equation (14) is suboptimal for a farmer seeking maximum profit because of the strict inequalities. By spending a dollar less on (not applying a marginal dollar's worth of) water during vegetative and maturation stages, the farmer affects total ET_A and rate of (ET_A)_v,m. This would cause loss in dry matter but gain in the production of corn grain. Net output will increase by a factor greater than zero for the same total cost. Shifting the amount spent on water from the less productive to the most productive plant growth stage can thus restore profit maximization. Going one more step, the dollars spent on water during maturation stage can be reduced to further enhance profit.

First Order Conditions

Using equation (13) to determine elasticity of production of the three different stages. Take the summation of \( W_v \), \( W_p \) and \( W_m \) and get an objective function which is linear and directly related to \( W \)'s, the data used in all regression. Taking equation (13) as the constraint, it has to be minimized to total product \( Y^* \). The constraint implies that optimum product is achievable given optimal allocation of factor inputs.

Optimal condition for water allocation demands that \( \text{MPP}_{w_v} = \text{MPP}_{w_p} = \text{MPP}_{w_m} \). To arrive at this optimality condition, the use of a Lagrangian approach to solve the problem is employed. Apart from determining \( \text{MPP}_{w_v, p, w} \) to be equal, the solution would given the shadow price which is defined as that possible price of water if a market exists. The shadow price tells how much maximum profit or minimum cost will be changed for a unit change in quantity of water.
A Lagrangian will be formed to minimize amount of water to be used in corn production, subject to a production function. Mathematically it is stated as:

Minimize \[ \sum_{i} W_i \] 
subject to \[ f(W) - Y^* \]  \hspace{1cm} (19)

where \[ Y^* = \lambda_0 W_m^{\lambda_1 + \lambda_4 \ln W_v} W_p^{\lambda_2 + \lambda_5 \ln W_p} W_m^{\lambda_3 + \lambda_6 \ln W_m} \]

\[ L = W_v + W_p + W_m - \phi(\lambda_0 W_v^{\lambda_1 + \lambda_4 \ln W_v} W_p^{\lambda_2 + \lambda_5 \ln W_p} W_m^{\lambda_3 + \lambda_6 \ln W_m} - Y^*) \]  \hspace{1cm} (20)

First Order Conditions will yield

\[ \frac{\partial L}{\partial W_v} = 1 - \phi \lambda_0 (\lambda_1 + \lambda_4 \ln W_v) W_v^{\lambda_1 - 1 + \lambda_4 \ln W_v} W_p^{\lambda_2 + \lambda_5 \ln W_p} W_m^{\lambda_3 + \lambda_6 \ln W_m} = 0 \]  \hspace{1cm} (21)

\[ \frac{\partial L}{\partial W_p} = 1 - \phi \lambda_0 (\lambda_2 + \lambda_5 \ln W_p) W_v^{\lambda_1 + \lambda_4 \ln W_v} W_p^{\lambda_2 - 1 + \lambda_5 \ln W_p} W_m^{\lambda_3 + \lambda_6 \ln W_m} = 0 \]  \hspace{1cm} (22)

\[ \frac{\partial L}{\partial W_m} = 1 - \phi \lambda_0 (\lambda_3 + \lambda_6 \ln W_m) W_v^{\lambda_1 + \lambda_4 \ln W_v} W_p^{\lambda_2 + \lambda_5 \ln W_p} W_m^{\lambda_3 - 1 + \lambda_6 \ln W_m} = 0 \]  \hspace{1cm} (23)

\[ \frac{\partial L}{\partial \phi} = \lambda_0 W_v^{\lambda_1 + \lambda_4 \ln W_v} W_p^{\lambda_2 + \lambda_5 \ln W_p} W_m^{\lambda_3 + \lambda_6 \ln W_m} - Y^* = 0 \]  \hspace{1cm} (24)

Solution for \( \phi \)

Equations (21) to (24) can be rearranged to solve for \( \phi \). 

\[ \phi = [\lambda_0 (\lambda_1 + \lambda_4 \ln W_v) W_v^{\lambda_1 - 1 + \lambda_4 \ln W_v} W_p^{\lambda_2 + \lambda_5 \ln W_p} W_m^{\lambda_3 + \lambda_6 \ln W_m}]^{-1} \]

\[ = [\lambda_0 (\lambda_1 + \lambda_5 \ln W_p) W_v^{\lambda_1 + \lambda_4 \ln W_v} W_p^{\lambda_2 - 1 + \lambda_5 \ln W_p} W_m^{\lambda_3 + \lambda_6 \ln W_m}]^{-1} \]

\[ = [\lambda_0 (\lambda_1 + \lambda_6 \ln W_m) W_v^{\lambda_1 + \lambda_4 \ln W_v} W_p^{\lambda_2 + \lambda_5 \ln W_p} W_m^{\lambda_3 - 1 + \lambda_6 \ln W_m}]^{-1} \]  \hspace{1cm} (25)

\[ \phi = \frac{(\lambda_1 + \lambda_4 \ln W_v)}{(\lambda_1 + \lambda_5 \ln W_p)} \frac{W_p}{W_m} = \frac{(\lambda_1 + \lambda_5 \ln W_p)}{(\lambda_1 + \lambda_6 \ln W_m)} \frac{W_m}{W_p} = \frac{(\lambda_1 + \lambda_6 \ln W_m)}{(\lambda_1 + \lambda_5 \ln W_p)} \frac{W_m}{W_p} \]  \hspace{1cm} (26)
Regression Analysis

A test using F and Dummy variables will be performed to test if we can justify applying our results to farm situations. The Economic Software Package was used in all regression analysis. Equation (10) was the original regression format and by stepwise regression, significant coefficients were obtained for the production function as:

\[ Y_G = 0.222W_{AV}^{0.377} W_p^{1.188} - 0.4271nW_p W_{Am}^{0.353} \] ........................ (27)

\[ (-3.260) (6.472) (-5.115) (10.774) \]

Figures in parenthesis are "t" ratios, they differ from 2.326, the "t" ratio from Tables at 1 percent confidence interval. Thus we reject the null hypothesis stated as irrigating phenologically has no impact on corn production. Where \( W_{AV} \) and \( W_{Am} \) are ETA observations. To obtain optimal allocation per given stage \( W_{0v}, W_{0p}, W_{0m} \), a solution has to be obtained for equation (27). Water applied at different stages will be the objective function: Minimize \[ \sum_{i,j} W_i \]

Subject to \( f(W) - Y* \) where \( Y* \) our actual production function, \( L = \sum_{i,j} W_i \)

\[ \phi \left[ C W_{AV}^{\beta - \lambda_21nW_p} W_{Am}^{\lambda_3} - Y* \right] \] ........................ (28)

\[ \frac{\partial L}{\partial W_{AV}} = 1 - \phi \left[ C W_{AV}^{\lambda_1} W_p^{\beta - \lambda_21nW_p} W_{Am}^{\lambda_3} \right] \] ........................ (29)

\[ \frac{\partial L}{\partial W_{Ap}} = 1 - \phi \left[ C (\psi - 2\lambda_21nW_{Ap}) W_{Ap}^{\psi - \lambda_21nW_{Ap}} (W_{pp} - \lambda_21nW_{Ap})^{-1} W_{AV} W_{Am}^{\lambda_3} \right]. \] (30)
\[
\frac{\partial L}{\partial W_{A_m}} = 1 - \phi [C \lambda_3 W_{A_V}^{\lambda_1} W_p^{\beta - \lambda_2 \ln W_p} W_{A_m}^{\lambda_3 - 1}] = 0 \quad \ldots \quad (31)
\]
\[
\frac{\partial L}{\partial \phi} = CW_{A_V}^{\lambda_1} W_p^{\beta - \lambda_2 \ln W_p} W_{A_m}^{\lambda_3} - Y = 0 \quad \ldots \quad (32)
\]
(* See appendix B for transformation)

Equations (29) to (31) can be written as
\[
C \lambda_1 W_{A_V}^{\lambda_1 - 1} W_p^{\beta - \lambda_2 \ln W_p} W_{A_m}^{\lambda_3} = \phi - 1 \quad \ldots \quad (33)
\]
\[
C W_{A_V}^{\lambda_1} (\beta - 2 \lambda_2 \ln W_p) W_p^{\beta - 1 - \lambda_2 \ln W_p} W_{A_m}^{\lambda_3} = \phi - 1 \quad \ldots \quad (34)
\]
\[
C \lambda_3 W_{A_V}^{\lambda_1} W_p^{\beta - \lambda_2 \ln W_p} W_{A_m}^{\lambda_3} = \phi - 1 \quad \ldots \quad (35)
\]

Solving equations (33) and (34) yielded
\[
\frac{\lambda_1}{W_{A_V}} = \frac{\beta - 2 \lambda_2 \ln W_p}{W_p} \quad \ldots \quad (36)
\]

Similarly solving equations (33) and (35) yielded
\[
\frac{\lambda_1}{\lambda_3} = \frac{W_{A_m}}{W_{A_V}} \quad \ldots \quad (37)
\]

While solving equation (34) and (35) yielded
\[
\frac{\lambda_3}{W_{A_m}} = \frac{\beta - 2 \lambda_2 \ln W_p}{W_p} \quad \ldots \quad (38)
\]

Before solving for optimal allocation, equation (27) can be further simplified. Using \( Y_p = 12.3 \), \( W_{p_V} = 243 \) and \( W_{m_p} = 207 \), we get
\[
Y = (0.936)(12.3) W_{p_V}^{0.377} W_{m_p}^{352} \left( \frac{W_{A_p}}{W_{p_m}} \right) 0.118 - 0.427 \ln W_{A_m} + W_{A_m}^{352}
\]
\[
Y = 0.222 W_{A_V}^{0.377} W_p^{2.249} 0.118 - 0.427 \ln W_{A_m} + W_{A_m}^{352} \quad \ldots \quad (39)
\]
where $\beta^* = 2.249$

Substituting transformed $W_{AV}$ and $W_{Am}$ into equation (32) yielded

$$\frac{\lambda_1 W_{Ap}}{\Psi - 2\lambda_2 \ln W_{Ap}} + \frac{\lambda_3 W_{Ap}}{\Psi - 2\lambda_2 \ln W_{Ap}} + W_{Ap} = W_T \hspace{1cm} (40)$$

Where $\Psi = \beta^* + 0.427 \ln 224 = 4.74$ (See appendix B for derivation)

Simplification yielded

$$W_{Ap} = \left[ \frac{\Psi - 2\lambda_2 \ln W_{Ap}}{\Psi - 2\lambda_2 \ln W_{Ap} + \lambda_1 + \lambda_3} \right] W_T \hspace{1cm} (41)$$

