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MEASURING NATURAL RESOURCE SCARCITY UNDER COMMON PROPERTY ENVIRONMENT AND UNCERTAINTY: AN INTERPRETIVE ANALYSIS

by

Soumendra N. Ghosh

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Economics

Approved:

UTAH STATE UNIVERSITY
Logan, Utah

1987

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ABSTRACT

Measuring Natural Resource Scarcity Under Common Property Environment and Uncertainty: An Interpretive Analysis

by

Soumendra N. Ghosh, Doctor of Philosophy

Utah State University, 1987

Major Professor: Dr. Donald L. Snyder
Department: Economics

The issue of natural resource scarcity has so far been addressed in the literature on the basis of various measures such as the unit cost of production, the relative market price, and the shadow price of a resource. Although it has been recognized that there exists some kind of jointness (sometimes inseparable) between an extractible resource and its surrounding environment, none of the measures, either theoretically or empirically, have included this concern.

In order to extract and use a natural resource (e.g., coal) the environment (air, water, etc.) must also be used as a repository of the discharged wastes (e.g., sulphur oxides, nitrous oxides, particulates, etc.). Moreover, if there is a mandated level of the environmental resource (e.g., clean air) that has to be maintained, then certain additional costs must be borne by society (firms utilizing the resource).

Thus, in evaluating the scarcity of an extractible resource, the relative position of the environmental resource also must be evaluated. The present study has incorporated such jointness in the evaluation of the measure of resource scarcity.

The theoretical model has been developed in an optimal control framework. It has been analytically shown that this new measure of resource scarcity would indicate a different trend compared to earlier ones. The measure of resource scarcity developed in this study captures previous measures as special cases. In an uncertain world, when the impacts of use of an extractible resource on the environment is not known the stock size of the environmental resource becomes uncertain. It has been analytically shown that in a situation of uncertain environmental stock the scarcity indicator would indicate a relatively slower extraction compared to that of a deterministic world.

Empirical investigations in this study suggest that coal in use might be becoming relatively scarce if one considers the use of it in the electricity industry as the major use, compared to a situation where no environmental concerns are in effect.

(162 pages)

CHAPTER I

INTRODUCTION

A resource is generally viewed as becoming scarce if the quantity demanded of it exceeds quantity supplied at a given (benchmark) price such that there is an upward pressure on the price through time (A. C. Fisher 1979). This is a pure economic concept and, as such, may or may not be synonymous with a physical measure of scarcity such as reflected in a "stock of reserve." Physical scarcity does not imply that a resource is becoming scarce in an economic sense. Similarly, economic scarcity does not necessarily mean a decline in the stock of reserve.

Fisher (1979, p. 252) provided a very simple but elegant answer to a question of resource scarcity: a measure of resource scarcity should summarize the "sacrifices, direct and indirect, made to obtain a unit of resource." This definition reflects both supply and demand. Demand or willingness to pay for a particular resource is implicit in the sacrifices made directly and indirectly by the purchasing party. The associated direct and indirect costs represent sacrifices made by those providing the resource in question. The direct costs may be represented by the total labor and capital costs used in obtaining a unit of a resource, whereas the indirect costs may involve evaluation of the trade-off between present and future consumption. This may be of particular significance if the indirect costs involve differences in the quality of life intertemporarily.

Section I.1: Statement of Problem

Various measures of resource scarcity, such as the unit cost, the shadow price (marginal user cost), and the market price of the resource, have been advocated as valid. Unit cost (as defined by Barnett and Morse 1963) reflects resource scarcity in terms of the relative size of the fixed natural resource with respect to the labor, capital, and sociotechnical knowledge available for utilizing that resource. On the other hand, the shadow price or the marginal user cost is defined as the opportunities foregone for using a unit of a resource. From a profit-maximizing firm's point of view, the shadow price would indicate the marginal loss in current profit due to future extraction. Conversely, it may represent tomorrow's income opportunities foregone if too much extraction occurs today. Market price has also been used as another measure of scarcity, since it captures both the demand and the supply of a resource.

From a historical perspective it does not appear that any concensus has been reached as to which measure has the greatest capacity in explaining resource scarcity. In fact, in a perfectly functioning market any of the measures may be appropriate. However, it has been recognized that each of these candidate measures, e.g., unit cost, shadow price (rent), and relative price are valid only if the common property aspects of the environment are nonexistent or has been incorporated into the model.

The environment itself is a natural resource and the flow of many extractible resources depends upon a stock of environmental services. Where such jointness holds, the true social opportunity cost of

extracting a natural resource cannot be reflected properly in any of the measures of resource scarcity as presently modeled. Typically, it has been implicitly assumed that the environment is a common property resource. No additional cost is imposed on the part of the profit-maximizing firms for use of that resource. However, if jointness occurs between the extractible natural resource and the environmental resource such that the latter has to be used as a repository of the waste products from the use of the former, then certain costs will be imposed on society. In order to facilitate an increased use of the primary resource, while maintaining the environment, additional costs must be incurred by the firm or society.

To the extent that resource scarcity is viewed as an intertemporal issue in a dynamic setting, it may be of interest to identify the optimal time paths of the stock of resources, the stock of environmental services (stock of environmental assets), and their respective shadow prices. Furthermore, uncertainty with respect to environmental impacts may also affect optimal time paths of the stock of resources, the stock of environmental services, and their respective shadow prices.

Section I.2: Purpose of the Study

The purpose of the present study is to develop a theoretical and empirical model of resource scarcity which explicitly accounts for the environmental effects of resource use. More specifically the objectives are to:

1. Identify and establish the role of a common property environment in the theory of natural resource scarcity.

2. Formulate a theoretical model incorporating the environment as a source of natural resources which has its own "laws of motion" and derive analytical results pertaining to resource scarcity.
3. Modify the previously developed theoretical model incorporating uncertainty with respect to the stock of environmental assets and derive analytical results having bearings on the measures of resource scarcity.
4. Formulate a model for empirical work incorporating environmental costs in order to test the hypothesis of increasing natural resource scarcity.
5. Indicate policy implications of the results derived in (2) through (4) and analyze their effects on social welfare.

CHAPTER II

LITERATURE REVIEW

The question of economic scarcity was first addressed by classical economists such as Malthus and Ricardo in the nineteenth century. Malthus predicted a doomsday for human beings in the face of continuous (exponential) population growth and arithmetic (or less) growth in natural resource output. Ricardo was somewhat more optimistic in visualizing plausible substitution between resources and technological progress. However, inevitable resource scarcity was the eventual conclusion in either case. These conclusions have been reasserted by recent predictions of impending scarcity and even exhaustion of extractive natural resources, such as metals and fuels, in the Club of Rome Study, "The Limits to Growth" (Meadows et al. 1972). This work brought renewed interest in the subject of resource scarcity and its measurement.

Barnett and Morse, in their pioneering work "Scarcity and Growth--The Economics of Natural Resource Availability" (1963), developed theoretical measures of resource scarcity. They then displayed various empirical measures of natural resource scarcity in the United States over the period 1870 through 1957. They distinguished between the physical availability of a natural resource and its economic scarcity with respect to the labor, capital, and sociotechnical knowledge available for putting the resource to work. Following Barnett and Morse, if the size of the resource is small relative to other factors,

then a resource could be said to be becoming scarce. Thus, Barnett and Morse selected Unit Cost (UC) of resources as a measure of scarcity and defined UC as:

$$UC = \frac{a(L_t/L_o) + b(C_t/C_o)}{O_t/O_o} \quad . . . \quad (2.1)$$

where L_t, C_t, O_t are current values of labor, capital, and output, respectively; L_o, C_o, O_o are base values of the same variables; and a and b stand for factor weights. This unit cost of Barnett and Morse seems to reflect the average cost of producing (extracting) a unit of a resource. Their hypothesis was, for an increasing resource scarcity the unit cost would indicate a positive trend. The empirical findings of Barnett and Morse established that during the period between 1870-1957 the unit cost of all extractive products declined monotonically, with the exception of forestry where an increasing trend in the UC was identified. Thus, Barnett and Morse concluded that, overall, the natural resources were being extracted efficiently over this period and there was no evidence of resource scarcity, with the possible exception of forestry.

Brown and Field (1978) criticized the application of unit cost as an index of natural resource scarcity for two reasons. Regarding the conceptual difficulties, Brown and Field stated that an increase in unit extraction cost is associated with a decrease in aggregate per capita output under static conditions. This would result in a decline in measured consumption (welfare) per capita. However, in a dynamic world of rapid technological change, the unit cost of extraction may fail to increase and thereby cannot be an unambiguous measure of scarcity.

Regarding the practical difficulty of its application, Brown and Field noted that as the price of a natural resource increases, the producers of extractive output tend to substitute from that resource into a relatively lower cost factor of production, which is widely known as the "backstop" technology in the literature. Consequently, the increase in the UC depends on the elasticity of substitution of capital and labor for the natural resource. To illustrate this, Brown and Field assumed that the extractive sector processing the natural resource is represented by a "CES" production function, such as:

$$Q = (aL^{-\beta} + bR^{-\beta})^{-1/\beta}, \quad \frac{1}{1+\beta} = \sigma; \ \beta > -1 \qquad \ldots \qquad (2.2)$$

where, L is a composite labor-capital input, R is a natural resource input, σ is the elasticity of substitution, a and b are factor intensities, and β is the substitution parameter.

