

EPICYCLE PARAMETER FILTER FOR LONG TERM ORBITAL PARAMETER ESTIMATION

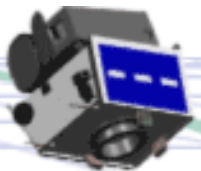
P L Palmer & S A A Gilani

Presenter:

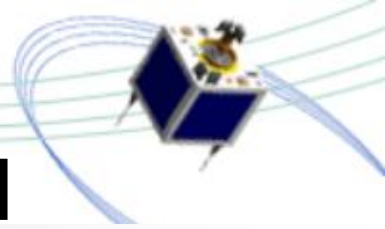
Andrew Auman

Surrey Space Centre, University of Surrey

United Kingdom



- **PROBLEM DESCRIPTION**
- **EPICYCLIC ORBIT**
- **SOLUTION METHODOLOGY**
- **RESULTS**
- **SUMMARY**



TWO BODY EQUATION OF MOTION

$$\ddot{\vec{r}} = -\frac{\mu}{r^2} \frac{\vec{r}}{r} + a_G$$

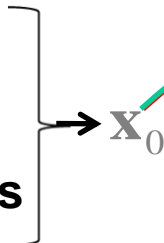
Acceleration due to Gravitational Harmonics

ANALYTICAL SOLUTION

- Brouwer Theory (1959)
- Kozai (1959)
- Epicyclic (2001)



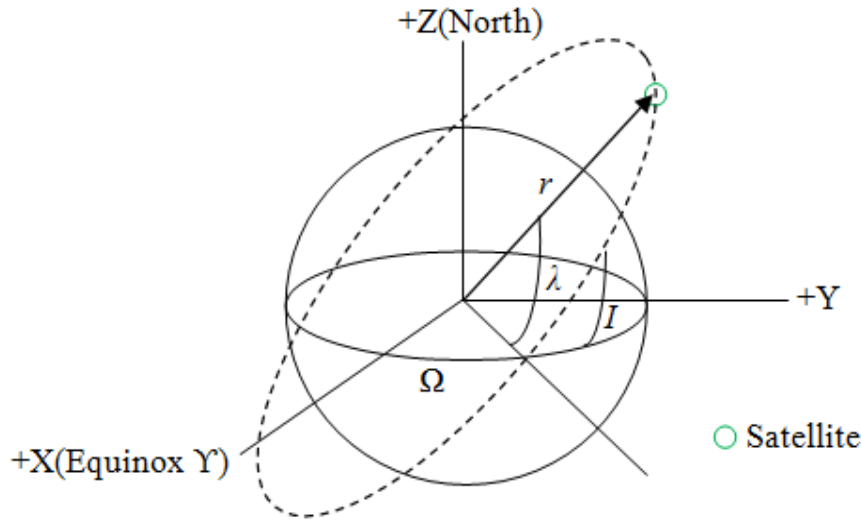
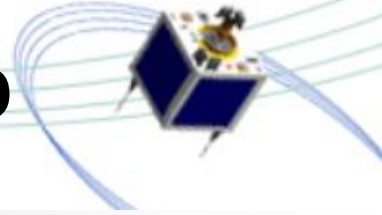
Initial Condition
Or
Orbital Parameters



Reference
Trajectory

Analytical
Solution

EPICYCLIC ORBIT AROUND OBLATE EARTH



Epicycle Coordinates

$$(r, \lambda, I, \Omega, v_r, v_\theta)$$

Epicycle Parameters

$$\mathbf{x}_0 = (a, \xi_P, \eta_P, I_0, \Omega_0, t_E)$$

a , Semimajor axis

I_0 , Inclination

Ω_0 , Right ascension of ascending node

$$\frac{r}{a} = 1 + \rho - (\xi_P \cos \alpha + \eta_P \sin \alpha) + \Delta r_2 \cos 2\beta$$

$$I = I_0 + \Delta I_2 (1 - \cos 2\beta)$$

$$\Omega = \Omega_0 + \mathcal{G}\alpha + \Delta \Omega_2 \sin 2\beta$$

$$\lambda = \beta + 2[\xi_P \sin \alpha + \eta_P (1 - \cos \alpha)] + \Delta \lambda_2 \sin 2\beta$$