Making the substitution $\Psi = 4.74$, $\lambda_1 = 0.377$, $\lambda_3 = 0.427$ and $\lambda_3 = 0.352$

we will get

$$W_{Ap} = \left[ \frac{4.74 - 0.854 \ln W_{Ap}}{5.469 - 0.854 \ln W_{Ap}} \right] W_T \hspace{1cm} (42)$$

Iterative solutions were obtained for $W_{Ap}$ given a certain amount of water. These values of $W_{Ap}$ then became the optimal amount of water to be applied given that an amount $W_T$ is available. $W_{Op}$ were then plugged in equations (43) and (44) to obtain optimal amount of water for vegetative ($W_{Ov}$) and maturation ($W_{Om}$) stages respectively.

$$\lambda_1 = \frac{4.74 - 2\lambda_2 \ln W_{Ap}}{W_{Ov}} \hspace{1cm} (43)$$

and

$$\lambda_3 = \frac{4.74 - 2\lambda_2 \ln W_{Ap}}{W_{Am}} \hspace{1cm} (44)$$

For $\phi$ values, equations (33) and (34) were solved to obtain

$$\phi = \frac{\lambda_1 W_{Am}}{\lambda_3 W_{Av}} \hspace{1cm} (45)$$

Table I shows optimal allocation of water per given stage of growth, shadow price $\phi$ and yield.
<table>
<thead>
<tr>
<th>Water at Vegetative stage ($W_V$) in mm</th>
<th>Water at Pollination stage ($W_P$) in mm</th>
<th>Water in Maturation stage ($W_M$) in mm</th>
<th>Total Amount of Water Used ($W_T$) during in mm</th>
<th>Grain Yield ($Y_G$) in Tons</th>
<th>Shadow Price for Water ($p$) in Growing Grain</th>
</tr>
</thead>
<tbody>
<tr>
<td>141.04</td>
<td>157.28</td>
<td>131.68</td>
<td>430</td>
<td>7.26</td>
<td>1</td>
</tr>
<tr>
<td>149.77</td>
<td>160.39</td>
<td>139.84</td>
<td>450</td>
<td>7.65</td>
<td>1</td>
</tr>
<tr>
<td>154.16</td>
<td>161.88</td>
<td>143.96</td>
<td>460</td>
<td>7.85</td>
<td>1</td>
</tr>
<tr>
<td>167.51</td>
<td>166.10</td>
<td>156.39</td>
<td>490</td>
<td>8.42</td>
<td>1</td>
</tr>
<tr>
<td>181.02</td>
<td>169.99</td>
<td>168.99</td>
<td>520</td>
<td>8.98</td>
<td>1</td>
</tr>
<tr>
<td>194.65</td>
<td>173.59</td>
<td>181.76</td>
<td>550</td>
<td>9.54</td>
<td>1</td>
</tr>
<tr>
<td>208.44</td>
<td>176.92</td>
<td>194.64</td>
<td>580</td>
<td>10.09</td>
<td>1</td>
</tr>
<tr>
<td>222.35</td>
<td>180.01</td>
<td>207.64</td>
<td>610</td>
<td>10.64</td>
<td>1</td>
</tr>
</tbody>
</table>
CHAPTER VI
DISCUSSION OF RESULTS

Effects of Increasing Water on Production

All results confirm the importance of the pollination stage for grain yield and vegetative stage for dry matter yield. This result indicates where irrigation management emphasis should be placed. Physical conditions are such that producers can only approach potential yield. Thus, rate of production due to increase in water applied is a product of a proportionality variable and potential yield.

\[ R_i = K Y_p \] (46)

where

- \( R_i \) = Increase in water applied
- \( K \) = Proportionality variable depending on ET, a fraction
- \( Y_p \) = Potential yield

For optimal solution during the growing season, amount of \( E_{T_A} \) was highest during pollination stage. Thus emphasis should be on pollination stage.

Determining Returns to Scale

Next we show the returns to scale associated with the Cobb-Douglas production functions: \( Y_G = 0.970 \bar{W}_v^{0.347} \bar{W}_p^{0.574} \bar{W}_m^{0.330} \) and \( Y_{dm} = 0.950 \bar{W}_v^{0.394} \bar{W}_p^{0.343} \bar{W}_m^{0.269} \).
To ascertain if \( \sum_{i=1}^{3} \lambda_i \geq 1 \) is increasing, decreasing or constant returns to scale we need to find \( \sum_{i=1}^{3} \lambda_i \) significant or not significant t-statistically.

To do this, the following hypothesis is required

\[ H_0: \sum \lambda_i = 1 \]
\[ H_A: \sum \lambda_i > 1 \]

Testing this linear combination of the \( \lambda \) coefficients would follow

\[
\frac{\hat{W} - W_0}{\sqrt{\hat{W}'S^2(\hat{X}'\hat{X})^{-1}W}}
\]

where \( W = \) fixed weight (unit) vector; \( \lambda = \) Least Square Estimator; \( W_0 = Unity (1) \); and \( S^2(\hat{X}'\hat{X})^{-1} = \) Estimated coefficient of variance-covariance matrix.

Solving, one gets

\[
\frac{1.451 - 1}{\sqrt{3.472 \times 10^{-3}}} = 7.598
\]

Following the t-statistic form of analysis employed earlier, we find 7.598 > 2.326, which is the book value for \( t \) at 1 percent level of significance. This result calls for a rejection of the null hypothesis stated as \( H_0: \sum \lambda_i = 1 \). Consequently for grain yield, we accept the alternate hypothesis stated as \( H_A: \sum \lambda_i > 1 \). Therefore we conclude that the production function stated exhibits increasing returns to scale. Furthermore, the major contributor to increasing returns is water applied during the pollination stage when considering grain yield. Since \( \sum \lambda_i > 1 \), as the amount of water applied is increased, its utilization also increases. The yield starts by increasing
at an increasing rate. With further increases in water application, 
the rate of increase declines. From our analysis, grain yield increase 
will come through pollination stage relatively more when compared to 
the other two stages.

The same procedure can be used for dry matter. It too has
\[ \sum_{i=1}^{3} \lambda_i > 1. \]
But, the t value is 0.168. Since 0.168 > 2.326 we cannot 
reject the null hypothesis \( H_0: \sum_{i=1}^{3} \lambda_i = 1 \). Thus for dry matter, the 
production function may yield constant returns to scale.
THE CONTROLLED EXPERIMENT ASSUMED INITIAL CAPITAL, LABOR AND FERTILIZER USED AS FACTOR INPUTS FOR DETERMINING THE PRODUCTION FUNCTION TO BE PROPORTIONALLY FIXED PER UNIT AREA, AND IN THIS SECTION THE SAME ASSUMPTION HOLDS ALSO.

ASSUME THE INPUTS CAPITAL, LABOR AND FERTILIZER USED DURING THE GROWING SEASON TO BE IN FIXED PROPORTIONS PER UNIT AREA AS WAS THE CASE WITH THE CONTROLLED EXPERIMENT. THESE TWO ASSUMPTIONS WILL BE PROVED USING AN ECONOMETRIC APPROACH.

WE ARE ABLE TO HOLD FERTILIZER, CAPITAL AND LABOR CONSTANT AND STATISTICALLY ACCOUNT FOR THE VALUE $T^{\lambda t}$ IN THE CONSTANT TERM $C$. THUS THE PRODUCTION FUNCTION CAN BE WRITTEN AS $Y = C W_{v}^{\lambda v} W_{p}^{\lambda p} W_{m}^{\lambda m} \ldots \ldots (48)$

TOPICS OF ANALYSIS

MORE DATA WAS OBTAINED FROM FARMERS. THE DATA OBTAINED FROM FARMERS WAS ADDED TO THE INITIAL CONTROLLED EXPERIMENT DATA AND A REGRESSION WAS RUN TO SEE IF:

(a) THERE WILL BE ANY STRUCTURAL CHANGE IN THE EQUATION.

(b) THERE WILL BE NO STRUCTURAL CHANGE, BUT ONLY AN INTERCEPT CHANGE.

(c) THERE WILL BE ANY CHANGE IN BOTH SLOPE AND INTERCEPT.

THE EQUATION IN LOG LINEAR FORM IS $\log Y = \log C + \lambda v \log W_{v} + \lambda p \log W_{p} + \lambda m \log W_{m}$. IF THERE IS NO STRUCTURAL CHANGE, THE REGRESSION
which will include sample from farmers should give

\[ \log Y = \log \overline{C'} + \lambda_v \log \overline{W}_v + \lambda_p \log \overline{W}_p + \lambda_m \log \overline{W}_m \ldots \ldots \tag{49} \]

The coefficients and the constant terms may not be identical because of intertemporal random sample variation.

**Analysis Procedure**

The question whether additional sample can be considered to come from the same sample population would be tested using an econometric hypothesis set up. The hypothesis being that the m additional observations obey the same relation as the controlled experiment data.

\[ F = \frac{(SSRP - SSRO)/m}{-SSRO/(n-k-1)} \]

where

- \( F \) signify F-test \((k+1, N-k-1)\)
- \( SSRP \) = Sum of square residual of pooled data \((n+m)\), 32.832.
- \( SSRO \) = Sum of square residual of lab data \((n)\), 32.663.
- \( k \) = degree of freedom of all variables except constant terms \((6)\).
- \( n = 493 \).
- \( m = 11 \).

\[ F = \frac{(32.832 - 32.663)/11}{32.663/(493-7)} = 0.0154 \]

From standard F tables, for \( F \) \((7,486)\) at 5 percent significance level one obtains 2.01. Since 0.229 < 2.01, we accept the hypothesis that the last 11 observations came from the same sample population. Thus the model is stable. Similar calculation for dry matter yielded \( F = 0.606, 0.606 < 2.01 \), again implying that the model is stable.

The F test is a quantitative test, it says the equation as a unit is stable. Qualitatively, it can be determined if the constant
term and the coefficients associated to dummy variables have changed. Without a structural test, averaging affect may hide parameter differences.