If "ω" is the price of "L" and "λ" is the price of "R" then the marginal products of the two factors equal their ratio of prices (in perfectly competitive markets) or

$$\frac{\partial Q/\partial L}{\partial Q/\partial R} = \frac{aL^{-\beta-1}}{bR^{-\beta-1}} = \frac{\omega}{\lambda}; \quad \text{or} \quad \frac{R}{L} = \left(\frac{\omega b}{\lambda a}\right)^{\sigma} \qquad \ldots \qquad (2.3)$$

Now, substituting equation (2.3) into (2.2) and expressing equation (2.2) in terms of L/Q gives:

$$\frac{L}{Q} = \left[a + \left[b\left(\frac{\lambda a}{\omega b}\right)^{\beta\sigma}\right]^{1/\beta}\right. \qquad \ldots \qquad (2.4)$$

Differentiating equation (2.4) with respect to λ in order to determine the change in average labor use (UC) with respect to a change in the

price of the natural resource, the following partial and cross partial derivatives are obtained:

$$\frac{\partial(L/Q)}{\partial \lambda} > 0 \quad \text{and} \quad \frac{\partial^2(L/Q)}{\partial\lambda\partial Q} > 0 \qquad \ldots \qquad (2.5)$$

These results indicate a perverse nature of UC since the "unit cost registers the greatest increase (signifying greatest scarcity) in the case where substitution is easiest" (Brown and Field, p. 232). Brown and Field also contended that the Barnett and Morse index involves a high degree of aggregation which may not be practically feasible. While noting that the real price of a natural resource or rental rate is a better measure of resource scarcity compared to the widely cited UC of Barnett and Morse, Brown and Field pointed out that data limitations may exclude the practical applicability of this index. They argued that there is no ideal index of relative scarcity.

In defense of Barnett and Morse, Johnson, Bell, and Bennett (1980) argued that the sign of the partial derivatives in equation (2.5), which Brown and Field claimed to be positive, are not unambiguously so and supported the view that UC is a valid measure of scarcity. They demonstrated that while the sign of the first partial derivative in equation (2.5) is positive, the sign of the second partial derivative is ambiguous. This can be shown by taking partial derivatives of equation (2.4) with respect to λ and Q successively,

$$\frac{\partial 2(L/Q)}{\partial\lambda\partial Q} = a + b\left(\frac{\lambda a}{\omega b}\right)^{\beta\sigma} \frac{1}{\beta-1} b\left(\frac{\lambda a}{\omega b}\right)^{\beta\sigma}\left(\frac{a}{\omega b}\right) x \left\{\left(\frac{1}{\beta} - 1\right)\left[a + b\left(\frac{\lambda a}{\omega b}\right)^{\beta\sigma}\right]^{-1}\right.$$

$$\left.\beta\sigma\left(\frac{\lambda a}{\omega b}\right)^{-1} b\left(\frac{\lambda a}{\omega b}\right)^{\beta} Ln\left(\frac{\lambda a}{\omega b}\right) + \beta\sigma\left(\frac{\lambda a}{\omega b}\right)^{-1} Ln\left(\frac{\lambda a}{\omega b}\right) + \frac{\omega b}{\lambda a}\right\}$$

. . . (2.6)

The sign of equation (2.6) is indeterminate. The actual sign can only be verified empirically. Furthermore, Johnson, Bell, and Bennett (1980) updated the works of Barnett and Morse up until 1970 and found out that the unit extractive costs declined for every major resource category except commercial fishing. They concluded that there had been a general decrease in relative scarcity rather than an increase.

In a note on increasing resource scarcity, V. Kerry Smith (1981) mentioned that any evaluation of scarcity should reflect the supply and demand conditions relevant to that resource. For extractive resources these are reflected by the "in situ" rents. With processing, the prices of the processed resource would be a relevant index. He then pointed out that the Johnson/Bennett (1980) study was misleading in the sense that it gives a false impression that their analysis has put to rest concerns over the importance of resource scarcity. He suggested that additional theoretical issues involving the interaction between extractive natural resources and environmental common property resources remain to be addressed in any evaluation of the significance of increasing limits to natural resource scarcity.

Margaret Slade (1982) demonstrated that the relative price can serve as a measure of resource scarcity since it captures the mechanics of supply and demand. Utilizing optimal control techniques Slade has

pointed out that the rate of change of price is equal to the rate of change of marginal cost due to changes in technology plus the discount rate times rent (pp. 125):

$$\dot{P} = \dot{K} + \rho\hat{\lambda} \qquad \ldots \qquad (2.7)$$

where P stands for time derivative of price, $\dot{K}$ represents the rate of change of marginal cost with respect to time, ρ is the discount rate, and $\hat{\lambda}$ is the shadow price or rent. Without technical change, i.e., $\dot{K} = 0$, $\dot{P} = \rho\hat{\lambda}$. Since $\hat{\lambda}$ is always positive, price increases with time. However, if the rate of technical change is sufficiently large such that K becomes sufficiently negative, then price might fall. If the marginal cost falls with time but at a decreasing rate while λ increases with time, the price path will generally be U-shaped. Using this technique, Slade then estimated the price paths for all the major metals and fuels by fitting linear and quadratic trends for annual time series data for the period 1870 to 1978. The estimated trend coefficients were both positive and negative in sign, indicating that no generalization could be made about natural resource scarcity from a linear model. However, the quadratic trend model indicated U-shaped price paths. Since the prices of all commodities have passed the minimum points on their fitted curves and have begun to rise, nonrenewable natural resources can be said to be becoming scarce.

Although "in situ" rents may be one of the best measures of resource scarcity, no attempt was made to estimate them because of data limitations until the recent work of Halvorsen and Smith (1984). Halvorsen and Smith have shown that the estimates of the prices of the

resources in situ, or rents, could be derived from data for vertically integrated natural resource industries by using "duality theory."

The production function for a vertically integrated natural resource firm is assumed by Halvorsen and Smith to be

$$Q = Q(X, Z, T) \qquad \ldots \qquad (2.8)$$

where Q is the quantity of final outputs, X is a vector of reproducible inputs, Z is the stock of exhaustible natural resource, and T (time) allows for the effects of technological change. Assuming that the quantities of inputs used in extraction are separable from those used in processing, equation (2.8) can be written as:

$$Q = Q[X^P, T, N(X^E, Z, T)] \qquad \ldots \qquad (2.9)$$

where superscripts P and E stand for processing and extraction activities, respectively, and N is the output of the extraction subproduction function, i.e., the quantity of ore actually extracted.

The firm is assumed to maximize wealth from its stock of a natural resource given input and output prices, the technological conditions governing extraction and processing, and laws of motion. The problem before the firm can be framed as:

$$\text{Maximize} \int_0^t e^{-rt}[P_Q Q(X^P, N, T) - \sum_i P_i X_i^P - C(N, P_x^E, Z, T)]dt \qquad \ldots \qquad (2.10)$$

$$\text{Subject to } \dot{Z} = \frac{dZ}{dt} = - Nt \qquad \ldots \qquad (2.11)$$

The current valued Hamiltonian for this problem is

$$H = P_Q Q(X^P, N, T) - \sum_i P_i X_i^P - C(N, P_X^E, Z, T) - \mu N$$

where $i = 1, \ldots, n$ . . . (2.12)

Equation (2.12) can be described as the net surplus or net profit that a firm maximizes at each instant of time. Here "μ" is a costate variable, r is the rate of discount, P_Q is the price of output, P_X is a vector of input prices, and C(.) is the minimal total cost function dual to the extraction subproduction function. Halvorsen and Smith argued that if a competitive market existed for the natural resource in situ, the market price in period "t" would be equal to "μ_t" because this represents the marginal opportunity cost to the firm of one unit of stock.

The procedure for estimating "μ" is based on the reproducible cost function dual to the production function for final output. From the primal problem, the optimal quantities of N and Q are obtained which are used in the dual, i.e., minimizing the total cost of reproducible inputs in each period given Z in order to derive the reproducible cost function. Thus, Halvorsen and Smith have derived the reproducible cost function as

$$CR = CR(Q, P_X, N, Z, T) \quad \ldots \quad (2.13)$$

The desired shadow price of the in situ resource is obtained by taking a partial derivative of this reproducible cost function with respect to the fixed input, N, i.e.,

$$\frac{\partial CR}{\partial N} = -\mu \quad \ldots \quad (2.14)$$

Equation (2.14) is referred to as "Hotelling's Lemma." Thus, Halvorsen and Smith have shown theoretically that estimates of the shadow price of the ore "in situ" can be obtained from the data for a vertically integrated natural resource industry by estimating its reproducible cost function and then differentiating the cost function with respect to the quantity of extraction.

By using a "translog" cost function for empirical estimation, Halvorsen and Smith have obtained the estimated values of the shadow price of the resource "in situ." While these estimated shadow prices of the resource "in situ" indicated a downward trend in natural resource scarcity, the market price of the resource indicated an upward trend. Thus, in their opinion, a true measure of resource scarcity critically depends on the concept of natural resource scarcity relevant in a particular application. The work by Halvorsen and Smith improves upon earlier measures of "in situ" resource prices and provides a sound technique for empirical work.

Hall and Hall (1984), in a recent article, dealt with some basic concepts related to resource scarcity and demonstrated that these earlier works were only special cases of a more general model. In their view, resource scarcity, if judged by the increase in relative prices, might originate from a number of factors, e.g., physical scarcity, international market links, government's regulation, imperfect market mechanism, common property environment (air, water), and so on. Hence, it becomes difficult to define scarcity and then compare the earlier studies with the later ones. The common property environment plays a special role in evaluating resource scarcity with specific reference to

the resource in use. Yet, up until their work, the interrelationship had been only tangentially mentioned. Hall and Hall viewed environment itself as a resource that provides a stream of inputs. However, the environmental sinks which receive wastes are limited in their assimilative capacity. When such capacity is exceeded, it becomes more costly to use the resource. Thus, the stock of environmental services limits the flow of natural resources. This concept of a stock of environmental services and flow or use of natural resources from within helped Hall and Hall in identifying four different types of scarcity.