Secular

Short Periodic

$$\xi_P = \frac{A}{a} \cos \alpha_P, \eta_P = \frac{A}{a} \sin \alpha_P$$

α_P , Epicycle phase at Perigee

t_E , Equator crossing time

$$\kappa_2 = \frac{3}{4} J_2 \left(\frac{R}{a} \right)^2 (5 \cos^2 I_0 - 1)$$

$$\mathcal{G}_2 = -\frac{3}{2} J_2 \left(\frac{R}{a} \right)^2 \cos I_0$$

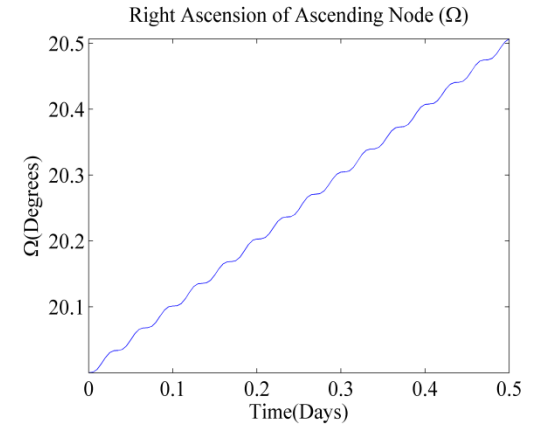
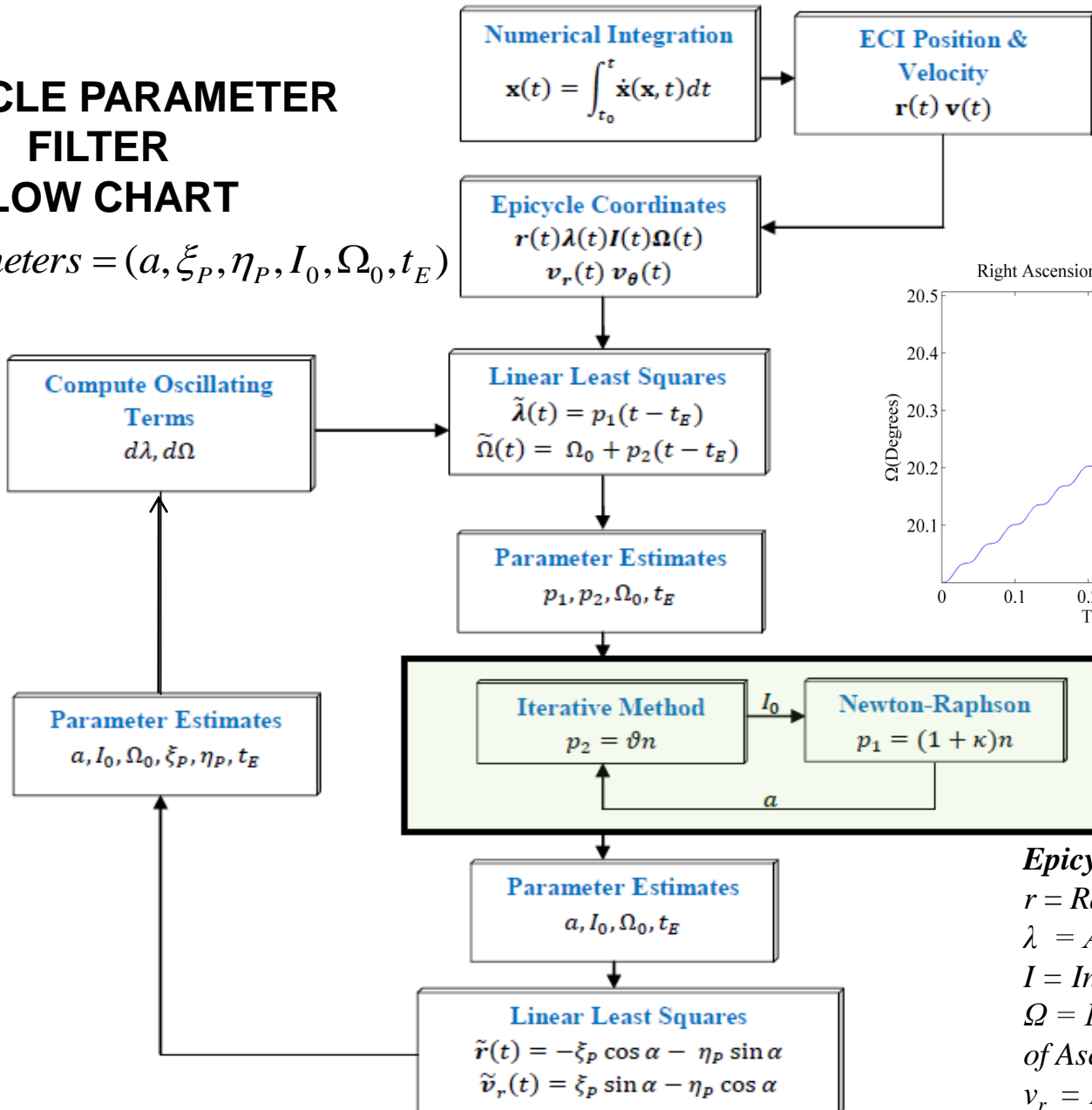
$$\beta = (1 + \kappa)\alpha$$

