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SIMULATION OF A PENETROMETRIC TEST ON APPLES USING  
VORONOI-DELAUNAY TESSELLATION

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Abstract

A penetrometer is very useful for evaluating fruit firmness, but no systematic study yet exists on the physical significance attributed to the curve variations. A tissue model, based on computer graphic methods, allows the simulation of penetrometric tests on apples. There are varying physical factors such as intercellular space, cell arrangement, cell stiffness and tissue cohesion. The real forward limit of the plunger is a hemispherical zone made of collapsed cells, whatever theoretical shape the plunger may take, and curve fluctuations are due to the periodic evacuation of those cells. The most influential factor in a penetrometric measurement is individual cell stiffness, its variation with depth can be evaluated with the curve mean slope.

Introduction

Firmness measurement using a penetrometer is a very popular method in food science and particularly in fruit analysis. Most people use it to obtain an index which is supposed to be a firmness value but they fail to take into account the penetration history.

In soil science, the apparatus size allows the incorporation of complex devices in the plunger itself (Olsen, 1988; Butterfield and Mahmoud, 1989; Tollner and Simonton, 1989), thus it is possible to directly measure compression and shear stress; however, in fruit analyses this possibility does not exist. Nonetheless, some researchers have tried to link penetrometer indices to geometrical parameters of the plunger to give a significance to these values (Morrow and Mohsenin, 1966; Bourne, 1975; 1982). Because these theories have not considered flesh as a cellular tissue, each cell having its own mechanical behavior, the results have only global significance.

Modelling is of great interest to better understand the response of apples to mechanical stresses. Most researchers tried to fit their data to mechanical (Tscheuschner and Du, 1988; Peleg, 1984) or statistical (McLaughlin, 1987) models of the whole apples, while some have generated particular empirical models (Roy and Peleg, 1989). They, very often, obtain good results in their particular field of research, but, because they do not take into account the cellular tissue structure, these models cannot be generalized. A new type of model considering the geometrical aspect of tissue structure seems to be more interesting to obtain a general theory of the mechanical behavior of apple flesh (Vincent, 1989).

We have already tried to graphically model apple tissue to compare impact and compression tests (Roudot and Duprat, 1990a). In this paper, we propose to simulate a penetrometric measurement and explain what happens at the cellular level during such a measurement.

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### Modelling a penetrometer

#### Penetrometer measurements

In food science, penetrometers are used to study firmness of fruits and vegetables at a given time or during ripening and so on, but it is impossible to find systematic analyses of their responses under simple mechanical conditions, such as pure compression or friction for instance, which would allow the interpretation of curve variations as physical or physiological changes. This type of study is partially done in soil science (Ayers and Perumpral, 1982; Harison, 1987), but the apparatuses differ significantly, being larger and most often having a conical end. Soils differ in their mechanical properties from foods and especially fruits because of their respective structures. However, these studies show that differences in the sample physical states or even in the measuring conditions can introduce important variations in the results obtained. So penetrometry could give much more information if more were known about penetrometric curve significance.

#### Graphic simulation

One method of analysing the consequences of physical changes in fruit on the final curve is modelling. Unfortunately fruit flesh with its cellular structure, its anisotropy, and its heterogeneous mechanical behavior is difficult to model mathematically (Philip, 1958; Pitt, 1982; Pitt and Davis, 1984).

Graphic simulation, with its graphical part used to help the mathematical and geometrical calculations, and to clearly visualize the results, allows the creation of a pseudo-random tissue, varying in cell shape, distribution and behavior without great difficulties. This method is very attractive in modelling heterogeneous media such as fruit tissues.

The method consists of creating an image of a cross-section of the tissue in one computer file, a second image of a section of the probe in another file, and progressively superimposing these two images. Superimposition is done step by step. Each time the second image covers one more line of the first image. According to the conditions and the cell behavior implemented, a new instantaneous force is calculated and new conditions can be created for the following steps: cell resistance, cell collapse, ... Then, all the conditions being variables, a complete description of a penetrometric curve is made.

The main limitations of this technique are as follows due to the limitations of

memory size of the computer used (an IBM AT compatible model):

- it is only a two-dimensional simulation,
- cell shape is relatively coarse because of the definition,
- real depth corresponding to the simulated one is low, because of the file size limitations, even if there is a good relationship between the simulated plunger diameter and a real one according to cell size.

Despite these limitations and lack of integration of different physical factors in the model, first results obtained in impact and compression assays show good agreement with experimental results (Roudot and Duprat, 1990b).

### Model definition

#### Tissue model

Apple flesh is considered to be an aggregation of cells, the dimensions of which are random (within limits). The tissue behavior is viscoelastic, the skin having a very high elasticity. Twenty to thirty percent of the tissue comprises intercellular space, which allows cells (isolated or within a block) to move to lower stressed zones.