Result of Analysis

A Chow test will be used on the equation transformed to show dummy variables. Thus equation (49) with dummy variables included becomes

$$\log Y = \log C + DC + \lambda_v \log \bar{W}_v + D\delta_v \log DW_v + \lambda_p \log \bar{W}_p + D\delta_p \log DW_p + \lambda_m \log \bar{W}_m + D\delta_m \log DW_m \ldots (50)$$

Assigning $D=1$ for observations collected from the controlled experiment and $D=0$ for the eleven observations obtained from farmers, the following results in log linear form were obtained.

$$\log \text{Y}_{\text{Grain}} = -0.342 + 0.310 - 0.957 \log \bar{W}_v + 1.304 \log (DW_v)$$

$$\begin{align*}
&(-1.898)^* (1.662)^* (-0.683)^* (0.931)^* \\
&-0.133 \log \bar{W}_p + 0.707 \log (DW_p) \\
&(-0.177)^* (0.942)^* \\
&0.859 \log \bar{W}_m + 0.529 \log (DW_m) \\
&(0.431)^* (-0.265)^* \ldots \ldots \ldots \ldots (51)
\end{align*}$$

where $\bar{W}_v$, $\bar{W}_p$, $\bar{W}_m$ shows variables whose coefficients were computed using dummy variables.

First test to see if intercept and coefficients of equation (49) in log-linear form are the same as those derived using dummy variables. Since coefficients associated with dummy variables are significant, we take the sum of coefficients associated to intercept and phenological stages to determine intercept and slope, i.e. for intercept we will have $-0.342 + 0.310 = -0.032$; for coefficient associated with vegetative stage we will have $-0.957 + 1.304 = 0.347$. Similarly for
pollination and maturation stages one gets 0.574 and 0.330 respectively. Thus it can be concluded that for grain, the eleven new observations come from the same sample population. Therefore, one can transfer the results of the controlled experiment to the farm. Similarly, if we assign \( D = 0 \) for observations collected from the controlled experiment and \( D = 1 \) for the eleven observations obtained from farmers, the same conclusion will be reached, that all data are from the same sample population.

Using the same procedure for dry matter, the conclusion reached for grain will be the same conclusion reached for dry matter, that all data are from the same sample population.

\[
\log Y_{\text{Grain}} = -0.033 - 0.310 + 0.347 \log \bar{W}_v - 1.304 \log \bar{D}_v
\]
\[
-1.654* (-1.662)* (5.886) (-0.930)*
\]
\[
+ 0.574 \log \bar{W}_p - 0.707 \log \bar{D}_p
\]
\[
12.092 (0.942)*
\]
\[
+ 0.330 \log \bar{W}_m + 0.529 \log \bar{D}_m
\]
\[
9.971 (0.265)*
\]

* shows that the t static in parenthesis is significant at 5 percent significance level.

The Cobb-Douglas production function precludes full utilization of land resources under certain conditions. We therefore tried other forms of production function like the Quadratic and the Cubic functions. These are power or polynomial functions with diminishing marginal returns for each factor input. Both Quadratic and Cubic functions take into account interaction between inputs. Two others that were tried were the square root function and the transcendental production functions. The Square Root function is a compromise between the Quadratic and the
Cobb-Douglas production functions. This function also takes into account interaction between factor inputs: A Transcendental production function combines characteristics of the power and exponential functions. The Transcendental power function also assumes input factors are limitational. It has the major disadvantage that solutions can only be ascertained by iterative procedures.

Thus, even though at times reference is made to the Cobb-Douglas production function, the tables, diagrams, results, discussion and conclusion are based on the Translog production function. The reference to the Cobb-Douglas production function is necessary in this study to indirectly contrast the two production functions.
CHAPTER VIII
POSSIBLE EFFECTS OF ADAPTING TO PROPOSED TECHNOLOGY

Since it has been shown that the technology is transferable to the farm, our attention is turned to irrigating by stage of growth and obtaining the derived demand for water. We shall also show possible economic reasons why farmers have not adopted the proposed technology.

Relative to current practices, water allocation based on pheno-
logy produced higher yield. Yet the proposed technology has not been widely accepted. Are there other economic principles that can show reasons why farmers have not adapted to this new technology? What are the benefits if any in adapting the proposed technology? How can the transition be made? First investigate the demand and supply characteristics of the input, water.

Derived Demand for Water

To obtain the derived demand equation for a given stage $W_{AV}$, $W_{AP}$, $W_{Am}$, a solution has to be obtained for equation (27), but now price of water will be included as an argument in the objective function. i.e.

\[ \text{Minimize } \sum_{i,j} W_i P_j \quad i = v, p, m \text{ and } j = 1, 2, 3 \]

Subject to $f(w) - Y^* \quad \ldots \quad (51)$

Where $Y^*$ is our actual production function. The Lagrangian is

\[ L = \sum_{i,j} W_i P_j - \phi [C_{AV}^\lambda W_p^{\beta_2 \lambda_2} \ln W_p \quad W_{Am}^{\lambda_3} - Y^*] \quad \ldots \quad (52) \]
where \( C = 0.936, \lambda_1 = 0.377, \beta = 2.429, \lambda_2 = 0.427 \) and \( \lambda_3 = 0.352 \)
and \( \gamma = 4.74 \)

\[
\frac{\partial L}{\partial W_{A_v}} = P_1 - \phi [C\lambda_1 W_{A_v}^{\lambda_1-1} W_p^{\beta - \lambda_2} \ln W_{A_p} W_{A_m}^{\lambda_3}] = 0 \quad \ldots \ldots \quad (53)
\]

\[
\frac{\partial L}{\partial W_{A_p}} = P_2 - \phi [C(\gamma - 2\lambda_2 \ln W_{A_p}) W_{A_p}^{\beta - \lambda_2} \ln W_{A_p} (W_p^{\beta - \lambda_2} \ln W_{A_p})^{-1} W_{A_v}^{\lambda_1}] = 0 \quad \ldots \ldots \quad (54)
\]

\[
\frac{\partial L}{\partial W_{A_m}} = P_3 - \phi [C\lambda_3 W_{A_v}^{\lambda_1} W_p^{\beta - \lambda_2} \ln W_{A_p} W_{A_m}^{\lambda_3}] = 0 \quad \ldots \ldots \quad (55)
\]

\[
\frac{\partial L}{\partial \phi} = C W_{A_v}^{\lambda_1} W_p^{\beta - \lambda_2} \ln W_{A_p} W_{A_m}^{\lambda_3} - \gamma \ast = 0 \quad \ldots \ldots \quad (56)
\]

Equations (53) to (55) can be rewritten as

\[
C\lambda_1 W_{A_v}^{\lambda_1-1} W_p^{\beta - \lambda_2} \ln W_{A_p} W_{A_m}^{\lambda_3} = P_1 \phi^{-1} \quad \ldots \ldots \quad (57)
\]

\[
C W_{A_v}^{\lambda_1} [C(\gamma - 2\lambda_2 \ln W_{A_p}) W_{A_p}^{\beta - \lambda_2} \ln W_{A_p} (W_p^{\beta - \lambda_2} \ln W_{A_p})^{-1}] W_{A_m}^{\lambda_3} = P_2 \phi^{-1} \quad (58)
\]

\[
C\lambda_3 W_{A_v}^{\lambda_1} W_p^{\beta - \lambda_2} \ln W_{A_p} W_{A_m}^{\lambda_3-1} = P_3 \phi^{-1} \quad \ldots \ldots \quad (59)
\]

and \( \beta = 2.429 \)

Equating equations (57) and (58) yielded the equilibrium result of

\[
\frac{W_{A_p} P_2}{(\gamma - 2\lambda_2 \ln W_{A_p})} = \frac{P_1 W_{A_v}}{\lambda_1} \quad \ldots \ldots \quad (60)\ast
\]

Similarly equations (57) and (59) yielded the second equilibrium result of

\[
\frac{P_1 W_{A_v}}{\lambda_1} = \frac{P_3 W_{A_m}}{\lambda_3} \quad \ldots \ldots \quad (61)\ast
\]

and equations (58) and (59) yielded the third equilibrium result of

\[
\frac{W_{A_p} P_2}{(\gamma - 2\lambda_2 \ln W_{A_p})} = \frac{P_3 \lambda_3}{\lambda_3} \quad \ldots \ldots \quad (62)\ast
\]

*See appendix F for derivation
Thus marginal conditions demand that
\[
\frac{W_{A_v}^p}{\lambda_1} = \frac{W_{A_p}^p}{\lambda_2} = \frac{W_{A_m}^p}{\lambda_3} \quad (63)
\]

Solving the production function and the marginal productivity conditions as set in equation (63) would lead to derived demand for the three stages of growth. General form of our production function is:

\[
Y = CW_{A_v}^{\lambda_1} W_{A_p}^{\beta - \lambda_2} \ln W_p W_{A_m}^{\lambda_3} \quad (64)
\]

and from equation (64) solve for

\[
W_{A_v}^{\lambda_1} = Y[C W_{A_p}^{\beta - \lambda_2} \ln W_p W_{A_m}^{\lambda_3}]^{-1} \quad (65)
\]

\[
W_{A_p}^{\beta - \lambda_2} \ln W_p = Y[C W_{A_v}^{\lambda_1} W_{A_m}^{\lambda_3}]^{-1} \quad (66)
\]

\[
W_{A_m}^{\lambda_3} = Y[C W_{A_p}^{\beta - \lambda_2} \ln W_p W_{A_v}^{\lambda_1}]^{-1} \quad (67)
\]

Combining equations (63) and (65) to (67) yielded the following derived demand curves.

\[
W_{A_v} = \frac{\left(\frac{W_{A_p}^{\beta - \lambda_2} \ln W_p}{1}\right)^{\frac{1}{\lambda_1 + \lambda_2 + \frac{\beta - \lambda_2}{\ln W_p}}}}{K(\psi - 2\lambda_2 \ln W_p)^{\beta - \lambda_2} \ln W_p} \times (68)**
\]

\[
W_{A_p} = \frac{\left(\frac{W_{A_v}^{\gamma_3} W_{A_p}^{\gamma_3} \ln W_p}{1}\right)^{\frac{1}{\lambda_1 + \lambda_2 + \frac{\beta - \lambda_2}{\ln W_p}}}}{K W_{A_p}^{-\frac{\beta + \lambda_2}{\ln W_p}} \ln W_p} \times (69)**
\]
\[ W_{A_m} = \left[\frac{Y P^{\lambda_1} (W_{p, p}^{\lambda_3} P^{2 \lambda_2})}{K (2 \lambda_2 \ln W_{A_p})} \theta_2 \lambda_2 \ln W_{A_p} \right] x \]

\[ p^{\frac{1}{3}} - \frac{\lambda_1 - \beta + \lambda_2 \ln W_{A_p} \lambda_3^{1 - \lambda_1}}{\lambda_1 + \lambda_3 + \beta - \lambda_2 \ln W_{A_p}} \frac{1}{1} \]

(70)**

(See Appendix G for derivation.) Figure 2 depicts the derived demand curves for water at a given stage.
Fig. 2. Grain's derived demand curve for the three stages of growth and their horizontal summation.
Effect of a Price Change on Demand

The assumption that the price of water remain the same throughout the irrigation season is true only if farmers do not construct ponds. Once a pond is constructed, other indirect cost must affect the stages for which the pond was constructed. The opportunity cost of water calculated as the cost of evaporated water during pollination stage will be added to price of water during pollination stage.