One such concept is "Malthusian Stock Scarcity" which is applicable in the case of resources with uniform quality having an ultimate limit. Another concept, "Malthusian Flow Scarcity," applies to resources for which, in addition to a binding constraint on the total availability of resource stock, the average extraction costs depend upon the rate of extraction. Both these scarcity concepts apply only to nonrenewable resources.

Similarly, Ricardian Flow and Stock Scarcity can be identified in case of nonrenewable resources. Average costs of extraction depend upon the rate of extraction when the former is considered and average costs depend upon the total amount extracted to date in addition to the rate of extraction when the latter is in perspective. With this distinction, Hall and Hall demonstrated that the "Malthusian Flow Scarcity" is the generalized scarcity concept and all others turn out to be special cases. From the model Hall and Hall used to identify these scarcity concepts:

Malthusian Stock Scarcity becomes: $P_t - C = \lambda_t$ . . . (2.15)

Ricardian Flow Scarcity is: Av. Costs = $ac(\dot{Q}_t)$, $ac_{\dot{Q}} > 0$

$$\ldots \quad (2.16)$$

Ricardian Stock Scarcity is: $P_t = ac + ac_{\dot{Q}}\dot{Q}_t + ac_Q\dot{Q}_{t/r}$

$$\ldots \quad (2.17)$$

and

Malthusian Flow Scarcity is: $P_t = ac + ac_{\dot{Q}}\dot{Q}_t + ac_Q\dot{Q}_{t/r} + \eta_t$

$$\ldots \quad (2.18)$$

where P_t is the price of the resources, $(ac + ac_{\dot{Q}}\dot{Q}_t)$ is defined as the average cost, $ac_Q\dot{Q}_{t/r}$ is the present value of increase in average cost borne in perpetuity, and η_t is user cost. Equation (2.18) captures equations (2.15) through (2.17) as special cases. Equation (2.18) illustrates the concept that price equals average cost plus the present value of the increase in average cost borne in perpetuity plus the user cost, η_t, which is nothing but the opportunity cost of present consumption equal to the value in use of future consumption. The unit cost of Barnett and Morse is equivalent to the second model, i.e., the Ricardian Flow Scarcity. The empirical works of Hall and Hall extends the earlier estimation of resource scarcity and they suggest that scarcity has increased in the 1970s for nonrenewable energy resources and for some renewable resources.

As a related issue, a majority of the literature on resource scarcity is concerned with resource scarcity in a deterministic world. However, the presence of uncertain stock size, uncertain environmental impacts, uncertainty in pricing of resources due to regulated or free market mechanism or international market adjustments, etc., may alter

the results of a deterministic model. While literature dealing with uncertainty and extraction of natural resources has expanded over the past few years, few articles have directly addressed the issue of resource scarcity under uncertainty. A major exception is the work of Devarajan and Fisher (1982).

By explicitly recognizing rents (which is equivalent to marginal discovery costs in a deterministic competitive equilibrium), Devarajan and Fisher developed a simple two-period model of exploration and extraction. They incorporated uncertainty through a stochastic exploration production function and found out that uncertainty does affect the behavior of a risk-averse firm that maximizes the expected utility of profits. Intuitively, the net results of uncertainty is a reduction in the level of an activity such that the marginal benefits exceeds marginal costs. The difference between the two is equivalent to a risk premium.

Contrary to the intuitive conclusion, Devarajan and Fisher found that the uncertain firm may explore to a point where expected marginal costs exceeds rents, the marginal benefit. So, in general, according to Devarajan and Fisher, the expected marginal discovery cost will not be equated to rent. There will, therefore, be significant differences in terms of estimates of resource scarcity in the two cases.

CHAPTER III

THE THEORETICAL MODEL AND ANALYTICAL RESULTS

Section III.1: Theoretical Model

It is often assumed that the competitive firm's objective is to maximize the discounted present value of net surplus, which is obtained by subtracting the production costs of the intermediate extracted input and final output from the value of the final output. This may be achieved by controlling the amount of production and/or controlling the amount of inputs.

Thus, the firm's objective is to:

$$\underset{\{K^E,\ K^P\}}{\text{Max}}\quad V = \int_0^\infty e^{-rt}[Py \cdot Y(N(x,\ e,\ K^E,\ T),\ e,\ K^P,\ T) - W\Sigma_i K_i^P$$

$$- \alpha C(N,\ X,\ e,\ W,\ T)]dt\ ,\ i = 1,\ 2,\ .\ .\ .,\ n \qquad . . . \quad (3.1.1)$$

subject to:

$$\frac{dx}{dt} = f(x) - N(x,\ e,\ K^E,\ T) \qquad . . . \quad (3.1.2)$$

$$\frac{de}{dt} = \gamma e - N(x,\ e,\ K^E,\ T) \qquad . . . \quad (3.1.3)$$

$$X(0) = X_0 > 0 \qquad . . . \quad (3.1.4)$$

$$e(0) = e_0 > 0 \qquad . . . \quad (3.1.5)$$

and for any time interval

$$[a,\ b],\ N(t) \not\equiv 0 \qquad . . . \quad (3.1.6)$$

Also, $0 < \alpha,\ \gamma \leq 1$

Based on the following assumptions:[1]

$C_N > 0$, $C_x < 0$, $C_{xx} < 0$; $C_e < 0$, $C_{ee} < 0$; $C_{Ne} > 0$; $C_W > 0$,

$C_T < 0$; $N_x > 0$, $N_{xx} > 0$; $N_e > 0$, $N_K E_e = 0$, $N_K E_T = 0$, $N_{ee} > 0$,

$N_K^E > 0$, $N_{KK}^{EE} < 0$; $N_T e = 0$, $N_T > 0$; $N_x e$, $N_{XK} E$, $N_{XT} = 0$,

$Y_{ee} > 0$, $Y_N > 0$; $Y_{Ne} > 0$; $Y_N > 0$, $Y_{NN} < 0$,

$Y_e > 0$, $Y_K^P > 0$, $Y_T > 0$, $Y_{TT} < 0$.

Note that a single subscript indicates a first partial derivative and a double subscript indicates a second partial derivative.

The function $Y(\cdot)$ is the production function for the final output which is assumed to be concave with respect to its arguments. The function $N(\cdot)$ is the extraction subproduction function and is treated separate because it is assumed that extraction and processing are two different economic activities. The cost function, $C(\cdot)$, is dual to the production function. It is separable in inputs and output and assumed to be concave with respect to output of the extraction subproduction function. It also is assumed that the hiring prices for inputs are the same for both processing and extraction. The biological growth function of the natural resources is represented by $f(x)$. In the case of an exhaustible resource, this is equal to zero. The stock of environmental asset, or environmental sink (is denoted by e), and suggests that the capacity to absorb waste disposal is assumed to be constant. Furthermore, for the sake of exposition, it is assumed that for production of the final output there is one-to-one discharge of waste in the air.

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[1] See Appendix A for a brief discussion of these partial derivatives.

Variables x, K^E, K^P, and T stand for the stock of the extractible natural resource, the composite capital-labor input used for extraction (K^E) and for processing (K^P), and the technology, respectively. The variable P_y is the product price and W is the hiring price for the composite input. Both are assumed to be exogenously determined. The firm can control either its extraction of the natural resource by controlling K^E and/or processing of the natural resource by controlling K^P. Hence, these are the control variables. The system has two state variables, X and e, and the corresponding costate variables are μ_1 and μ_2. Equations (3.1.2) and (3.1.3) can be described as the laws of motion of this dynamic system. These two equations describe how the stocks of the extractible and the environmental resource change with respect to time due to human actions upon them. Equations (3.1.4) through (3.1.6) can be regarded as initial conditions of this system.

In order to maximize the present value of the functional, the current value Hamiltonian is constructed as follows:

$$H = P_y Y[N(x, e, K^E, T), e, K^P, T] - W\sum_i K_i^P - \alpha C(N, X, e, W, T)$$
$$+ \mu_1[f(x) - N(x, e, K^E, T)] + \mu_2[e - N(x, e, K^E, T)] \quad (3.1.7)$$

Equation (3.1.7) can be interpreted as the current value of the discounted net present benefits or surplus at time t. Here,

$$\mu_1 = \mu_1(t) = \lambda_1 e^{rt} \quad \ldots \quad (3.1.8)$$

and

$$\mu_2 = \mu_2(t) = \lambda_2 e^{rt} \quad \ldots \quad (3.1.9)$$

are the current value costate variables which yield the shadow prices of the extractible natural resource and the environmental resource at time t, respectively. Note that in equations (3.1.8) and (3.1.9), μ_1 and μ_2 are the present value co-state variables corresponding to the state variables x and e.

In order to show that the first-order conditions are necessary and sufficient, assume that the current value Hamiltonian function, $H(K^E, K^P, x, e)$, is concave with its arguments.