These different elements are taken into account in our tissue model whose creation is precisely described elsewhere (Roudot and Duprat, 1990b). Strictly speaking, this model is a mathematical one: coordinates of all singular points and equations of all lines are calculated and saved in the computer memory. During model creation, the graphic display only helps the user.

Representations of this kind of cellular tissue are obtained by geometrical means, using a tessellation of a bidimensional domain, i.e., a subdivision of the plane into a finite number of convex irregular polygons. Among the different methods, the Voronoi-Delaunay tessellation (Voronoi, 1908) is the simplest to implement and gives a good description of apple tissue.

Suppose that we are given a set of  $n$  points in the plane, which are not all collinear. Voronoi polygons may be viewed as the result of a growth process. Assume all  $n$  points simultaneously start a uniform outward growth along a circular frontier. The growth stops at points of contact between any two circles which then expand into straight line segments along which growth frontiers meet and freeze. An edge continues elongating until it encounters the

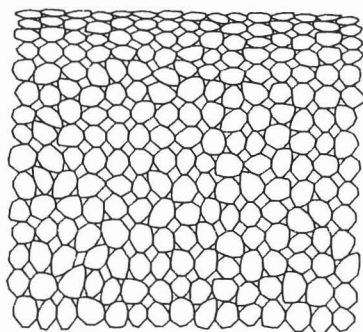


Fig. 1: Model of apple cellular tissue obtained using Voronoi polygons. Direct algorithm is modified a little bit to create intercellular voids and to smooth the angles, to obtain a more realistic representation of apple flesh from a geometrical point of view.

border of a third expanding circle. At the end of the process, each of the  $n$  points is contained in exactly one convex polygon. The set of complete polygons is called the Voronoi diagram (Fig. 1).

The individual cell behavior when submitted to a penetrometric test is as follows:

- When the plunger and a cell come close together, the latter, if possible, moves downward to a less stressed zone. However, that displacement is always small compared to the cell size.

- If movement is impossible, the cell area can reduce in size until it reaches half its unstressed area; it then collapses and disappears.

One important parameter involved in the simulation takes into account the tissue structure: cell cohesion. It is a factor which is calculated for each cell, a function of the ratio between cell wall length in contact with walls of other cells, and whole wall of that cell. Cohesion is a multiplicative factor which reinforces each cell stiffness, which is used to calculate the force created by the cell.

#### Plunger test simulation

The plunger moves downward line after line, with a constant speed.

Calculation of the force is made, for each step, for all the points  $i$  limiting the

plunger inside the tissue. Three different cases can occur:

- Point  $i$  is close to an intercellular void:

$$VF_i = A_1 \times g(\text{cell depth})$$

$$HF_i = B_1$$

- Point  $i$  is close to a cell which cannot move:

$$VF_i = A_2 \times f(\text{remaining cell area}) \times g(\text{cell depth}) \times \sin \Theta \times CC$$

$$HF_i = B_2 \times f(\text{remaining cell area}) \times \cos \Theta \times FC$$

- Point  $i$  is close to a cell which can move:

$$VF_i = A_3 \times g(\text{cell depth}) \times \sin \Theta$$

$$HF_i = B_3 \times \cos \Theta$$

$$\text{At the end } VF = \sum VF_i \text{ and } HF = \sum HF_i$$

with:  $VF_i$  and  $HF_i$ , respective vertical and horizontal forces at point  $i$ ,  $VF$  and  $HF$ , total vertical and horizontal forces at the end of the considered step,  $FC$  and  $CC$ , friction and compression coefficients,  $A_1$ ,  $A_2$ ,  $A_3$ ,  $B_1$ ,  $B_2$  and  $B_3$ , multiplicative coefficients,  $\Theta$ , angle between horizontal and the normal of the plunger at point  $i$ .

Factors  $FC$  and  $CC$  differ for each cell and are linked with their stiffness. The more densely placed the cells are, the greater the influence of stiffness,  $g(\text{cell depth})$  allows the implementation of "firmness" variations with depth.

The test is completed after 200 downward advances (or steps), i.e., when the plunger is 200 lines lower than it was at the beginning (just in contact with the skin), corresponding to an experimental penetration of about 4 to 6 mm depending on the randomization. Considering that a relaxation phenomena exists, after each step, a part of the total force is decreased before a new advance.