$$\alpha = n(t - t_E)$$

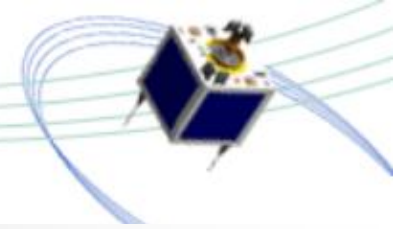
n , mean - motion

EPICYCLE PARAMETER FILTER FLOW CHART

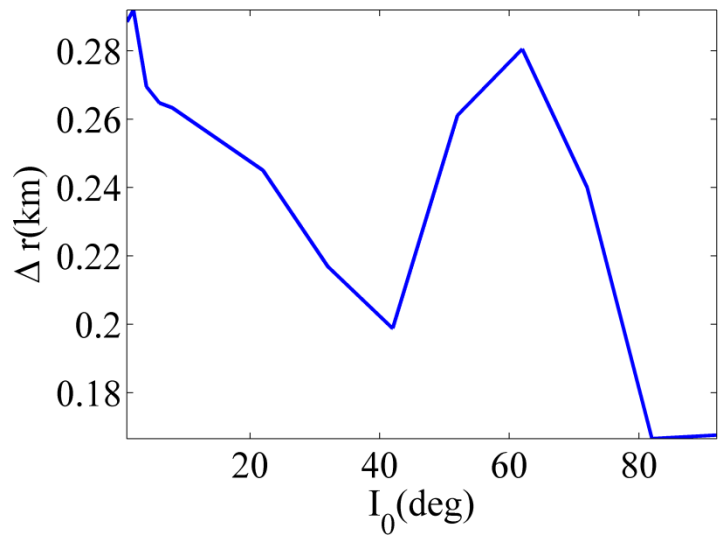
Epicycle Parameters = $(a, \xi_P, \eta_P, I_0, \Omega_0, t_E)$



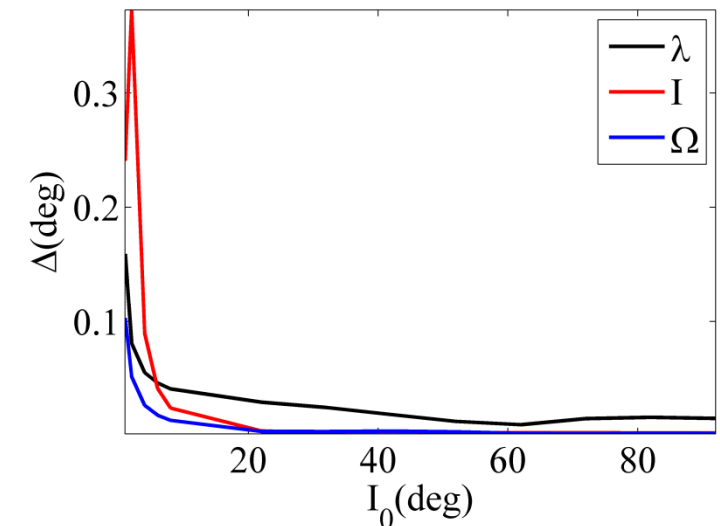
Epicycle Coordinates
r = Radius
λ = Argument of latitude
I = Inclination
Ω = Right Ascension of Ascending Node
v_r = Radial Velocity
v_θ = Azimuthal Velocity