The Hessian matrix of the Hamiltonian function is:

$$\begin{bmatrix} H^{EE}_{KK} & H^{EP}_{KK} & H^{E}_{Kx} & H^{E}_{Ke} \\ H^{PE}_{KK} & H^{PP}_{KK} & H^{P}_{Kx} & H^{P}_{Ke} \\ H_{xK^E} & H_{xK^P} & H_{xx} & H_{xe} \\ H_{eK^E} & H_{eK^P} & H_{ex} & H_{ee} \end{bmatrix} \qquad \ldots(3.1.10)$$

If it can be shown that $H^{EE}_{KK} < 0$ and that the following quadratic form is greater than or equal to zero, i.e.,

$$H^{EE}_{KK}H^{PP}_{KK}H_{xx}H_{ee} - H^{2}_{xe}H^{EE}_{KK}H^{PP}_{KK} \geq 0 \qquad \ldots(3.1.11)$$

then the necessary conditions are sufficient for this problem (Kamein and Schwartz 1981, p. 165). This is shown in Appendix B.

The first-order necessary conditions of this maximization problem are:

i) $\frac{\partial H}{\partial K^E} = P_y Y_N N_K E - \alpha C_N N_K E - \mu_1 N_K^E - \mu_2 N_K^E = 0$

or

$\mu_1 N_K^E = P_y Y_N N_K^E - \alpha C_N N_K^E - \mu_2 N_K^E$

$\mu_1 = P_y Y_N - \alpha C_N - \mu_2$ . . .(3.1.12)

ii) $\frac{\partial H}{\partial K^P} = P_y Y_K^P - W = 0$

or

$P_y Y_K^P = W$ . . . (3.1.13)

These conditions are derived by taking the first derivative of the Hamiltonian function with respect to the control variables, K^E and K^P, respectively.

In order to complete the necessary first-order conditions of this maximization problem, the first derivative of the shadow prices, μ_1 and μ_2 (co-state variables) must be taken with respect to time. These are obtained in equations (3.1.14) and (3.1.15).

iii) $\frac{d\mu_1}{dt} = r\mu_1 - H_x = r\mu_1 - [P_y Y_N N_x - \alpha(C_N N_x + C_x) + \mu_1(f'(x) - N_x)$

$- \mu_2 N_x]$

$= r\mu_1 + \alpha(C_x + C_N N_x) - N_x(P_y Y_N - \mu_1 - \mu_2) - \mu_1 f'(x)$

. . . (3.1.14)

iv) $$\frac{d\mu_2}{dt} = r\mu_2 - H_e = r\mu_2 - [P_yY_NN_e + Y_e - \alpha(C_NN_e + C_e) - \mu_1N_e + \mu_2\gamma$$

$$- \mu_2N_e] = r\mu_2 - \mu_2\gamma + \alpha(C_e + C_NN_e) - N_e(P_yY_N - \mu_1 - \mu_2) - Y_e$$

$$= \mu_2(r - \gamma) - Y_e + \alpha(C_e + C_NN_e) - N_e(P_yY_N - \mu_1 - \mu_2) \qquad \ldots (3.1.15)$$

Equations (3.1.12) through (3.1.15) describe the conditions necessary for the Hamiltonian function maximization. From equation (3.1.12), the following equation is obtained:

$$\mu_2 = P_yY_N - \alpha C_N - \mu_1 \qquad \ldots (3.1.16)$$

By substituting equation (3.1.16) into equation (3.1.14), equation (3.1.17) is obtained:

$$\frac{d\mu_1}{dt} = r\mu_1 + \alpha(C_x + C_NN_x) - N_x(P_yY_N - \mu_1 - P_yY_N + \alpha C_N + \mu_1)$$

$$- \mu_1f'(x) = r\mu_1 + \alpha C_x - \mu_1f'(x) \qquad \ldots (3.1.17)$$

Similarly, by substituting μ_1 from equation (3.1.12) in (3.1.15), we derive the following:

$$\frac{d\mu_2}{dt} = \mu_2(r - \gamma) - Y_e + \alpha C_e \qquad \ldots (3.1.18)$$

Thus, the following system of differential equations emerge:

$$(A) < \begin{cases} \dfrac{dx}{dt} = f(x) - N(x, e, K^E, T) & \ldots (3.1.19) \\[2ex] \dfrac{d\mu_1}{dt} = r\mu_1 + \alpha C_x - \mu_1f'(x) & \ldots (3.1.20) \end{cases}$$

$$\frac{de}{dt} = \gamma e - N(x, e, K^E, T) \qquad \ldots (3.1.21)$$

(B) <

$$\frac{d\mu_2}{dt} = \mu_2(r - \gamma) - Y_e + \alpha C_e \qquad \ldots (3.1.22)$$

and the initial conditions contained in equations 3.1.23 through 3.1.26:

$$X(0) = X_0 \qquad \ldots (3.1.23)$$

$$\mu_1(0) = \mu_1 \qquad \ldots (3.1.24)$$

$$e(0) = e_0 \qquad \ldots (3.1.25)$$

$$\mu_2(0) = P_y Y_N - \alpha C_N - \mu_1 \qquad \ldots (3.1.26)$$

by equation (3.1.12).

The pair of differential equations in (A) describe the time paths of the stock of the extractible natural resource and its shadow price. Similarly, the pair of differential equations in (B) depict the time paths of the stock of the environmental asset and its shadow price. Before further analysis of the time paths is undertaken, certain key equations need be interpreted. It is also necessary to establish the sufficiency conditions for this maximization problem.[2]

From the necessary conditions of the Hamiltonian, equation (3.1.12) has been obtained.

In that equation, μ_1 and μ_2 are the shadow prices of the stocks of extractible natural resource and the environmental resource, respectively. The shadow price of the stock of the extractible resource is equated to the value of the marginal product (P_y is the product price and Y_N is the marginal physical productivity of the input) minus the marginal cost of extraction of the in situ resource (αC_N) and the shadow

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[2] See Appendix B.

price of the environmental asset. While it has been argued that the shadow price or the marginal user cost of a factor (e.g., coal) would be the same as its price if there exists perfect competition in the factor market, this may not necessarily be true in a natural resource market. Rewriting equation (3.1.12) yields

$$P_y Y_N = \alpha C_N + \mu_1 + \mu_2 \qquad \text{. . (3.1.12a)}$$

The market price of the resource (since the price of the factor, coal, is equated to its value of the marginal product) is equated to the sum of the marginal cost of current extraction and the marginal loss in profit due to removal of the extractible and environmental resources from their stocks in the future. The results in equation (3.1.12a) would have been different had it been assumed that the firm chooses K^E and K^P to maximize current net return. In that case, the resulting necessary condition would have been conventional $P_y Y_N = \alpha C_N$, or the price of the resource equals its marginal cost.

The divergence of price from marginal cost in a natural resource market does not necessarily arise from any market imperfection in a static sense. It may be due to the absence of evaluations of the transactions in all futures market in the construction of the discounted present value of net surplus, which is to be maximized. In other words, if the social opportunity costs of depleting (augmenting) a unit of stock of reserve on the stream of net surpluses could have been evaluated over an infinite time horizon at each instant of time under profit-maximizing and perfectly competitive conditions then and only then would the divergence between the price and the marginal cost be eliminated.

Conventional results, i.e., price equals marginal cost, which assumes the existence of a competitive market, are obtained in equation (3.1.12a) if μ_1 and μ_2 are zero. However, μ_1 and μ_2 could be treated as zero only under the strong assumption that there is no stock effect, i.e., the marginal cost of extraction remains constant whatever be the level of the stocks of the extractible and the environmental resources under a given technology. This could occur only under perfect foresight (ultrarationality) with respect to all future cost adjustments relating to stock effect on the part of the firm. The standard equilibrium condition in the resource market is that the price in any period equals marginal cost plus rent (shadow price) (see Fisher 1979).

In the model under study, since an attempt has been made to capture the jointness between an extractible and an environmental resource, the true marginal user costs (shadow price) of the extractible resource would be different from those obtained from traditional models. Here, in equation (3.1.12) the shadow price of an extractible resource (μ_1) is equated to the differences between the price of the resource ($P_y Y_N$) and the marginal cost of extraction (αC_N) and the shadow price of the environmental resource (μ_2). However, the true shadow price of the extractible resource is just not μ_1, it is rather $\mu_1 + \mu_2$. In other words, the marginal loss in profit from not extracting a unit of the extractible resource and at the same time not utilizing a unit of the environmental resource is higher than that of just not extracting. This phenomenon may further be explained from the profit-maximizing firm's point of view in the following way. Assume that due to some regulatory reason (nonmarket forces) the users of an extractible resource (e.g.,

coal) are completely barred from discharging the waste products into the air which has so far been treated as a free (common property) good. This would immediately increase the cost of use of that resource (coal) in the sense that the firms have to internalize all the waste products. This in turn implies that there will be additional marginal loss in profit due to not being able to use the so far previously "free" resource any longer. This, however, suggests that even if the firms are allowed to make use of the environmental resource at a positive cost, the marginal loss in profit will be higher compared to a situation where there is no cost for utilizing the environmental resource. Thus, the shadow price of an extractible resource in the light of this modified explanation of natural resource use will be higher along an optimal extraction path.

The hiring price for the composite capital-labor input is equated to the value of the marginal physical product as noted in equation (3.1.13). This result is consistent with derivations obtained in neo-classical production/growth economics. The intertemporal behavior of the shadow prices are described in equations (3.1.14) and (3.1.15) which are further modified to yield equations (3.1.20) and (3.1.22), respectively. Equation (3.1.20) suggests that the time path of the shadow price of the stock of the extractible resource depends on the interest rate used for discounting the present value of the net benefits, the marginal cost of extraction due to change in the stock of the resource, and the biological growth of the natural resource. In the case of an exhaustible resource, $f'(x)$ does not exist, and if it is assumed that

there is no stock effect, i.e., $c_x = 0$, then the rate of growth of shadow price is given by the following:

$$\frac{d\mu_1}{dt} \cdot \frac{1}{\mu_1} = r \qquad \ldots (3.1.20a)$$

This is known as "Hotelling's rule" within economics literature. Equation (3.1.22) can be similarly interpreted.