The skin, whose firmness was shown to be of interest (Nicolas et al., 1986; Philouze et al., 1988), is assumed to be a perfect elastoplastic medium, so the simulation is different for that part. Elasticity is believed to be a factor not only in surface but also in the subsurface cells. The deeper the plunger moves into the fruit, the larger the subsurface area involved in the reaction. So the skin elasticity is not only a surface phenomena, but also a volume phenomena. When the skin breaking force is

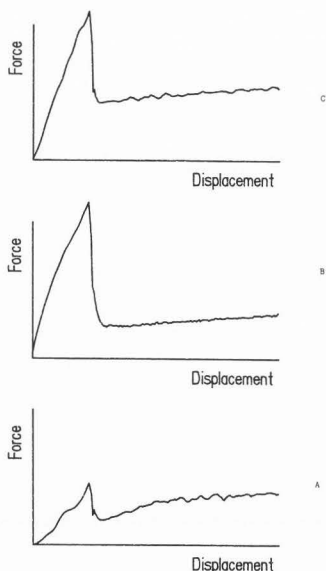


Fig. 2 : Simulated curves of a penetrometric test with a conical (A), a cylindrical (B) and a hemispherical probe (C). Large differences can be seen between peak levels, and also between mean values of the horizontal part of the curve. Note the regularity of the latter. The scale is arbitrary but the same for the three curves.

reached, the normal plunger advance takes place.

### Results

#### Shape of the plunger

We have simulated three different plunger shapes: conical, cylindrical and hemispherical. The main difference in the penetration curve occurs in the peak linked with the skin penetration: the hemispherical plunger gives a peak somewhat lower than the cylindrical one, with the conical one being very low. In any case, the curve, once it has passed that peak is very regular (Fig. 2).

The mean value obtained for that part of the curve is very much lower for the cylindrical plunger than for the hemispherical. This is absolutely unrealistic, compared with real results where almost no difference exists (Fig. 3).

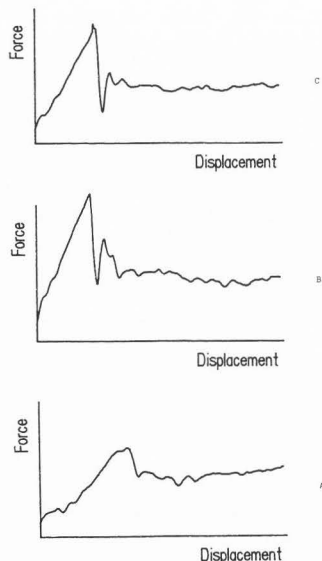


Fig. 3 : Experimental curves obtained with our penetrometer (Duprat et al., 1986) with a conical (A), a cylindrical (B) and a hemispherical probe (C). Probe diameter is 2 mm, and advance is 20 cm/min. The "horizontal" parts are more irregular than in Fig. 1, but they have almost the same mean level. Relative differences between peak heights are fairly similar with those of Fig. 1. The scale is arbitrary but the same for the three curves.

To explain this fact, let us suppose that, during a penetration test, numerous squashed cells stay just in front of the very end of the plunger (Mulqueen et al., 1977), and can be considered as its real forward limit (as it is well known in machining (Roudot, 1982)). So, whatever the theoretical plunger shape may be, the real one will always be the same, immediately after the beginning of the penetration test (Fig. 4). We can suppose then, that a hemispherical model is the more realistic: the nearer the cells are to the edge of the probe, the easier they are evacuated.

#### Influence of the mechanical behavior of cells

The notion of cell rearrangement is introduced by various theories which try to explain the mechanical behavior of apple

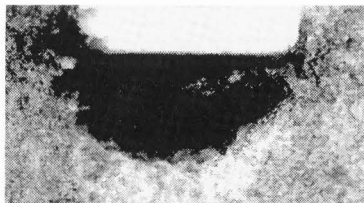


Fig. 4 : Photonic image of cross section of an apple showing the bottom of the hole obtained after penetration (and removal) of a cylindrical plunger into an apple. One can see a hemispherical conglomerate of squashed cells (dark gray) just in front of the forward limit of the plunger (white). Intact tissue is light gray. This image was obtained by a computer vision system, and modified to enhance contrasts.

flesh. This factor seems to be very marginal in penetrometric tests. Almost no variation occurs in curve shape or typical values. Two other elements appear to be interesting in confirming this result and in questioning these theories: the occurrence of strong bonds between apple cells (Matz, 1962; Trakoontivakorn et al., 1988), which could forbid displacement, and fracture mechanics which can explain some mechanical behavior similar to that of apple, in such different materials as rock or clay without displacement (Tchalenko, 1968; Petit, 1988).

Conversely individual cell stiffness (defined as a parameter used to calculate the response of the cell to a mechanical stress) seems to be one of the most important factors having an influence on the result. Cell cohesion (defined as a multiplicative factor which reinforces each cell's real stiffness) is another, although it appears less influential on the final curve (Tables 1 and 2).

#### Irregularity of the curve

Whatever parameter is modified (plunger shape, cohesion, cell stiffness, intercellular space,...), the curve obtained using this simulation is very regular whereas actual curves are very irregular.