A third of water during pollination stage, VP, is the relevant quantity to be considered for evaporation. The one third of VP does not all evaporate. According to research, as much as a quarter of the one third will evaporate. Thus the cost of water during pollination stage becomes $P_p + \frac{P_p}{12}$. To make arithmetic easier, use price of water during pollination stage as $P_p + \frac{P_p}{10}$. For the three stages, price of water will be $P_v$ for vegetative stage, $P_p$ for pollination stage and $P_m$ for maturation stage. Now $P_v = P_m$ and $P_p > P_v$ by $\frac{P_v}{10}$. Taking a range of prices, a derived demand schedule will be made for each productivity level, taking into account the price differential during pollination stage.

With price differentiation during stages of growth, there is a reduction in quantity of water demanded during pollination stage because, water price at pollination stage is a tenth higher than water price at the other two stages. When price of water was the same throughout the growing season, about 428 mm of water was optimally used and the demand was 197 mm for vegetative stage, 164 mm for pollination stage and 67 mm for maturation stage. When a 10 percent
price difference (from 50¢ to 55¢) during pollination stage is taken into account, the allocative demand becomes 202 mm of water for vegetative stage, 150 mm of water for pollination stage and 70 mm of water for maturation stage. Price differentiation caused an increase of 5 mm of water during vegetative stage, a decrease of 4 mm of water during pollination stage and an increase of 3 mm of water during maturation stage. The increase represent 2 percent increase for vegetative stage and for maturation stage a 5 percent increase. Pollination stage showed a decrease of 3 percent.

Cost Function

Proposing a new technology is only half the problem. The other half deals with what costs are involved in adapting such a new technology. First determine cost relationships. Solving $W_1$, the derived demand equations with the cost equation yields a cost relationship of

$$C = P_1W_{AV} + P_2W_{Ap} + P_3W_{Am} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (71)$$

where $W_{AV}$, $W_{Ap}$, and $W_{Am}$ are defined as in equation (68), (69), and (70) respectively.

Marginal Cost Functions

Marginal cost was derived using equation (71), and MC curves are infinitely elastic. Since we have assumed both input and output markets to be perfectly competitive, then $MC_i = P_i$. This marginal cost conform to that defined in textbooks as $MC = \frac{\delta TC}{\delta Q}$. 
Adapting the proposed production function has some implications. First compare yields of:

(a) A production using the newly derived production function, and
(b) A production as practiced by farmers today. In the experiment, phenological studies were based on three stages, with each stage having a defined number of days. Calculations for yield will be made on the basis of even water applications through the life of the crop, taking note of number of days in a given stage. While for the new method, optimal allocation of water will be used to determine yield. Table 2 shows higher yield from corn associated to optimally allocated water according to research findings. Average amount used in the valley is 580 mm of water. "Average" is based on amount of ET observed in corn growing by farmers. At 580 mm, there is a difference of 1.20 tons per hectare between the two methods being compared.

At 1978 market price of corn $2.47/bushel, and assuming the same cost for the two methods, a farmer using traditional method of irrigating will be loosing $106 per hectare. Figure 3 compares revenue from the two methods used in irrigation. If the profit is as shown, why have farmers not adopted a system as the one proposed here?

Given the present institutional set up, additional structures will be required to cater for peak demand during pollination stage. If farmers construct small holding ponds and other attaining structures, there may or may not be profit. Of course pond construction is the
<table>
<thead>
<tr>
<th>Total Water $W_T$</th>
<th>Price of Corn $P_Y$ (Grain)</th>
<th>Corn Yield Using Optimal Allocation $Y_G$ in Tons</th>
<th>Revenue of Corn from Optimal Allocation of Water $P_Y, Y, R$ in $$$</th>
<th>Corn Yield No Optimal Allocation of Water $Y_n$ in Tons</th>
<th>Revenue of Corn from Non-Optimal Allocation of Water $P_Y, Y_n, R_2$</th>
<th>Net Revenue $R_1 - R_2$ $$$</th>
</tr>
</thead>
<tbody>
<tr>
<td>430</td>
<td>88</td>
<td>7.26</td>
<td>640</td>
<td>5.6</td>
<td>493</td>
<td>147</td>
</tr>
<tr>
<td>450</td>
<td>88</td>
<td>7.65</td>
<td>674</td>
<td>6.04</td>
<td>532</td>
<td>142</td>
</tr>
<tr>
<td>460</td>
<td>88</td>
<td>7.85</td>
<td>691</td>
<td>6.26</td>
<td>550</td>
<td>141</td>
</tr>
<tr>
<td>490</td>
<td>88</td>
<td>8.42</td>
<td>741</td>
<td>6.92</td>
<td>609</td>
<td>132</td>
</tr>
<tr>
<td>520</td>
<td>88</td>
<td>8.98</td>
<td>791</td>
<td>7.59</td>
<td>668</td>
<td>123</td>
</tr>
</tbody>
</table>
$R_1 =$ Revenue Using Optimally Allocated Water
$R_2 =$ Revenue Using Current Practices

Fig. 3. Revenue comparison for $R_1$ and $R_2$. 
high cost limiting case. There may be cases where a pond may not be necessary. Additional costs have to be calculated and the new costs are to be added to one computed earlier. If cost of water is assumed constant throughout a growing season, then imputed cost would make $P_2$ increase such that the new cost would be higher than the first one. Thus, $C_1 > C$ and the corresponding marginal cost curves would maintain the ordering $\frac{\partial C_1}{\partial P_1} > \frac{\partial C}{\partial P_i}$.

Cost Associated with Building a Pond

To solve for cost associated with pond building, the following are costs explicitly considered.

Pond cost*. Pond cost is that cost incurred for earthwork in constructing the pond. Farm size will affect pond size. The larger the farm, the larger the volume of water to be held in the pond and the larger should the pond size be. Volume of pond in cubic meters is divided by amount charged per cubic meter to obtain the relevant earth volume (REV) cost. See Figure 4.

Cost due to evaporation loss*. Evaporation loss is a critical factor. Extensive studies on open water evaporation made by Dr. Trevor Hughes, Mr. Arlo Richardson and Mr. James Franchiewicz (13) shows that Logan looses about 0.69 m (27 in.) of water to evaporation. This works out to be 0.25 of 2.74 m (9 ft.), which is the depth recommended by Soil Conservation Service in building small ponds. Evaporation loss is multiplied by cost of water per volume to obtain cost due to evaporation loss.

*See Appendix G for detail cost calculation.
Example:

Excavation volume = 0.2 Hectare meter (2000 M$^3$)
Unit excavation cost = 50¢ per cubic meter
Total excavation cost = $1000.00

Fig. 4. Excavation cost in dollars per cubic meter
Opportunity cost of land and management*. Ponds are built on land taken off from producing other crops. Dr. Lynn Davis et. al. (3) have made recommendations on what net returns to land and management can be. The net returns to land and management is based on land class and crop type. Opportunity cost is labeled OC.

Pump cost (PPC). The average stream size of farmers' head ditch or lateral is 2.5 cfs. A pump powerful enough to deliver 2.5 cfs costs $324 (in 1978 dollars), a quoting from a local retail store in Logan. Thus, costs associated with building a pond are stated in equation form as $PC = REV + EL + OC + PPC$. The cost per hectare was calculated, linear extrapolation was made for any area greater than one hectare.

Capital Recovery Factor and Yearly Payment

The capital required for building a new pond will be borrowed and the Farmers' Home Administration (FHA) in 1978 loaned at an interest rate of 8 1/2 percent. Depending on the life, $n$, of the pond, a capital recovery factor, $CRF$, was computed using exact equation as shown by Grand and Iresen (8).

$$CRF = \frac{i}{(1+i)^{n-1}} + i.$$ For $n$, the life of the structure, 25 and 50 years were used. The CRF was used to determine yearly payment. See Table 3. While figures 6 and 7 show unit excavation cost given different field capacity.

From revenue earned in growing corn, a farmer makes his yearly payments as shown in Table 3. The payments depend on capital borrowed to finance the capital investment as well as the interest rate charged on that amount borrowed. After considering additional costs, it has to be determined if adopting corn production based phenology is profitable.

*See appendix G for detail cost calculation.
### Table 3

**Yearly Payments in Dollars for Different Soil Types at Varying Excavation Costs Given an Interest Rate and Life Span of Project**

<table>
<thead>
<tr>
<th>Field Capacity Depleted FCD in Cubic Meters</th>
<th>Relevant Earth Volume (REV) in Cents per cubic meters (In parenthesis in cents per cubic yard)</th>
<th>Interest Rate in percent</th>
<th>Capital Recovery Factor (CRF for 25 years (For 50 years in parenthesis)</th>
<th>Yearly payment in $ For 25 years (For 50 years in parenthesis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCD</td>
<td>0.145</td>
<td>0.15</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>0.145</td>
<td>2227</td>
<td>1180</td>
<td>1448</td>
<td>1671</td>
</tr>
<tr>
<td>0.15</td>
<td>2313</td>
<td>1226</td>
<td>1504</td>
<td>1735</td>
</tr>
<tr>
<td>0.16</td>
<td>2460</td>
<td>1304</td>
<td>1599</td>
<td>1845</td>
</tr>
<tr>
<td>0.18</td>
<td>2780</td>
<td>1474</td>
<td>1807</td>
<td>2085</td>
</tr>
<tr>
<td>0.20</td>
<td>3080</td>
<td>1633</td>
<td>2002</td>
<td>2310</td>
</tr>
</tbody>
</table>

Relevant Earth Volume Cost plus water cost, plus pump and opportunity cost

<table>
<thead>
<tr>
<th>Field Capacity Depleted FCD in Cubic Meters</th>
<th>Relevant Earth Volume (REV) in Cents per cubic meters (In parenthesis in cents per cubic yard)</th>
<th>Interest Rate in percent</th>
<th>Capital Recovery Factor (CRF for 25 years (For 50 years in parenthesis)</th>
<th>Yearly payment in $ For 25 years (For 50 years in parenthesis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCD</td>
<td>0.145</td>
<td>0.15</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>0.15</td>
<td>2227</td>
<td>1507</td>
<td>1794</td>
<td>2017</td>
</tr>
<tr>
<td>0.16</td>
<td>2313</td>
<td>1572</td>
<td>1850</td>
<td>2080</td>
</tr>
<tr>
<td>0.18</td>
<td>2460</td>
<td>1651</td>
<td>1945</td>
<td>2191</td>
</tr>
<tr>
<td>0.20</td>
<td>2780</td>
<td>1822</td>
<td>2154</td>
<td>2432</td>
</tr>
<tr>
<td>0.20</td>
<td>3080</td>
<td>1980</td>
<td>2351</td>
<td>2658</td>
</tr>
</tbody>
</table>
Net Profit or Net Loss?