In order for a first-order condition to be both necessary and sufficient, it must be shown that the conditions specified in equation (3.1.11) are satisfied. This is shown in Appendix B.

In order to evaluate the time paths in state and co-state space, further manipulation of the set of equations in (A) and (B) is needed. The sets of differential equations must be expressed in terms of x, μ_1 and e, μ_2.

Rewriting the equations in (A) and (B) help illustrate the required manipulation. In equations (3.1.19) and (3.1.20), the extraction subproduction function $[N(X, K^E, e, T)]$ needs to be modified using equation (3.1.12); this can be done as follows:

$$\mu_1 = P_y Y_N - \alpha C_N - \mu_2 \quad \text{or} \quad \mu_1 = f(N, \mu_2) \qquad \ldots (3.1.27)$$

Now, replacing N in equation (3.1.27) by N(x) yields:

$$\mu_1 = f[N(x), \mu_2] \qquad \ldots (3.1.28)$$

which can further be written as

$\mu_1 = f(N, x, \mu_2)$ for given levels of K^E, e, and T.

Now again for a given $\bar{\mu}_2$:

$$\mu_1 = f(N, x) \qquad \ldots (3.1.29)$$

assuming there exists one-to-one mapping between the functional relationship described in equation (3.1.29), or,

$$N = g(\mu_1, x) \qquad \ldots (3.1.30)$$

where $g_{\mu_1} > 0$ and $g_x > 0$.

Thus, substituting equation (3.1.30) in equation (3.1.19) yields:

$$(A') \left\{ \begin{array}{ll} \dfrac{dx}{dt} = f(x) - g(x, \mu_1) & \ldots (3.1.19a) \\ \dfrac{d\mu_1}{dt} = r\mu_1 + \alpha C_x - \mu_1 f'(x) & \ldots (3.1.20) \end{array} \right.$$

Similarly,

$$(B') \left\{ \begin{array}{ll} \dfrac{de}{dt} = \gamma e - h(e, \mu_2) & \ldots (3.1.21a) \\ \dfrac{d\mu_2}{dt} = (r - \gamma)\mu_2 - Y_e + \alpha C_e & \ldots (3.1.22) \end{array} \right.$$

where $h_e > 0$ and $h_{\mu_2} > 0$ and $N = h(\mu_2, e)$. From (A') and (B'), the time paths of the stock of resources and their shadow prices can be derived. For this, the signs of such time paths in the state/co-state spaces need first be evaluated at a point or at steady states. Note that at steady state, $\frac{dx}{dt}$, $\frac{d\mu_1}{dt}$, $\frac{de}{dt}$, and $\frac{d\mu_2}{dt}$ are all equal to zero. This can be accomplished using the implicit function rule, i.e., assuming that there exists an explicit solution to these implicit functions and all the first partial derivatives exist.

From (A')

$$\left.\frac{d\mu_1}{dx}\right|_{X(t) = \overset{*}{X}(t)} = -\frac{f'(x) - g_x}{-g_{\mu_1}} \gtrless 0 \qquad \ldots (3.1.31)$$

$$\text{or, } \frac{dx}{dt} = 0$$

For an exhaustible resource, $f'(x) \neq 0$ since $f(x)$ is zero (no biological growth). As g_x and g_{μ_1} are positive by assumption, the sign of equation (3.1.31) is negative. However, for a renewable resource, the sign could be $\gtrless 0$, depending on the sign of numerator and denominator.

Also,

$$\left.\frac{d\mu_1}{dx}\right|_{\mu_1(t) = \overset{*}{\mu}_1(t)} = -\frac{\alpha C_{xx} - \mu_1 f''(x)}{r - f'(x)} \qquad \ldots (3.1.32)$$

$$\text{or, } \frac{d\mu_1}{dt} = 0$$

Again, for an exhaustible resource, $f''(x) \neq 0$ and since $C_{xx} < 0$ and $r > 0$, by assumption, the sign becomes positive.

However, for a renewable resource, the following possibilities exist:

(i) A positive numerator (since $f''(x) < 0$, assuming that the biological growth function is a concave one); and a positive denominator would yield a negative sign.

(ii) A positive numerator and a negative denominator would yield the sign of equation (3.1.32) positive.

(iii) A positive numerator but zero denominator would result in a sign (slope) that is infinite.

(iv) A negative numerator but a positive denominator gives rise to a sign that is a positive one.

(v) A negative numerator and a negative denominator would yield a negative sign.

(vi) A zero numerator (whatever be the sign of the denominator) would yield the net result as zero. Thus,

$$\frac{d\mu_1}{dx} = [\cdot] \quad 0 \qquad \text{otherwise } [\cdot] \infty \qquad \ldots (3.1.33)$$

$$\frac{d\mu_1}{dt} = 0$$

where $[\cdot] = -\dfrac{\alpha C_{xx} - \mu_1 f''(x)}{r - f'(x)}$.

In order to determine whether there exists at least one equilibrium point for this problem, various shapes of the time paths in the $x - \mu_1$ spaces (denoted by equations (3.1.31) and (3.1.33)) are evaluated in the following set of Figures (3.1 through 3.5).

<u>Case I: Exhaustible Resource</u>

$$\left.\frac{d\mu_1}{dx}\right|_{\substack{X(t) = \overset{*}{X}(t) \\ \text{or, } \frac{dx}{dt} = 0}} < 0$$

and

$$\left.\frac{d\mu_1}{dx}\right|_{\substack{\mu_1(t) = \overset{*}{\mu_1}(t) \\ \text{or, } \frac{d\mu_1}{dt} = 0}} > 0$$

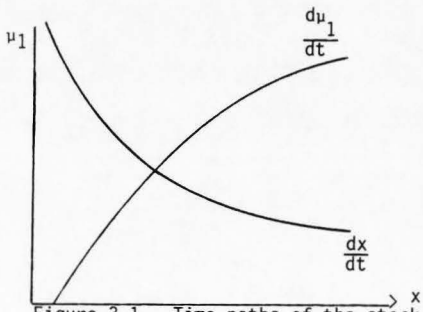

Figure 3.1. Time paths of the stock of exhaustible resource and its shadow price.

Figure 3.1 illustrates one such possibility in case of an exhaustible resource. Here, the slopes of the time paths in the $(x - \mu_1)$ spaces have been derived by assuming that there exists optimal solutions. In other words, if the competitive firm can control its use of composite capital-labor input either in the process of extraction and/or in the processing stage, then there may exist an optimal solution indicated by the intersection of the two curves $\frac{dx}{dt}$ (time rate of change in resource stock) and $\frac{d\mu_1}{dt}$ (time rate of change in resource shadow price). Note that the two curves $\frac{dx}{dt}$ and $\frac{d\mu_1}{dt}$ indicate that any point along these two curves would represent a solution level of stock of resource and its shadow price, respectively. In case of an exhaustible resource, since there is no net addition to the stock, depletion occurs continuously. Hence, it is logically consistent to have a negatively sloped time path of the stock of extractible resource. By the same logic, the slope of the time path of shadow price should be positive since it is assumed that as stock of the resource gets depleted, the cost of extraction increases (i.e., $C_{xx} < 0$). Thus, the opposite signs of the slopes of the two time paths are logically consistent. In case of a nonrenewable resource, steady state solutions might never be reached, since that would mean no extraction. If the intersection of the curves $\frac{dx}{dt}$ and $\frac{d\mu_1}{dt}$ is interpreted as a steady state solution, then that solution can only be reached at infinity, when all the stocks are exhausted or no extraction occurs.

Case II: Renewable Resource

(i) $\left.\frac{d\mu_1}{dx}\right|_{\frac{dx}{dt}=0} > 0$

and

$\left.\frac{d\mu_1}{dx}\right|_{\frac{d\mu_1}{dt}=0} > 0$

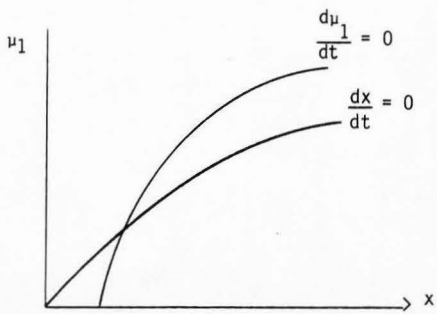

Figure 3.2. Time paths of the stock of renewable resources and its shadow price when both are positively sloped.

Since various possibilities for the shapes of these two time paths exist, it is necessary to evaluate which of such cases yield equilibrium solutions. While this is done more rigorously in the next section under stability analysis, an examination of equations (3.1.31) and (3.1.33) would indicate the economic significance of the various shapes in Figures 3.2 through 3.5.

In Figure 3.2, both the time paths are positively sloped. This might occur if the biological growth of the stock, $f'(x)$, is greater than the rate of extraction, g_x (see equation (3.1.31)), yielding a positively sloped time path of the stock of resource. Under the assumption of a concave biological growth function (wherein the rate of change of growth declines implying $f''(x)$ less than zero) and also under an assumption that the interest rate being greater than the biological growth rate (e.g., r being 5 percent and growth rate being 3 percent),

growth rate (e.g., r being 5 percent and growth rate being 3 percent), the shape of the time path of shadow price would be a positively sloped one. The intersection, such as depicted here, is possible under the assumption that the absolute value of the slope of $\frac{dx}{dt} = 0$ is less than that of $\frac{d\mu_1}{dt} = 0$.