Table 1 - Variations of firmness versus cell stiffness.

Cell stiffness	Firmness
2*	1288*
7	1391
10	1472

\* - Numerical values are in arbitrary units.

Table 2 - Variations of firmness versus cell cohesion.

Cell cohesion	Firmness
1*	1430*
4	1448
10	1483

\* - Numerical values are in arbitrary units.

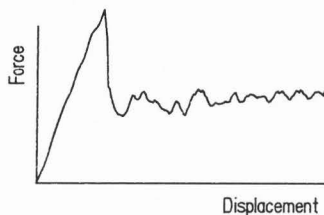


Fig. 5 : Final simulated curve obtained, considering that the real forward limit of the probe is made of numerous squashed cells having a hemispherical shape, whatever real shape the plunger has. Variations on the right side of the curve are due to semi-periodical evacuation of these cells.

Considering that collapsed cells, gathered together just in front of the plunger, are evacuated from time to time, analogously with metal cuttings during machining, and not continuously (Roudot, 1982), the apparent relaxation coefficient will change at each step. When that new condition is simulated, the penetration curve has a shape very similar to a real one (Fig. 5).

#### Mean slope of the curve

It can be seen on the actual penetrometric curves, that the mean slope of the

curve just after the skin peak can rise, fall or stay constant (Bourne, 1966).

With the model, this can happen in only two cases : if the shear force is important compared with the compression one, or if cell stiffness changes with the penetration depth. The simulation does not allow to choose between these two hypothesis.

However, other results show that shear force in apple is only a few percent of compression force (Bourne, 1975). On the other hand, tissue structure (cell size, shape and arrangement) is linked with the depth (Tukey and Oran Young, 1942; Reeve, 1953). So it can be taken that the curve slope is linked with a variation of cell stiffness.

### Conclusions

We have simulated a penetrometer test on apple flesh. Apple tissue was modelled previously and only a few details were modified since the earlier experiments. That simulation gives curves very similar to real ones and so confirms our tissue model.

This study shows that the very irregular shape of the curve when the plunger sinks into the flesh is linked with the evacuation of collapsed cells just before the plunger. It also shows that the plunger shape has almost no influence on the result because the real zone which crushes the cells is made of collapsed cells of a globally hemispherical shape, whatever plunger is used.

Cell stiffness is the most influential factor in the final result, and its change with depth can be evaluated using the curve slope. Because of this, firmness measurement using a penetrometer could give at least two numerical values: firmness (the mean value of the curve after skin peak) and firmness variation with depth (the slope of the curve).

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#### Discussion with reviewers

J.M. De Man: It is assumed that cells behave as individual entities and can collapse and disappear. In actual fact, the cell walls are all joined together and it is doubtful that a cell could just behave as a separate entity. How could this fact be taken into account in your model?

Authors: Different assumptions are made for individual cells, but we also assume a possible interaction between cells with the cohesion parameter. This is calculated for each cell looking at its surroundings. This cohesion is used to calculate cell stiffness. So cohesion, which is a tissue characteristic, is one of the most important parameters in the simulation.

D.D. Hamann: Is cell rupture involved in the concepts of cohesion and stiffness?

Authors: No, cohesion and stiffness are only used to calculate the force created by each cell on the plunger. A cell can only collapse when its stressed area reaches half of its unstressed area.

J. Abbott: Don't you think the model is greatly simplified by the lack of true estimates of internal pressure, cellular adhesion, wall rigidity and so on?

Authors: The aim of this work was to have qualitative information to better understand penetrometric curves. A quantitative study needs better knowledge and integration of numerous parameters in the model. However, as already shown (Roudot and Duprat, 1990b), geometry of cells and cell arrangement are the main factors in mechanical behavior of apple flesh. Then this geometrical model is a good approximation of the reality.

J.M. De Man: Do you think the cells are really as compressible as the assumptions indicate?

Authors: When submitted to a force, a cell will change its shape, being flatter than unstressed, but its volume will not change a lot, because fluids are only slightly compressible. In our model, we decided to



create far more compressible cells, because we calculate the force created by cells using their volume variations. Although it was simple to have less compressible cells and to use a multiplicative factor to calculate the force and have the same range of variations, we prefer our solution which increases the resolution.

J. Abbott: Why don't you use photomicrographs of real tissue instead of pseudo-random cells generated mathematically?

Authors: A photomicrograph is certainly the best solution to make a two-dimensional simulation. However, if geometry has a great importance in the results, the final aim is to use a three-dimensional model. The advantage of Voronoi-Delaunay tessellation is that it is a general n-dimensional theory, and so our model can be generalized to a three-dimensional one. This was the main reason for using a mathematical model, which gives a good approximation of the apple structure.