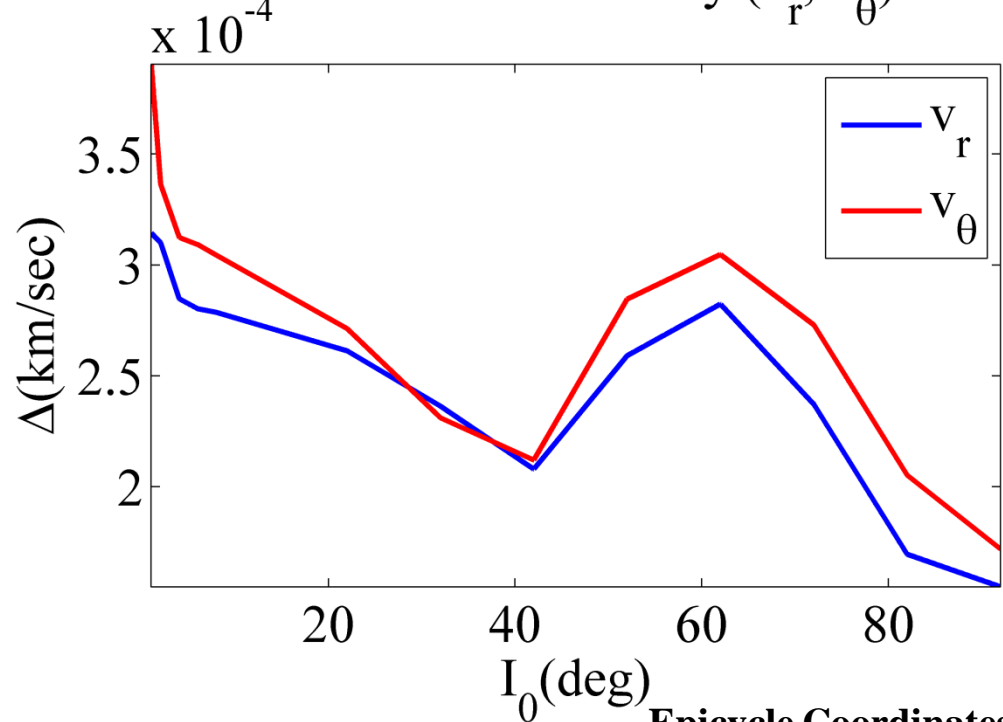
Maximum Radial Errors



Maximum Errors in λ , I , Ω



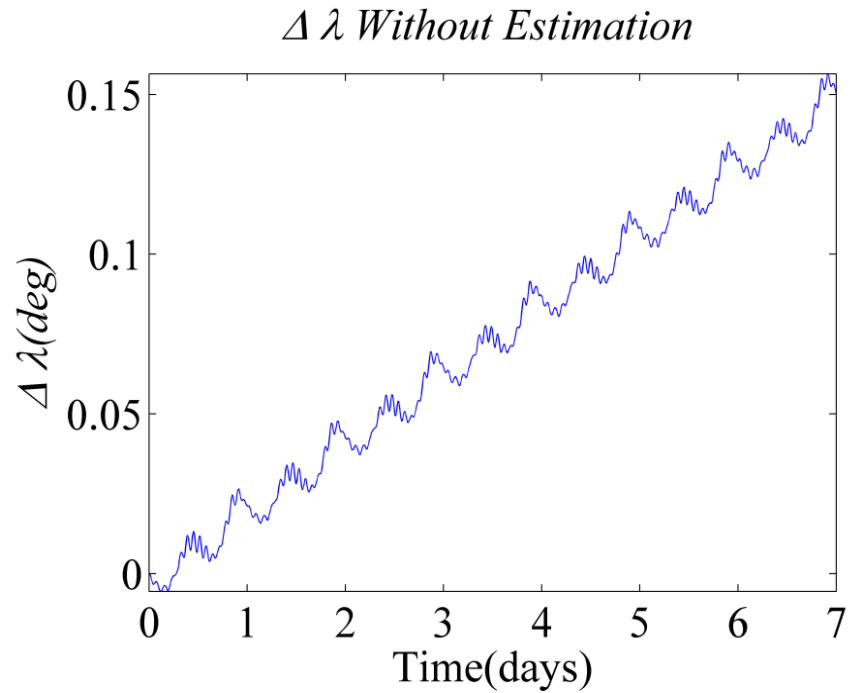