(ii) $\left.\frac{d\mu_1}{dx}\right|_{\frac{dx}{dt} = 0} < 0$

and

$\left.\frac{d\mu_1}{dx}\right|_{\frac{d\mu_1}{dt} = 0} < 0$

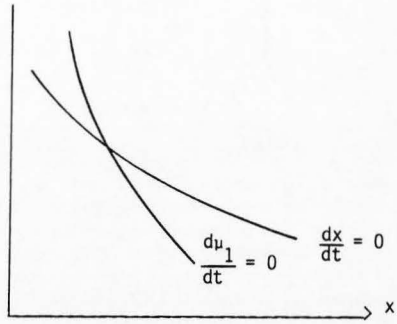

Figure 3.3. Time paths of the stock of renewable resource and its shadow price when both are negatively sloped.

Figure 3.3 depicts both time paths, which are negatively sloped. However, the intersection shown here is possible under the assumption that the absolute value of the slope of the time path of the stock of resource is less than that of the time path of its shadow price. In terms of economic logic, this means that as the extraction rate outweighs the biological growth (i.e., $g_x > f'(x)$), a negatively sloped $\frac{dx}{dt} = 0$ will result. The time path of shadow price which depends on the rate of change of extraction cost with respect to the level of stock (C_{xx}) outweighs the rate of change of biological growth ($f''(x)$). If it

also is assumed that the interest rate is less than the biological growth rate, then a negatively sloped time path of the shadow price results. In terms of absolute magnitude of these two negatively sloped paths, it may be assumed that the rate of change in the extraction cost (net of rate of change of biological growth) is greater than the rate of extraction (net of biological growth). Thus, the absolute value of $\frac{d\mu_1}{dt} = 0$ is greater than that of $\frac{dx}{dt} = 0$; which guarantees at least an intersection of these two curves in the $x - \mu_1$ spaces.

(iii) $\left.\frac{d\mu_1}{dx}\right|_{\frac{dx}{dt} = 0} = 0$

and

$\left.\frac{d\mu_1}{dx}\right|_{\frac{d\mu_1}{dt} = 0} = 0$

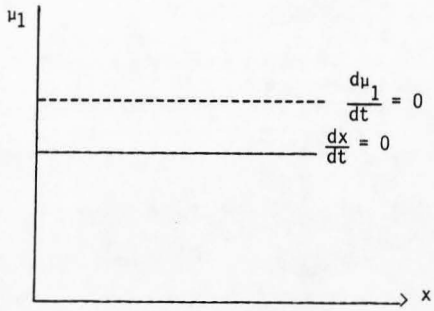

Figure 3.4. Time paths of the stock of renewable resource and its shadow price when both have zero slopes.

Figure 3.4 describes that the slope of these two time paths are zero. These two time paths also may overlap with each other yielding infinite solutions. However, that case is not analyzed here.

Also,

(iv) $\left.\dfrac{d\mu_1}{dx}\right|_{\frac{dx}{dt}=0} > 0$

and

$\left.\dfrac{d\mu_1}{dx}\right|_{\frac{d\mu_1}{dt}=0} \infty$

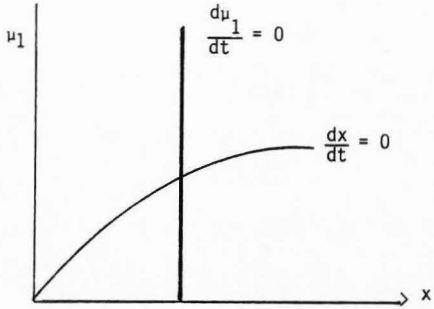

Figure 3.5. A positively sloped time path of the stock of renewable resource but infinitely sloped time path of the shadow price.

The graph in Figure 3.5 illustrates that the slope of the time path of the shadow price may be infinite whereas that of the stock of the resource may be a positive one. There are many other possibilities which can be obtained by any possible combination of the cases above.

Similarly, the shapes of the time paths of the stock of environmental resource and its shadow price need to be evaluated in the state/co-state space. This is accomplished using the implicit function rule.

From (B')

$$\left.\frac{d\mu_2}{de}\right|_{\substack{e(t)=e(t)^* \\ \text{or, } \frac{de}{dt}=0}} = -\frac{\gamma - h_e}{-h_{\mu_2}} \qquad \ldots (3.1.34)$$

Since γ, h_e, and h_{μ_2} all are positive, the sign of equation (3.1.34) can be positive, negative, or equal to zero depending upon the absolute values of the numerator.

Also,

$$\left.\frac{d\mu_2}{de}\right|_{\substack{\mu_2 = \overset{*}{\mu}_2(t) \\ \text{or, } \frac{d\mu_2}{dt} = 0}} = -\frac{-Y_{ee} + \alpha C_{ee}}{r - \gamma} \qquad \ldots (3.1.35)$$

The sign in equation (3.1.35) could be $\gtrless 0$ or ∞, depending upon the sign of the denominator because the numerator is always positive. The time paths in the $(e - \mu_2)$ state/co-state space are drawn below.

Case III: Environmental Resource

The slopes of the time paths of the stock of environmental resource, and its shadow price are depicted in Figures 3.6 through 3.8.

(i) $$\left.\frac{d\mu_2}{de}\right|_{\frac{de}{dt} = 0} > 0$$

and

$$\left.\frac{d\mu_2}{de}\right|_{\frac{d\mu_2}{dt} = 0} > 0$$

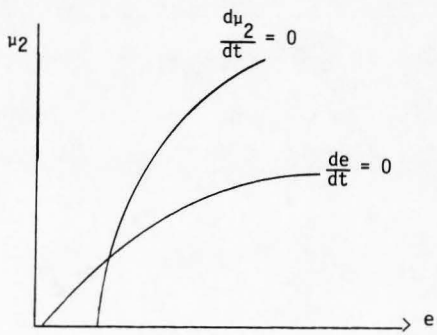

Figure 3.6. The positively sloped time paths of the stock of environmental resource and its shadow price.

(ii) $\left.\frac{d\mu_2}{de}\right|_{\frac{de}{dt}=0} < 0$

and

$\left.\frac{d\mu_2}{de}\right|_{\frac{d\mu_2}{dt}=0} < 0$

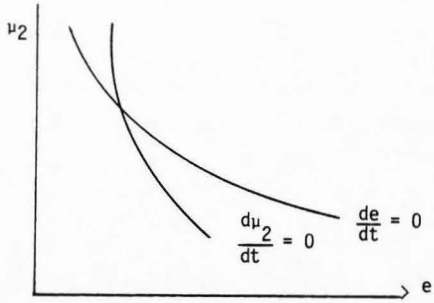

Figure 3.7. The time paths of the stock of environmental resource and its shadow price when both are negatively sloped.

(iii) $\left.\frac{d\mu_2}{de}\right|_{\frac{de}{dt}=0} > 0$

and

$\left.\frac{d\mu_2}{de}\right|_{\frac{d\mu_2}{dt}=0} \infty$

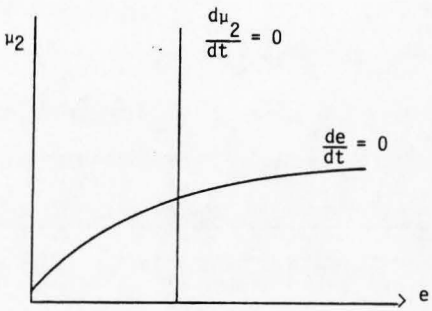

Figure 3.8. The positively and infinitely sloped time paths of the stock of environmental resource and its shadow price, respectively.

Here again various other possibilities exist depending upon the combinations in signs. However, it is interesting to note that in the case of the environmental resource there is no distinction between the nonrenewable and the renewable resources. This is primarily due to the

assumption that it does not matter whether the extractible resource is renewable or nonrenewable, the environmental resource is affected.

Thus, the two sets of curves in Figures 3.1 through 3.8 depict the steady state levels of the stock of the extractible and the environmental resources and their shadow prices. In order to determine whether there exists stable solution or optimal steady state time paths for this dynamic problem, the phase diagram technique of Takayama (1974) is used in the following section.

Section III.2: Stability Analysis

In order to investigate whether there exists at least one equilibrium point for this problem one has to examine the dynamic behavior of the system that leads to equilibrium values of the variables in the limit as time becomes infinite, i.e., if $\lim_{t \to \infty} x_i(t) = x_i$, where x_i could be a steady state level of the stock of extractible natural resource, regardless of the initial conditions (Samuelson 1947). Alternatively, it can be stated that an equilibrium is stable if a dispacement from equilibrium is followed by a return to it through time. The stability of an equilibrium is particularly important if comparative static analysis is to be made. In other words, if one has to predict how the behavioral postulates lead to certain analytical or even empirical conclusions, then the equilibrium conditions (e.g., first-order maximization conditions) must lead the system to some converging values of the variables under examination. In essence, assuming there exists at least an equilibrium, it is customary to show that the equilibrium is stable either globally or, at least, locally.

To evaluate the phase diagram and stability, the signs (the direction of the movements) in the half spaces must be examined. Moreover, the same exercise needs to be repeated for all possible cases in order to determine the uniqueness of a solution(s).