To find out if it is profitable or not, revenue from adopting the new technology minus costs incurred due to constructing a pond will be compared to revenue earned from corn production using traditional methods. Table 4 shows the profit possible. Using 1978 corn grain prices, net revenue is shown as RR\textsubscript{1}.

With additional costs, farmers having a soil with field capacity of 29 percent by volume which implies a refillable soil water volume of 14.5 percent of field capacity will make profit only if pond construction costs are less than 75 cents per cubic meter (60 cents per cubic yard). From planting to harvesting, corn uses an average of 580 mm of water as ET\textsubscript{A}. At 580 mm of water, a loss of $14 per hectare is achievable on a 25 year loan at 8 1/2 percent interest rate. And profit of $1 per hectare is achievable on a 50 year loan at 8 1/2 percent interest rate. At 430 mm of water a profit of $29 at 8 1/2 percent for 25 years is achievable, while for 50 years $43 is achievable.

Excavation costs obtained from the Bureau of Reclamation ranged from 53¢ to $3.98 per cubic meter (40¢ to $3.00 per cubic yard). A farmer with difficult terrain and a topography on which it is difficult to have construction may find the cost of pond construction prohibitive. If the pond site is subject to leakage, the cost of lining a pond may make adoption of the proposed technology unprofitable.
<table>
<thead>
<tr>
<th>Price of Corn Per P\text{\text{ton}} \text{\text{in} $ (grain)}</th>
<th>Total Water W_t</th>
<th>Corn Yield from Opt. Allocation of water Y_g</th>
<th>Revenue from Opt. Allocation of Water in $ P_y \cdot Y_g = R_1</th>
<th>Corn Yield of Non Opt. Allocation of Water Y_{NG}</th>
<th>Revenue from Non Opt. Allocation of Water in $ P_y \cdot Y_{NG} = R_2</th>
<th>Net Revenue $R_1 - CFR_{R2} (R_1 - CFR_{R2})</th>
<th>Profit (loss) $R_1 - R_2 \text{ 25 years}</th>
<th>Profit (loss) $R_1 - R_2 \text{ 50 years}</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>430</td>
<td>7.26</td>
<td>640</td>
<td>5.604</td>
<td>493</td>
<td>522(535)</td>
<td>29</td>
<td>43</td>
</tr>
<tr>
<td>88</td>
<td>450</td>
<td>7.65</td>
<td>674</td>
<td>6.04</td>
<td>532</td>
<td>556(570)</td>
<td>24</td>
<td>38</td>
</tr>
<tr>
<td>88</td>
<td>460</td>
<td>7.85</td>
<td>691</td>
<td>6.26</td>
<td>550</td>
<td>573(587)</td>
<td>23</td>
<td>37</td>
</tr>
<tr>
<td>88</td>
<td>490</td>
<td>8.42</td>
<td>741</td>
<td>6.92</td>
<td>609</td>
<td>623(637)</td>
<td>14</td>
<td>28</td>
</tr>
<tr>
<td>88</td>
<td>520</td>
<td>8.98</td>
<td>791</td>
<td>7.59</td>
<td>668</td>
<td>673(687)</td>
<td>5</td>
<td>19</td>
</tr>
</tbody>
</table>

At 65¢ per cubic meter (60¢ per cubic yard)

| 88 | 430 | 7.26 | 640 | 5.604 | 493 | 496(513) | 3 | 34 |
| 88 | 450 | 7.65 | 674 | 6.04 | 532 | 530(547) | -2 | 15 |
| 88 | 460 | 7.85 | 691 | 6.26 | 550 | 547(564) | -3 | 14 |
| 88 | 490 | 8.42 | 741 | 6.92 | 609 | 597(614) | -12 | 4 |
| 88 | 520 | 8.99 | 791 | 7.59 | 668 | 647(664) | -21 | -4 |

At 75¢ per cubic meter (60¢ per cubic yard)

| 88 | 430 | 7.26 | 640 | 5.604 | 493 | 474(493) | -19 | 0 |
| 88 | 450 | 7.65 | 674 | 6.04 | 532 | 508(527) | -24 | -5 |
| 88 | 460 | 7.85 | 691 | 6.26 | 550 | 525(544) | -25 | -6 |
| 88 | 490 | 8.42 | 741 | 6.92 | 609 | 575(594) | -34 | -15 |
| 88 | 520 | 8.89 | 791 | 7.59 | 668 | 625(644) | -43 | -24 |
CHAPTER X

POSSIBLE PROBLEMS WITH PROPOSED TECHNOLOGY

Possible Reasons for not Adapting to the Proposed Technology

Apart from possible cost increases on building ponds and other structures it can be reasoned that farmers have not adapted the proposed technology for several other reasons. It is not because of an absence of rational behaviour on the part of farmers, rather, it is because of a combination of several reasons which may include:

(1) Farmers may incur losses if present irrigation institutions are maintained

(2) Institutional restraints

(3) Even if farmers desire to adapt, there will be problems if correct information is not available

(4) Opportunity cost of learning the new technology

(5) With modern technological change on input of the production function, the educated farmers adapt easier, while the less educated find available information more difficult to decode. In their attempt to decode, farmers incur "additional costs", which acts as a barrier in adapting the new technology

(6) Adjustment lag in the availability and adoption of the technique indicates that when new technology is available, its adoption takes time because of adjustment lag

On the profit incentive theory, a case can be made as to why farmers have not readily adapted to the proposed technology.
Institutional restraints remain the most powerful barrier to adapting the proposed technology. Irrigators are assigned a certain amount of water depending on their shares in the irrigation company. The amount mentioned also comes only once a week, on a preassigned day. The climate and the crops behave independent of any pre-arranged schedule. Thus a farmer willing to raise a certain crop differently, may find it impossible because of pre-arranged methods and procedures.

Within the framework of institutional restraint is legal restraint. The problem of who has water rights and when water rights can be sold is rather complicated, farmers try to avoid costs, including opportunity cost. The time it takes to go through legal hassles is better spent doing an agricultural operation.

Limits on capital is also a retardant to adapting the new technology. Only the Farmers' Home Administration loan at a low interest rate of 8 1/2 percent. Most commercial banks loan at between 12 1/2 to 18 percent! At such interest rate, building a pond to take care of peak demand for optimum production would only lead to the farmer incurring losses at 1978 prices. Indeed, a crop on a soil that leads to a refillable soil water volume of 25 percent of field capacity will lead to losses at 1978 prices, even given FHA's low interest rate. Thus the proposed technology can be more beneficial if present irrigation institutions can be modified to relax some of the institutional restraints.

In 1978 irrigation season, any profit from sale of irrigation water or from the use of it, is accrued to owners of shares in the irrigation company. If for example water rights can be easily sold, farmers may find one more incentive to adopt the proposed technology.
Social Welfare Implications

The idea behind the decision to do research is to tie research findings to policy implications. The implication of such policy should be clearly stated with respect to the society in general and those closely involved in growing corn. This demands that social gain (loss) and private gain (loss) have to be analyzed. It can be proven that the adoption of the new technology in corn production may be profitable and a change may be effected on corn producers.

The main vehicle of change will be the profit motive. The change can be accelerated by use of extension service which is expected to reduce the cost of seeking information.

Specific attention will be focussed on possible benefits (losses)

(1) If the proposed technology is adopted
(2) Is there a better way of collecting data or are there other approaches that can further shed light on the phenological approach to irrigating corn.

The social welfare implication of adapting or not adapting the proposed technology include welfare loss if most farmers have an output less than 9 tons per hectare. Depending on demand for corn and corn products, farmers may make a $25 profit per hectare if excavation costs are 53¢ per cubic meter. And with any increase in excavation costs, the profit margin will dwindle. Phenology approach to growing corn will have the effect of increasing supply from $S_0$ to $S_1$ shown in Figure 5, and the price of corn will drop from $P_c^0$ to $P_c^1$.

A measure of welfare change is shown by an equivalent variation or the amount of money that can be taken away from a consumer and still leave the consumer at the same utility level. Due to drop in price
Consumer Surplus = \( P_G Q_G \)

Producer Surplus = \( OP^1 \)

Resources used in corn production = \( OP^1 GQ^1 \)

Fig. 5. Possible welfare effects due to a downward shift of corn supply curve.
of corn, and assuming other prices remain constant, the society is made better off by the amount.

\[ C(P_c^0, P_{AOG}, U^0) - C(P_c^1, P_{AOG}, U^0) \]  

(72)

where \( C \) represents corn consumption as a function of price of corn \( P_c \) and price of all other goods \( P_{AOG} \) and a given utility function that stays constant. In a more recognizable form, equivalent variation is given by the area represented by

\[ W = \int_{P_c^0}^{P_c^1} \frac{dC}{dP_c} (P_c^0, P_{AOG}, U^0) \, dP_c \]  

(73)

In this case, change in consumer surplus is given by \( P_c^0 G P_c^1 - P_c^1 B P_c^0 = P_c^0 B G P_c^1 \). Producer surplus defined by the area \( P_c^1 \times Q_c^1 = OP_c^1 G Q_c^1 \) and due to change in price of corn, the change in producer surplus is given by \( P_c^0 B Q_c^0 - P_c^1 G Q_c^1 \). Resources used in corn production or cost of corn production is given by \( OP_c^1 G Q_c^1 \).
CHAPTER XI
MANAGEMENT RECOMMENDATIONS AND CONCLUSION

Management Recommendations

Irrigation practices used by farmers generally follow "rule of thumb" decision making for frequency and amount of water applied. Many follow the practice of running the water to the end of the row every two weeks without concern for infiltration rates, lengths of row or other determinants of the amount of water applied. Such practices could hardly be expected to achieve optimal water application practices in amounts or timing.

For grain production we found that the optimal allocation of water would give the vegetative stage highest ET_A value. ET_A was 0.461 of total ET_A for vegetative stage, 0.383 of total ET_A for pollination stage and 0.156 of total ET_A for maturation stage. (This means 46.1 percent, 38.3 percent, and 15.6 percent of the water applied in the respective stages.) Thus, the pollination stage needs 38 percent of total ET_A in a 26 day period as compared to vegetative stage needing 46 percent in a 63 day period and maturation stage needing 16 percent in a 43 day period.