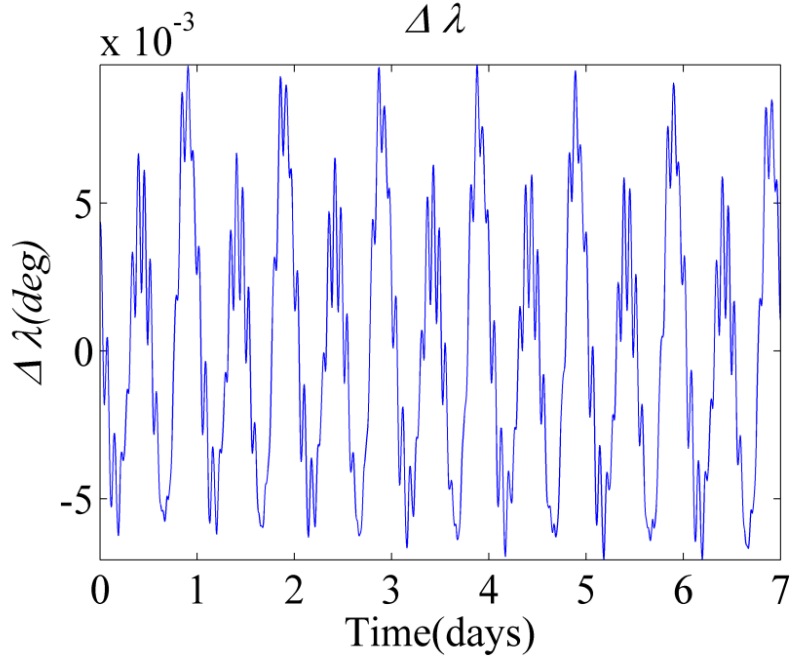
Maximum Errors in Radial & Azimuthal Velocity (v_r , v_θ)