Case I: Nonrenewable Resource

Given,

$$\left.\frac{d\mu_1}{dx}\right|_{x(t) = x^*(t)} < 0 \quad \text{and} \quad \left.\frac{d\mu_1}{dx}\right|_{\mu_1(t) = \mu_1(t)} > 0$$

or the shapes of the time paths in $x - \mu_1$ spaces, the signs in the half spaces can be identified as follows:

$$d\left(\frac{d\mu_1}{dt}\right) = \frac{\partial\left(\frac{d\mu_1}{dt}\right)}{\partial\mu_1} d\mu_1 = [r - f'(x)]d\mu_1 \qquad \ldots (3.2.1)$$

The sign of equation (3.2.1) is positive since $f'(x)$ does not exist for an exhaustible resource and r and $d\mu_1 > 0$. Similarly,

$$d\left(\frac{dx}{dt}\right) = \frac{\partial\left(\frac{dx}{dt}\right)}{\partial x} dx = [f'(x) - g_x]dx \qquad \ldots (3.2.2)$$

The sign of equation (3.2.2) is negative for a positive dx. With the help of these results, phase diagrams can be drawn by dividing the $x - \mu_1$ space into four half spaces (quadrants). Then, the directions (vector movements) in each of such quadrants can be identified to determine whether the optimal steady state time path is converging or diverging. For this, the sign of dx, holding $d\mu_1$ constant, above the curve $\frac{dx}{dt}$ is assumed to be > 0 and below < 0. Similarly, the sign of $d\mu_1$, holding

dx constant, above $\frac{d\mu_1}{dt}$ is assumed to be > 0 and below < 0. It is shown in Figure 3.9 that there is some force that could lead the system to have a convergent stable solution. In other words, there is an optimal stable solution in case of an exhaustible resource which is depicted in the time path S^eS^e. Note that this stable solution is not a steady state solution, since economically that would be meaningless because at steady state there will be no extraction.

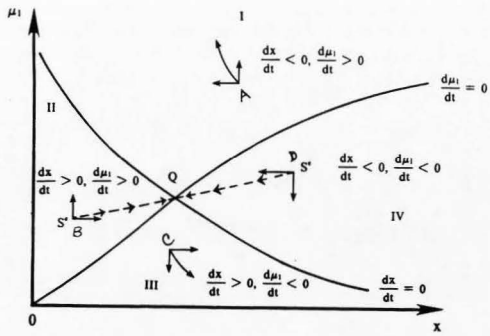

Figure 3.9. A phase diagram of the stock of exhaustible resource and its shadow price when $d(\frac{dx}{dt}) < 0$ and $d(\frac{d\mu_1}{dt}) > 0$.

The mechanism of the phase diagram in Figure 3.9 can be illustrated as follows. The curves $\frac{dx}{dt}$ and $\frac{d\mu_1}{dt}$ indicate that there may be any point along these two curves which would correspond to a solution level of the stock of the extractible resource and its shadow price. Alternatively, any point along $\frac{dx}{dt}$ indicates that the excess demand or supply

of the stock of resource is zero and same for its shadow price along $\frac{d\mu_1}{dt}$. Thus, any point above $\frac{dx}{dt}$ means that there exists a positive excess demand such that its price (shadow price) must increase (Walrasian sense) leading to an adjustment (fall) in the excess demand. Note that in deriving the signs in each quadrant, i.e., in order to find out the adjustment process, it is customary to hold one of the variables constant and evaluate the other. Similarly, any point above $\frac{d\mu_1}{dt}$ would indicate that there is an upward pressure on the shadow price such that there will be a quantity adjustment (Marshallian sense). Thus, combining these two forces, the four quadrants are identified such that in any quadrant, the joint movement (vector movement) is examined. Consider a point A in quadrant I of Figure 3.9. Here the dynamic forces are such that the vector movement is divergent from the equilibrium point Q. At A, the adjustment process is such that there is an upward pressure on the shadow price and at the same time, a decreasing pressure operates on the use of the stock of resource. However, the pressure in the shadow price is so much that the use rate declines below an optimal level and, thus, the system cannot lead itself to the optimal trajectory that reaches the equilibrium point at Q. Any point in quadrant I would lead the system away from a stable equilibrium. Consider now a point, such as B in quadrant II. Point 'B' is below $\frac{dx}{dt}$ curve and above $\frac{d\mu_1}{dt}$. Hence, the adjustment process is such that there will be an increase in the use rate of the stock of extractible resource since at 'B' the excess demand is negative and at the same time there is upward pressure on the shadow price. However, the pressure on the use rate must outweigh the pressure on the shadow price, such that the vector movement leads the system to

converge to Q. However, any point above Q in quadrant II would not lead to convergence. Thus, it is a matter of chance that the system would start at B. Similarly, if the system starts at point D in quadrant IV, then the vector movement leads to convergence. However, any parallel point below Q in quadrant IV would not lead to convergence. Point 'D' in quadrant IV indicates that there is excess demand and so there will be downward pressure in the use rate of the stock of extractible resource. In terms of the pressure on the shadow price, there also is a downward thrust such that the vector movement leads the system to converge to Q. Thus, S^eS^e is the optimal trajectory in this $x - \mu_1$ phase space, along which there exist dynamic forces such that the steady state equilibrium 'Q' is attained. Note that 'Q' is not globally stable since the dynamic forces in quadrants I and III lead the system totally away from the equilibrium point. Also, in quadrants I and IV, the attainment of the optimal phase path (S^eS^e) is conditional upon the initial starts (i.e., B and D). So, one can say that the conditions (signs) in equations (3.2.1) and (3.2.2) are sufficient for the system to have local stability.

Case II: Renewable Resource

As was mentioned in the earlier section, there are several possibilities regarding the shapes of the two time paths on the basis of various combinations; one such case is explored here and the rest are pursued in Appendix C.

(i) $\left.\frac{d\mu_1}{dx}\right|_{\frac{dx}{dt}=0} > 0$ and $\left.\frac{d\mu_1}{dx}\right|_{\frac{d\mu_1}{dt}=0} > 0$

The sign in the half spaces are

$$d\left(\frac{d\mu_1}{dt}\right) = \frac{\partial\left(\frac{d\mu_1}{dt}\right)}{\partial\mu_1} d\mu_1 = [r - f'(x)]d\mu_1 \qquad \ldots (3.2.3)$$

For $d\mu_1 > 0$ the sign of equation (3.2.3) can be $\gtrless 0$. Similarly,

$$d\left(\frac{dx}{dt}\right) = \frac{\partial\left(\frac{dx}{dt}\right)}{\partial x} dx = [f'(x) - g_x]dx \qquad \ldots (3.2.4)$$

which again can be positive, negative, or zero. One of these cases is examined below, while others are examined in Appendix C.

Figures 3.10 and then C.1 through C.5 describe phase diagrams of the stock of renewable resource and its shadow price under different sets of conditions obtained from equations (3.2.3) and (3.2.4).

A phase diagram is drawn in Figure 3.10 for the following conditions. It is shown that there does not exist any optimal steady state time path.

(i-a) $d\left(\frac{d\mu_1}{dt}\right) > 0$ and $d\left(\frac{dx}{dt}\right) > 0$

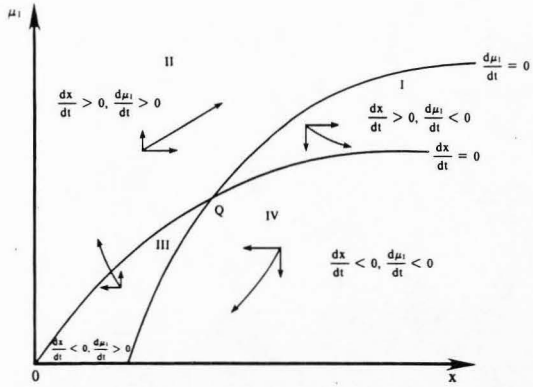

Figure 3.10. A phase diagram of the stock of extractible resource and its shadow price when $d(\frac{dx}{dt}) > 0$ and $d(\frac{d\mu_1}{dt}) > 0$.

Note that Figures 3.10 and C.1 through C.5 in Appendix C examine under which condition(s) there exists an optimal steady state solution. The set of conditions necessary to have at least local stability in the renewable resource case are determined.

Other possibilities with respect to the slopes of the time paths need also to be examined. However, only two cases [(i-b) and (i-e)], for which it has been found that there exists an optimal steady state solution, will be analyzed under each other possible shapes of these two time paths. These are actually examined in Appendix D.

The phase diagrams in Figures 3.10 and then C.1 (Appendix C) through D.4 (Appendix D) help in identifying the conditions under which there exists an optimal steady state solution. It may be pointed out that for the other cases as well, the conditions in (b) and/or (e) would

guarantee optimal steady state solutions. The necessary and sufficient condition for this dynamic problem to have an optimal (steady state) solution is $[r - f'(x)]$ and $[f'(x) - g_x]$ having the opposite sign.

The economic interpretation of this necessary and sufficient condition is given below. The interest rate, used in this model to discount the stream of future net benefits, is represented by r. The biological growth rate is captured in $f'(x)$. So, $[r - f'(x)]$ can be interpreted as the net discount rate (net of biological growth of the stock of resource). Similarly, $[f'(x) - g_x]$ can be viewed as the net biological growth rate (net of extraction). If $[f'(x) - g_x]$ is positive, there is net addition to the stock, which is equivalent to a negative net discount rate. If the stock (net) increases, then its shadow price decreases and conversely.

In order to investigate whether there exists an optimal steady state time path for the environmental resource and its shadow price, the same techniques as used above again would be employed. To draw the phase diagrams in the state/co-state $(e - \mu_2)$ space, the signs in the half spaces need to be evaluated.