The relevant question for management is how to optimally allocate ET_A. A transfer of units of water from one stage to another is an attempt to change the unequal marginal physical product of water during the three stages. By transferring units of ET_A (input) from the less efficient stages to those established as the most efficient, a farmer can approach an optimal allocation of water.
Possible alternatives for a farmer to employ are:

1) Varying irrigation frequency is the key to obtaining optimal yield. For example, a farmer should vary the number of days between irrigations so as to get the 38 percent of total $ET_A$ in a 26 day irrigation period during the pollination stage.

2) The schedule in terms of amount of water and irrigation frequency should allow for important characteristics such as soil, land slope, and so on.

3) On some types of soils, it may be better to vary duration of irrigation while maintaining the same number of days between irrigations.

4) Regardless of irrigation frequency, irrigating above field capacity at any given irrigation would waste water. If the irrigation schedules calls for irrigating when moisture content is down to a desired field capacity fraction, irrigation should not be delayed.

5) Transferring irrigation water to another stage at a particular time can save water and labor cost. Such management would increase yield if the water was shifted from a lower utilization stage to a higher one. Eliminating waste will reduce costs.

6) To design a pond, soil type and field capacity associated with soil type must be known.

7) Interest rates are critical in adapting to the proposed technology. For example interest rate of 10 percent or more inevitably will lead to losses given the 1978 market price of corn at $4.40 per 100 pounds of corn (close to $2.54 per bushel).

8) When assumption of price equality during the three stages of growth is relaxed, results show that water allocated to pollination stage is decreased by 3 percent where as in the other two stages,
derive demand for water is increased by 2 percent at vegetative stage and 5 percent at maturation stage

9) Effect of price differential on marginal cost is that less water is bought from the initial quantity used during pollination stage when the price was lower

10) The higher the marginal cost of water, the more water that is forth coming from suppliers.

Conclusion

Phenologically timed irrigation of corn can lead to profits provided certain conditions as explained in the study are observed. For a given soil, field capacity is reached only after a certain quantity of water has been applied at a suitable intake rate. It would be wasteful to exceed field capacity.

One way to enhance yields is to be sure the plant does not go through stress. This can be done by increasing the irrigation frequency, reducing the time period between any two consecutive irrigation, or by increasing the amount per irrigation. This is a practical management option to be decided on the basis of relative costs and physical factors.

Care must be taken during vegetative and pollination stages. The data show that the level of water applied at a particular stage of growth can affect yield. More research is necessary to ascertain precisely which stage of growth is more important when using the Translog function.

From what was learned the results obtained from the research are applicable on the farm. Thus adoption of the proposed technology
must be done if and only if costs of adopting the new technology have been considered. Profits are possible and as explained earlier the society welfare is bettered due to an increase in consumer surplus.


5 de Wit, C. T., "Transpiration and Crop Yields," Institute of Biological and Chemical Research on Field Crops and Herbage, Wageningen, the Netherlands, Verse-Landbouwk, order Z, No. 64.6-S Gravenhage.


APPENDIX A

FARMERS' QUESTIONNAIRE AND AN IMPROVED QUESTIONNAIRE
Farmers' Questionnaire

Name of Farmer:
Area planted under corn:
Corn yield per acre:
Type of irrigation used:
How much water was applied at each irrigation:
Total water used to grow corn:
Price of water per acre foot:
Price of corn per bushel (the year corn was sold):
Amount of gasoline used:
Amount of hours put in by labor:
How many hours spent in moving
  a) Pipes:
  b) Opening gates for water:
  c) Siphons:
How many days did corn take before harvesting:
Number of days between irrigation:
Suggested Improved Questionnaire For Farmers

Name of farmer:
Address:
Acres planted to corn:
Ton of silage per acre:
Bushel of grain per acre:
How many second feet did your canal carry to corn field?:
How many streams did your canal carry to corn field?:
Type of irrigation used:
   a) Flooded (furrow):
   b) Sprinkled:
   c) Other:
How many times did you flood irrigate?:
How many times did you sprinkle?:
How many hours sets did you use (hours per irrigation)?:
How many days between irrigation?:
Did you change number of days between irrigations?:
   a) Yes
       How many days between irrigations in the first 50 days?
       How many days between irrigations in the second 50 days?
       How many days between irrigations in the third 50 days?
   b) No
Total water used to grow corn in acre feet:
How many days between planting and harvesting:
Price of water per acre foot:
Price of water per stream:
Price of grain corn per bushel and year grain was sold:
Price of silage per ton and year silage was sold:
Amount of diesel used from planting to harvesting:
Amount paid to hired help:
How many hours spent in:

(a) Disking an acre of land:
(b) Harrowing an acre of land:
(c) Land planning an acre of land:
(d) Spreading an acre of land:
(e) Planting an acre of land:
(f) Cultivation and furrowing an acre of land:
(g) Ditching:
(h) Hauling:
(i) Drying:
APPENDIX B

DERIVATIVES AND RELATED CONSTANTS
The regression equation obtained is actually in the form

\[ Y = \left( \frac{W_A}{W_V} \right)^{0.377} \left( \frac{W_A}{W_P} \right)^{\beta-\lambda_2 \ln(\frac{W_A}{W_P})} \left( \frac{W_A}{W_M} \right)^{\lambda_3} \]

This simplifies to

\[ Y = \frac{(0.936 \times Y_p)}{(W_V)^{0.377}(W_M)^{0.352}} \left( \frac{W_A}{W_P} \right)^{0.377} \left( \frac{W_A}{W_P} \right)^{0.118-0.427 \ln(\frac{W_A}{W_P})} \frac{W_A}{W_M} \]

where \( W_V, W_P, \) and \( W_M \) are potential values for vegetative pollination and maturation stages respectively. Thus \( 0.936 \times Y_p \) is a constant C.

And the pollination stage value can be further simplified to

\[ \left( \frac{W_A}{W_P} \right)^{0.118-0.427 \ln(\frac{W_A}{W_P})} + 0.427 \ln(\frac{W_A}{W_P}) = 0.427 \ln(224) = 2.31. \]

Thus \( (\frac{W_A}{W_P})^{0.118+2.31-0.427 \ln(\frac{W_A}{W_P})} \)

and our original equation becomes

\[ Y = C W_A^{\lambda_1} \left( \frac{W_A}{W_P} \right)^{\beta-\lambda_2 \ln(\frac{W_A}{W_P})} W_A^{\lambda_3} \]

now \( \beta^- \) has the value 2.429.

The derivative for \( Y = \left( \frac{W_A}{W_P} \right)^{\beta-\lambda_2 \ln(\frac{W_A}{W_P})} \) will be taken using the
chain rule. Our function is the same as \( Y = u^v \), where \( u = \frac{W_{Ap}}{W_{Pp}} \)

and \( v = \beta - \lambda_2 W_p \), thus

\[
\frac{dy}{dw_p} = \frac{\partial y}{\partial u} \frac{du}{dw_p} + \left( \frac{\partial y}{\partial v} \frac{\partial u}{\partial dw_p} \right) \left( \frac{\partial lny}{\partial v} \right) \frac{dv}{dw_p} + \left( \frac{\partial lny}{\partial v} \right) \frac{dv}{dw_p} \frac{dv}{dw_{Ap}}
\]

Using this formula, the derivative is

\[
(\beta - \lambda_2 \ln W_{Ap}) \left[ \frac{W_{Ap}}{W_{Pp}} \right] \cdot \frac{1}{W_{Pp}} + \frac{W_{Ap}}{W_{Pp}} \ln \left[ \frac{W_{Ap}}{W_{Pp}} \right] \left( -\frac{\lambda_2}{W_{Ap}} \right)
\]

\[
= \frac{(\beta - \lambda_2 \ln W_{Ap}) (\beta - \lambda_2 \ln W_{Ap}) - \lambda_2 \ln W_{Ap} - \lambda_2 \ln \left( \frac{W_{Ap}}{W_{Pp}} \right) + \ln \left( \frac{W_{Ap}}{W_{Pp}} \right) (\beta - \lambda_2 \ln W_{Ap})}{(W_{Pp})^{\beta - \lambda_2 \ln W_{Ap}}} - \frac{(\beta - \lambda_2 \ln W_{Ap})}{(W_{Pp})^{\beta - \lambda_2 \ln W_{Ap}}}
\]

For this problem, the constants are

\( \lambda_1 = 0.377, \lambda_2 = 0.427, \lambda_3 = 0.352, \beta = 0.118, \beta' = 2.429, \psi = \beta + \lambda_2 \ln W_{Pp} = 4.74, \) and \( C = 0.222. \)
APPENDIX C

STATISTICAL DATA AND MAXIMUM VALUES
### TABLE 5

**STATISTICAL DATA AND MAXIMUM VALUES FOR STATES IN 1974 AND 1975**

Number of Observations of ET for Two Years in Three Locations (a)

<table>
<thead>
<tr>
<th>Year</th>
<th>Location</th>
<th>Number of Observations</th>
<th>Year</th>
<th>Location</th>
<th>Number of Observations</th>
<th>Year</th>
<th>Number of Observations</th>
<th>Grand Total for</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td>Davis</td>
<td>120</td>
<td>1975</td>
<td>Davis</td>
<td>96</td>
<td>1974</td>
<td>216</td>
<td>Davis, Fort Collins</td>
</tr>
<tr>
<td></td>
<td>Fort Collins</td>
<td>0*</td>
<td></td>
<td>Fort Collins</td>
<td>123</td>
<td></td>
<td>123</td>
<td>and Fort Collins</td>
</tr>
<tr>
<td></td>
<td>Logan</td>
<td>56</td>
<td></td>
<td>Logan</td>
<td>98</td>
<td>1975</td>
<td>154</td>
<td>Logan</td>
</tr>
</tbody>
</table>

**Maximum Values (b)**

<table>
<thead>
<tr>
<th>Year</th>
<th>Grain in Tons/Hec.</th>
<th>Dry Matter in Tons/Hec.</th>
<th>ET in mm Vegetative</th>
<th>ET in mm Pollination</th>
<th>ET in mm Maturation</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td>12.3</td>
<td>25.0</td>
<td>243</td>
<td>224</td>
<td>207</td>
<td>Davis</td>
</tr>
<tr>
<td>1975</td>
<td>12.1</td>
<td>23.4</td>
<td>222</td>
<td>187</td>
<td>207</td>
<td>Davis</td>
</tr>
<tr>
<td>1974*</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Fort Collins</td>
</tr>
<tr>
<td>1975</td>
<td>6.3</td>
<td>16.2</td>
<td>267</td>
<td>110</td>
<td>157</td>
<td>Fort Collins</td>
</tr>
<tr>
<td>1974</td>
<td>8.0</td>
<td>17.9</td>
<td>289</td>
<td>189</td>
<td>166</td>
<td>Logan</td>
</tr>
<tr>
<td>1975</td>
<td>8.9</td>
<td>19.2</td>
<td>257</td>
<td>115</td>
<td>190</td>
<td>Logan</td>
</tr>
</tbody>
</table>

*Neutron Probe Malfunctioned.
APPENDIX D

NON OPTIMAL PLOT OF MPP FOR EVAPOTRANSPIRATIONS OF A GIVEN STAGE OF GROWTH
For total water, $W$, applied at 50 cm distribution is, $a = 90\text{mm}$, $b = 102\text{mm}$, $308\text{mm} = W_p(e), (a = W_m, b = W_v)$. 