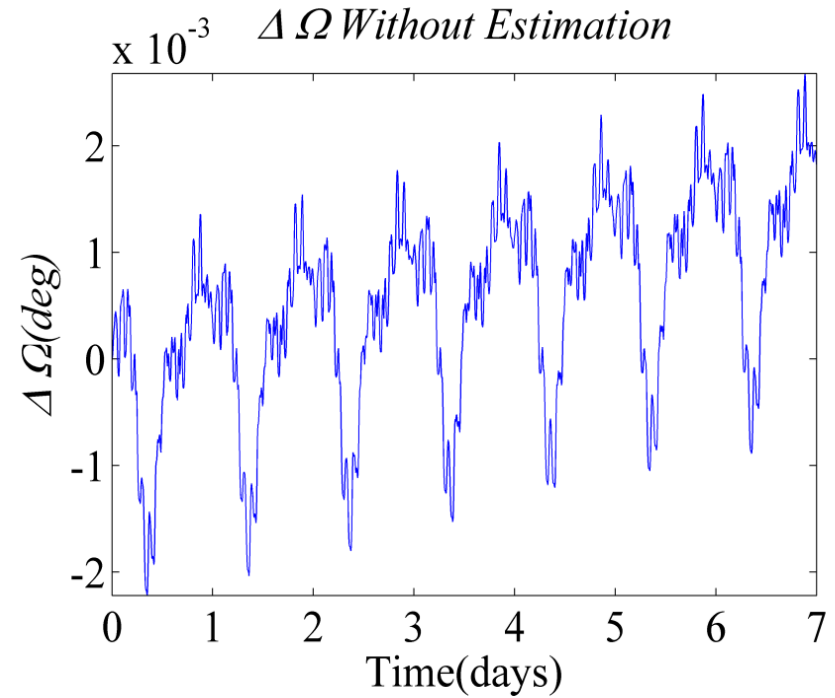
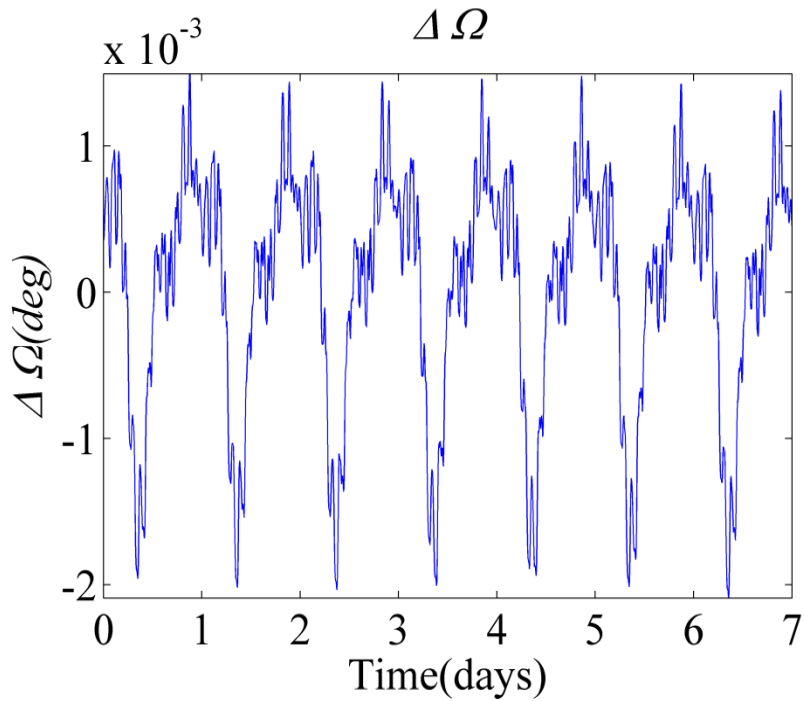
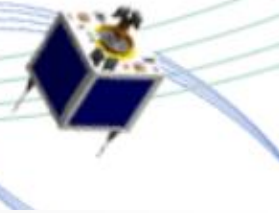
Epicycle Coordinates

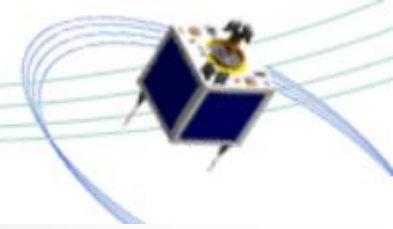
- r = Radius
- λ = Argument of latitude
- I = Inclination
- Ω = Right Ascension of Ascending Node
- v_r = Radial Velocity
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RESULTS – SUN SYNCHRONOUS LEO TIME HISTORY OF ERRORS EPICYCLE COORDINATES



RESULTS – SUN SYNCHRONOUS LEO TIME HISTORY OF ERRORS EPICYCLE COORDINATES





- Developed Epicycle Parameter Filter (EPF) for high precision Epicyclic orbit propagation
- Includes J_4 – can be extended to higher order
- Avoids calculation of Jacobian or Hessian matrices used in differential correction methods
- No filter initialization parameters or initial conditions required
- Reduction in errors:

Epicycle Coordinate	% Reduction in Maximum Errors
$r = \text{radius}$	26
$\lambda = \text{argument of latitude}$	93
$I = \text{inclination}$	45
$\Omega = \text{right ascension of ascending node}$	20
$v_r = \text{radial velocity}$	16
$v_\theta = \text{azimuthal velocity}$	15



Questions?