For $W = 70\text{ cm}$, $c(W_m) = 110\text{mm}$, $d(W) = 130\text{mm}, f(W_p) = 470\text{mm}$. 

Fig. 6. Marginal physical product of water holding the value of two stages constant while the other stage varies.
APPENDIX E

DERIVATION OF MARGINAL PRODUCTIVITY CONDITIONS
The Marginal Physical Product were the first order conditions as shown in equations (53) to (56) and equating equation (57) and (58); yielded:

\[
\frac{\lambda_1 W_{A_v}^{\lambda_1 - 1} W_p^{\beta - \lambda_2 \ln W_{A_p}} W_{A_m}^{\lambda_3}}{p_1} = \frac{C_{W_{A_v}}(\beta - \lambda_2 \ln W_{A_p})(W_{A_p}^{\beta - \lambda_2 \ln W_{A_p}})}{p_2} \times \left(\frac{W_p^{\beta - \lambda_2 \ln W_{A_p}} - 1}{W_m^{\lambda_3}}\right)
\]

Rearranging yields:

\[
\frac{p_2}{p_1} = \frac{C_{W_{A_v}}(\beta - \lambda_2 \ln W_{A_p})(W_{A_p}^{\beta - \lambda_2 \ln W_{A_p}})}{C_{W_{A_v}}(\beta - \lambda_2 \ln W_{A_p})(W_{A_p}^{\beta - \lambda_2 \ln W_{A_p}})} \quad \text{and it simplifies to}
\]

\[
\frac{p_2}{p_1} = \frac{W_{A_v}(\beta - \lambda_2 \ln W_{A_p})}{\lambda_1 W_{A_p}} \quad \text{and required condition is}
\]

\[
\frac{W_{A_p} p_2}{(\beta - \lambda_2 \ln W_{A_p})} = \frac{p_1 W_{A_v}}{\lambda_1}
\]

Similarly equating equations (57) and (59) yields:

\[
\frac{C_{\lambda_1 W_{A_v}^{\lambda_1 - 1} W_p^{\beta - \lambda_2 \ln W_{A_p}} W_{A_m}^{\lambda_3}}}{p_1} = \frac{C_{\lambda_3 W_{A_v}^{\lambda_1 - 1} W_p^{\beta - \lambda_2 \ln W_{A_p}} W_{A_m}^{\lambda_3 - 1}}}{p_3}
\]

Rearranging yields:

\[
\frac{p_3}{p_1} = \frac{C_{\lambda_3 W_{A_v}^{\lambda_1 - 1} W_p^{\beta - \lambda_2 \ln W_{A_p}} W_{A_m}^{\lambda_3 - 1}}}{C_{\lambda_1 W_{A_v}^{\lambda_1 - 1} W_p^{\beta - \lambda_2 \ln W_{A_p}} W_{A_m}^{\lambda_3}}}
\]

\[
\frac{p_3}{p_1} W_{A_v} = p_3 W_{A_m} \quad \text{further rearrangement gives marginal condition of}
\]

\[
\frac{p_1 W_{A_v}}{\lambda_1} = \frac{p_3 W_{A_m}}{\lambda_3}
\]

Using equations (58) and (59) and similar solution methodology yields a marginal condition of: \((W_{A_p} p_2)(\beta - \lambda_2 \ln W_{A_p})^{-1} = (p_3 W_{A_m})\lambda_3^{-1}\)
APPENDIX F

DERIVATION OF DERIVED DEMAND
Substituting the marginal conditions into equation (65) and (67)

\[ W_{AV}^{\lambda_1} = \frac{Y}{C W_p^{\beta_1 - \lambda_3 + \lambda_2} \ln W_{Ap} [ (P_1 W_{Av}^{\lambda_3} ) (P_3^{\lambda_1} )^{-1} ]^{\lambda_3} } \quad \text{and rearrangement yields} \]

\[ W_{AV}^{\lambda_1 + \lambda_3} = \frac{\lambda_1 - \lambda_3 + \lambda_3 Y P_3^{\lambda_3} p^{\lambda_3} - \lambda_3}{C W_p^{\beta_1 - \lambda_2} \ln W_{Ap}} \quad \text{remembering} \quad W_p = \frac{W_{Ap}}{W_{pp}} \quad \text{and substituting} \]

\[ W_{AV}^{P_1 (\psi - 2\lambda_2 \ln W_{Ap}) (\lambda_1 P_2)^{-1}} \quad \text{for} \quad W_p \quad \text{we'll get} \]

\[ W_{AV}^{\lambda_1 + \lambda_3 + \beta_1 - \lambda_2 \ln W_{Ap}} = \frac{\lambda_1 - \lambda_3 + \lambda_3 Y P_3^{\lambda_3} p^{\lambda_3} - \lambda_3}{C W_{Av}^{P_1 (\psi - 2\lambda_2 \ln W_{Ap})} } \quad \text{simplification yields} \]

\[ W_{AV}^{\lambda_1 + \lambda_3 + \beta_1 - \lambda_2 \ln W_{Ap}} = \frac{\lambda_1 - \lambda_3 + \lambda_3 Y P_3^{\lambda_3} p^{\lambda_3} - \lambda_3}{C W_{Av}^{P_1 (\psi - 2\lambda_2 \ln W_{Ap})} } \quad \text{simplification yields} \]

The derived demand for \( W_{Av} \) is

\[ W_{Av} = \left[ \frac{\lambda_1 - \lambda_3 + \lambda_3 Y P_3^{\lambda_3} p^{\lambda_3} - \lambda_3}{C (\psi - 2\lambda_2 \ln W_{Ap})} \right]^{\lambda_1 + \lambda_3 + \beta_1 - \lambda_2 \ln W_{Ap}} \]

Solving for \( W_{Ap} \)

\[ W_p^{\beta_1 - \lambda_2 \ln W_{Ap}} = \frac{Y}{C W_{Av}^{\lambda_1} W_{Av}^{\lambda_3}} \quad \text{substituting} \quad W_{Ap}^{P_1 (\psi - 2\lambda_2 \ln W_{Ap})} \quad \text{for} \quad W_{Am} \]

\[ W_p^{\beta_1 - \lambda_2 \ln W_{Ap}} = \frac{Y P_3 (\psi - 2\lambda_2 \ln W_{Ap})^{\lambda_2}}{C W_{Av}^{\lambda_1} W_{Ap}^{P_1 (\psi - 2\lambda_2 \ln W_{Ap})}} \quad \text{and substituting} \quad W_{Ap}^{P_1 (\psi - 2\lambda_2 \ln W_{Ap})} \quad \text{for} \quad W_{Av} \quad \text{and we will get:} \]

\[ W_p^{\beta_1 - \lambda_2 \ln W_{Ap}} = \frac{Y P_3^{\lambda_2} (\psi - 2\lambda_2 \ln W_{Ap})^{\lambda_2} P_1^{\lambda_1} (\psi - 2\lambda_2 \ln W_{Ap})^{\lambda_1}}{C W_{Av}^{\lambda_1} W_{Av}^{\lambda_3} W_{Ap}^{P_1 (\psi - 2\lambda_2 \ln W_{Ap})}} \]
simplification yields

\[ \lambda_1 + \lambda_2 W_p^{\beta - \lambda_2 \ln W_p} = \frac{Y P_3^{\lambda_1 P_2^{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_2 \lambda_1} (\psi - 2 \lambda_2 \ln W_p) \lambda_2 + \lambda_1 (W_p^{\beta - \lambda_2 \ln W_p})}{K W_p^{\beta - \lambda_2 \ln W_p}} \]

and the derived demand for pollination stage is

\[ W_p = \frac{\left[ Y P_3^{\lambda_1 P_2^{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_2 \lambda_1} (\psi - 2 \lambda_2 \ln W_p) \lambda_2 + \lambda_1 \right]}{K W_p^{\beta - \lambda_2 \ln W_p}} \]

For \( W_m \)

\[ \lambda_3 = \frac{Y}{K P_p^{\beta - \lambda_2 \ln W_p W_v^{\lambda_1}}} \], substituting \( W_m P_3^{\lambda_1} \) for \( W_v \), one obtains

\[ W_m^{\lambda_3} = \frac{Y (P_1^{\lambda_1})^{\lambda_1}}{K W_p^{\beta - \lambda_2 \ln W_p (W_m P_3^{\lambda_1})^\lambda_1}} \], simplification yields

\[ \lambda_3 + \lambda_1 = \frac{Y (P_1^{\lambda_1})^{\lambda_1 (P_3^{\lambda_1})^{\lambda_1}}}{K W_p^{\beta - \lambda_2 \ln W_p}} \]

and substituting \( W_m P_3 (\psi - 2 \lambda_2 \ln W_p) (P_2^{\lambda_3})^{-1} \) for \( W_p \) the equation becomes

\[ W_m^{\lambda_3 + \lambda_1} = \frac{Y (P_1^{\lambda_1})^{\lambda_1 (P_3^{\lambda_1})^{\lambda_1}} (P_2^{\lambda_3 W_p})^{\beta - \lambda_2 \ln W_p}}{K [W_m P_3 (\psi - 2 \lambda_2 \ln W_p)]^{\beta - \lambda_2 \ln W_p}} \]. Further simplification yields

\[ W_m^{\lambda_1 + \lambda_3 + \beta - \lambda_2 \ln W_p} = \frac{Y P_3^{\lambda_1 (W_p P_2^{\lambda_3})^{\beta - \lambda_2 \ln W_p} P_3^{\lambda_1 - \beta + \lambda_2 \ln W_p} \lambda_3^{\lambda_1 - \lambda_1}}{K (\psi - 2 \lambda_2 \ln W_p)^{\beta - \lambda_2 \ln W_p}} \]

and the derived demand curve for maturation stage becomes

\[ W_m = \left[ \frac{Y P_3^{\lambda_1 (W_p P_2^{\lambda_3})^{\beta - \lambda_2 \ln W_p} P_3^{\lambda_1 - \beta + \lambda_2 \ln W_p} \lambda_3^{\lambda_1 - \lambda_1}}{K (\psi - 2 \lambda_2 \ln W_p)^{\beta - \lambda_2 \ln W_p}} \right] \]

\[ \lambda_1 + \lambda_3 + \beta - \lambda_2 \ln W_p \]} ^{\lambda_1 + \lambda_3 + \beta - \lambda_2 \ln W_p} \]}
APPENDIX G

DETAILED COST CALCULATION
Detailed Cost Calculations

Pond:

The initial water in pond will be that amount required during the vegetative stage, \( VV \). To determine \( VV \), a unit area, the hectare (2.47 acres) will be used. Soil characteristics for Logan allow for 105 cm (3.44 ft.) depth as the extent of soil moisture depletion during vegetative stage. From data available, field capacity, \( FC \), by volume was 29 percent for Logan and 32 percent for Davis. Present irrigation practice allow for a 50 percent field capacity depletion before wilting point is reached.

\[
VV = \frac{105 \text{ cm}}{100 \text{ cm}} \times m \times 1 \text{ hectare} \times 0.29 \text{ FC} \times \frac{50}{100} \times 0.152 \text{ hectare-meter}
\]

or 1.28 acre feet. That is, \( VV \) depicts replaceable water in soil profile.

Similar procedure apply for finding the amount of replaceable water in soil profile for pollination stage, \( VP \). Soil characteristics and plant rooting system allow for 225 cm (7.38 ft.) depth as the extent of soil moisture depletion during pollination stage.

\[
VP = \frac{180 \text{ cm}}{100 \text{ cm}} \times m \times 1 \text{ hectare} \times 0.39 \text{ FC} \times \frac{50}{100} = 0.261 \text{ hectare-meter}
\]

or 2.741 acre feet.

For adequate pond size, evaporation \( E \), a major source of loosing water must be accounted for.

\[
E = (VV + VP/3) \text{ hectare meter}. \quad \text{VP is divided by three because farmers can get all the water they need once a week, depending on how many shares they have in the irrigation company. Since in Logan, pollination stage overaged 26 days, then a farmer can refill his pond at least 3 times during pollination stage. Therefore, for evaporation}
\]
only one third of VP is relevant. Trevor Hughes et al. (13) suggested 0.69 meters (27 inches) for Logan as the May to October open lake evaporation. May to October correspond to the growing season. According to the Soil Conservation Service, small ponds should be about 2.74 meters (9 ft.) deep. 2.74 meters is deep enough to prevent algae and other aquatic growth in the pond. Also, 2.74 meters is not deep enough to require special design that will take into account water pressure variance with depth. Thus, real loss due to evaporation is:

\[
E = (VV + VP) \times 0.69 \text{ hectare meters, and for Logan it is } (0.152 + \frac{2.74}{3}) x 0.25 = 0.060.
\]

Critical evaporation during pollination stage EVA, is VV + E and relevant volume, RV, is given by RV = \( \frac{EVA + VV}{3} \) hectare meter and for Logan it is 0.2227. EVA is divided by three because there are three stages of growth, and pollination stage is a third of the three stages. The relevant stage to use for an optimal pond volume is the pollination stage. The volume computed at pollination stage can accommodate the peak demand volume, and pond size is tailored to peak demand volume.

Relevant earth volume REV, computation becomes REV = RV x 10,000 cubic meter. Thus, for the conditions stated earlier, Logan's REV = 0.2227 x 10,000 cubic meters; which is 2227 cubic meters.

Using price ranges given by Bureau of Reclamation of 53¢ per cubic meter (40¢ per cubic yard) to 92¢ per cubic meter (70¢ per cubic yard), a cost table can be made reflecting earth work cost. But there are other costs. Pump cost as quoted by a local retail store in Logan $324 to deliver average stream head used by farmers.
The cost of a share of water is $1 per share, which is equivalent to $8.104 per hectare meter. Evaporation cost EC was calculated by multiplying EVA by $8.104.

Table 10 shows a range of replaceable field capacity and relevant volumes for cost calculations.
# TABLE 6

RANGE OF REPLACEABLE FIELD CAPACITY AND RELEVANT VOLUMES FOR COST CALCULATIONS

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Replaceable Field Capacity and Volume for Cost Calculations</th>
<th>Row No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>VV to 105 cm depth in Hectare meter</td>
<td>.145 0.150 0.160 0.180 0.200</td>
<td>1</td>
</tr>
<tr>
<td>VP to 180 cm depth in Hectare meter</td>
<td>.261 0.270 0.288 0.324 0.360</td>
<td>2</td>
</tr>
<tr>
<td>$E = (VV + VP/3) \times 0.25$ in Hectare meter</td>
<td>.060 0.062 0.066 0.074 0.082</td>
<td>3</td>
</tr>
<tr>
<td>EVA = $VV + E$ in Hectare meter</td>
<td>.212 0.220 0.234 0.263 0.293</td>
<td>4</td>
</tr>
<tr>
<td>$RV = \frac{EVA}{3} + VV$ in Hectare meter</td>
<td>.2227 2213 2460 2780 3080</td>
<td>5</td>
</tr>
<tr>
<td>REV = $RV \times 10,000$ in Cubic meter</td>
<td>2227 2313 2460 2780 3080</td>
<td>6</td>
</tr>
<tr>
<td>EC = EVA $\times$ $8.104$ in $$</td>
<td>2 2 2 2 3</td>
<td>7</td>
</tr>
</tbody>
</table>

To get volume in foot-pound system, multiply rows 1 to 5 by 8.104, and 8 to get volume in cubic feet, multiply row 6 by 1.308. To get EC, multiply new row 5 in foot pound system by $1.$
APPENDIX J

BIBLIOGRAPHY


APPENDIX I

COMPUTER PROGRAMS AND OUTPUT
** PRIMARY SOFTWARE PACKAGE**

**UCSD VERSION OF 1/21/72**

**THIS JOB WAS RUN AT 13:49 HOURS ON 04/17/79**

**PROGRAM**

```
LINE 0

**-----------------------------------------------**

LINE 1

LOADS
SMPL 1 493
GENR SETEPV = RETPV**2
GFNP SETEPV = RETFPP**2
GENR SETEPM = RETEPM**2
GENR PETEPM = RETEPV*RETEPP
GFNP PETEPM = RETFPV*RETEPM
GENR PETEPM = RETEPP*RETEPM
OLDQ RYGYP C RETEPV RETEPP RETEPM RETEPM SETEPM
OLDQ RYDYPM C RETEPV RETEPP RETEPM RETEPM SETEPP
END *
```
** FLUOROMETRIC SOFTWARE PACKAGE - UCSF VERSION OF 12/12 **

THIS JOB WAS RUN AT 1424 HOURS ON 04/17/74

** EQUATION **

** *********** **

** SMALL VECTOR **

<table>
<thead>
<tr>
<th>C</th>
<th>RETEPP</th>
<th>KETEPP</th>
<th>PTEPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.58828E-01</td>
<td>5.5268E-01</td>
<td>1.0658E+00</td>
<td>9.3510E-01</td>
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</tbody>
</table>

** STANDARD ERROR **

<table>
<thead>
<tr>
<th>C</th>
<th>RETEPP</th>
<th>KETEPP</th>
<th>PTEPP</th>
<th>SETEPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6425E-01</td>
<td>5.8268E-01</td>
<td>1.0658E+00</td>
<td>9.3510E-01</td>
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</tr>
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</table>

** t-STATISTIC **

<table>
<thead>
<tr>
<th>C</th>
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<th>KETEPP</th>
<th>PTEPP</th>
<th>SETEPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.32576E+01</td>
<td>0.14376E+01</td>
<td>0.11672E+01</td>
<td>0.10773E+01</td>
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</table>

** R-SQUARED **

<table>
<thead>
<tr>
<th>R-SQUARED</th>
<th>F-STATISTIC (4,486)</th>
<th>DWIGHT-WATSON STATISTIC (4,486)</th>
<th>NUMBER OF OBSERVATIONS</th>
<th>SUM OF SQUARED RESIDUALS</th>
<th>STANDARD ERROR OF REGRESSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5622</td>
<td>0.15694E+03</td>
<td>1.06374</td>
<td>493</td>
<td>0.31001E+02</td>
<td>0.15292E+01</td>
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</table>
**ECONOMETRIC SOFTWARE PACKAGE: DEC SUB VERSION OF 12/12**

**EQUATION 2**

**SUMMARY VECTOR**

1943

**ORDINARY LEAST SQUARES**

**VARIABLES...**

<table>
<thead>
<tr>
<th>P</th>
<th>Y</th>
<th>YP</th>
<th>PY</th>
<th>P</th>
<th>Y</th>
<th>P</th>
<th>Y</th>
<th>P</th>
<th>Y</th>
<th>P</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
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<td>.253524</td>
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<td>2</td>
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<td>.01</td>
<td>.01</td>
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</table>

**P-SUMMARY = 0.6265**

**F-STATISTIC (5, 457) = 1.0640**

**DURBIN-WATSON STATISTIC (ADJ. FOR 9 GARS) = 1.0440**

**NUMBER OF OBSERVATIONS = 493**

**SUM OF SQUARED RESIDUALS = 1.114714E+02**

**STANDARD ERROR OF THE REGRESSION = 1.107170E+00**
VITA

Dawuda Tsalhatu Gowon

Candidate for the Degree of
Doctor of Philosophy

Dissertation: Economic Implications of Phenologically Timed Irrigation in Corn Production.

Major Field: Economics

Bibliographical Information:

Personal Data: Born April 21, 1944 to Pa Yohanna Gowon and Madam Saraya Kuryan in Wusasa, Zaria, Zaria Province of Nigeria.

Education: Attended St. Bartholomews Primary School, Wusasa. Graduated from St. Paul's College, Wusasa, Zaria. Attended School of Agriculture, Samaru, Zaria. Received Bachelor of Science degree in 1969 from New Mexico State University with a major in Agricultural Engineering. Received a Master of Engineering degree in 1974 from Utah State University and in 1978 received a Bachelor of Science in Journalism; completed requirements for the Doctor of Philosophy degree in Economics at Utah State University in 1979.


Responsibility: Chairman of several campus organizations and member of several constitution writing committees. Mentioned two years in a row in the Who's Who Among Students in American Universities and Colleges. Was chairman of the Invited Graduate Students to the National Academy of Sciences 1976 Bicentennial Symposium. Was a secretary of an Interdisciplinary Research team at Utah State University. Married Chileshe Hilda Wabo.