From Case III above, the shapes of the time paths are obtained, with the conditions rewritten.

<u>Case III: Environmental Resource and Stability</u>

(i) Given

$$\left.\frac{d\mu_2}{de}\right|_{\frac{de}{dt} = 0} > 0 \quad \text{and} \quad \left.\frac{d\mu_2}{de}\right|_{\frac{d\mu_2}{dt} = 0} > 0$$

The sign in the half spaces are obtained as follows:

$$d\left(\frac{de}{dt}\right) = \frac{\partial\left(\frac{de}{dt}\right)}{\partial e} de = \gamma - h_e \qquad \ldots (3.2.5)$$

The sign of equation (3.2.5) can be positive, negative, or zero. Similarly,

$$d\left(\frac{d\mu_2}{dt}\right) = \frac{\partial\left(\frac{d\mu_2}{dt}\right)}{\partial \mu_2} d\mu_2 = (r - \gamma) d\mu_2 \qquad \ldots (3.2.6)$$

Again, the sign of equation (3.2.6) can be $\gtrless 0$, depending upon the absolute values of r and γ. However, as done in Case II, the signs for which there exists an optimal steady state time path must be determined.

Figures 3.11 and then E.1 through E.5 (in Appendix E) describe phase diagrams for a positively sloped time path of the stock of environmental resource and its shadow price under different sets of conditions. Under the conditions that

(i-a) $\quad d\left(\frac{de}{dt}\right) > 0 \quad$ and $\quad d\left(\frac{d\mu_2}{dt}\right) > 0$

the phase diagram in Figure 3.11 shows that there does not exist an optimal steady state time path which is convergent at Z.

Similarly, it can be shown that if any of the sign(s) in the half spaces (i.e., $d\left(\frac{de}{dt}\right)$ or $d\left(\frac{d\mu_2}{dt}\right)$) is(are) zero, then there is no solution. Now, in case of the other possible slopes of the time paths, only the two conditions will be employed for which there exists an optimal steady state time path. These are shown in Appendix F.

In the case of the other possible slopes of the time paths, there may exist optimal steady state time paths for the conditions in (b) and

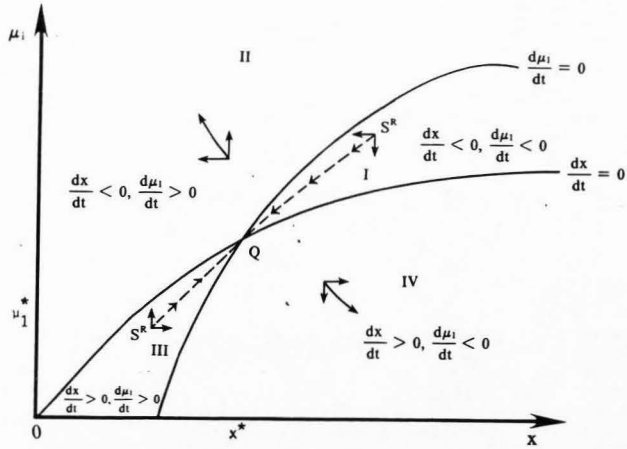

Figure 3.11. A phase diagram of stock of environmental resource and its shadow price when $d(\frac{de}{dt}) > 0$ and $d(\frac{d\mu_2}{dt}) > 0$.

(e). However, this may be stated that the conditions in (b) and (e), i.e., $(\gamma - h_e)$ and $(r - \gamma)$ having the opposite sign are the necessary and sufficient conditions for a stable optimal steady state solution for the environmental resource. The necessary and sufficient (n - s) conditions can be interpreted in the following way. The variable γ is the proportion of the environment that is used as a stock of environmental resource for this problem and h_e is the rate of change of extraction with respect to the stock of the environmental resource. Hence, $(\gamma - h_e)$ can be interpreted as the net change in the stock of the environmental resource. Similarly, $(r - \gamma)$ can be interpreted as the net discount rate (net of the proportion that is constantly used as the determining factor of the stock of environmental resource). If $(\gamma - h_e)$ is positive, implying that there is a net increase in the environmental asset, then the change in its shadow price should be negative.

In summary, it can be stated that for both the extractible (renewable and nonrenewable) and the environmental resource, the existence of optimal steady state time paths crucially depend on the signs of $(r - f'(x))$, $(f'(x) - g_x)$ and $(\gamma - h_e)$, $(r - \gamma)$, respectively. Optimal steady state time paths exist only when these conditions have the opposite sign.

It can be stated further that the solutions are locally unique depending upon the particular slopes of the two time paths for the stocks of resources and their shadow prices. However, global uniqueness depends on the assumptions employed in the model.

Thus, in an evaluation of a natural resource scarcity, it is instructive to analyze the time paths of the stock of resources (both extractible and environmental) and their shadow prices. As it was defined in Chapter I, resource scarcity would be reflected in the time path of the shadow price of the natural resource. The above analysis illustrates the necessity of determining the slopes of such time paths and whether a stable solution exists. For example, $s^R s^R$ in Figure C.1 (Appendix C) is an optimal steady state time path for the stock of extractible resource and its shadow price. Any point on this time path would eventually lead the system to reach Q, i.e., the optimal steady state levels of the stock of the extractible resource and its shadow price. If the extraction rate is controlled so that the stock remains at x^*, which is an optimal one, then the corresponding shadow price also remains at μ_1^* or a steady state. The level of scarcity is reflected at that level of the shadow price.

However, in a practical evaluation of resource scarcity, it follows that if the shadow price μ_1^* changes over time, then the level of scarcity would be changing in the same direction.

Section III.3: Comparative Statics

The theoretical constructs of Sections III.1 and III.2 have been exposed to the traditional comparative static analysis in this section in order to evaluate the built-in rationality of the model. In other words, how well the model, developed above, can predict in the face of an exogenous or parametric shift is the object of investigation in this section.

Case I: An Exogenous Change in the Price of the Final Product

In order to examine the impacts of an exogenous change in the price of the final product on the stocks of extractible and environmental resources and their shadow prices, the solutions at steady state must now be evaluated.

Using equation (3.1.16a) in (A') above

$$\left.\frac{dx}{dP_y}\right|_{\frac{dx}{dt}=0} = -\frac{-g_{\mu_1} Y_N}{f'(x) - g_x} \qquad \ldots (3.3.1)$$

Since $\mu_1 = P_Y Y_N - \alpha C_N - \mu_2$ or

$$\frac{d\mu_1}{dP_y} = Y_N \qquad \ldots (3.3.2)$$

The sign of equation (3.3.1) could be $\gtrless 0$ or ∞, depending on the sign of the denominator. However, in the case of an exhaustible resource, the sign is negative since $f'(x)$ is zero and all other terms are positive by assumption.

A more complete interpretation follows. If there is an increase in the price of the final output due to an increase in the demand for it relative to its supply, then the demand for the extractible resource will increase. Since the stock of this resource is fixed (for an exhaustible resource), the stock size will be depleted. However, for a renewable resource, the stock size may increase or decrease. The change may be positive if the biological growth is greater than the extraction rate that yields a positive sign of equation (3.3.1). It is economically infeasible for the sign of equation (3.3.1) to be infinite (∞) because it is not difficult to conceive that the biological limit of the growth of stock size would likely occur much before the stock size reaches infinity.

Similarly,

$$\left.\frac{de}{dP_y}\right|_{\frac{de}{dt}=0} = -\frac{\gamma - h_{\mu_2} Y_N}{\gamma - h_e} \qquad \ldots (3.3.3)$$

where $\mu_2 = P_y Y_N - \alpha C_n - \mu_1$ and

$$\frac{d\mu_2}{dP_y} = Y_N \qquad \ldots (3.3.4)$$

The sign of this equation could be $\gtrless 0$. However, since γ is a fixed proportion as utilized in this model in determining the stock of environmental resources (capacity of the sink), it is reasonable to assume the sign of the numerator will normally be positive. As a result, equation (3.3.3) will usually have a negative sign. This suggests that as the price of the final product increases, the demand for the extractible and the environmental resources also increases. Since the stock of the environmental resource has been treated as a constant (due to a fixed proportion of the entire environment appearing here as a stock), it gets depleted. However, other possibilities do exist.

In order to evaluate the impact on the shadow prices, the same technique as above is employed:

$$\left.\frac{d\mu_1}{dP_y}\right|_{\frac{d\mu_1}{dt} = 0} = -\frac{Y_N(r - f'(x))}{(r - f'(x))} < 0 \qquad \ldots (3.3.5)$$

since $\frac{d\mu_1}{dt} = [(P_y Y_N - \alpha C_n - \mu_2)(r - f'(x))] + \alpha C_x$.

The negative sign of this equation suggests that as the price of the product increases, the shadow price or the marginal user cost decreases. In the case of a renewable resource, as the price of the final product increases, the demand for the extractible resource increases faster than the biological growth rate. Thus, the stock is depleted. As the stock is reduced, the marginal cost of extraction will likely increase. In fact, it may rise faster than the increase in price, resulting in a negative sign for equation (3.3.5).

Similarly,

$$\left.\frac{d\mu_2}{dP_y}\right|_{\frac{d\mu_2}{dt}=0} = -Y_N < 0 \qquad \ldots (3.3.6)$$

The same line of argument is also applicable in this case.

Case II: A Parametric Shift in the Interest Rate "r"

In order to evaluate the effects of a change in "r" on the stock, some modifications in the structural equations (A)' and (B)' are required.

From equation (3.1.9), Section (III.1),

$$\mu_1 = P_y Y_N - \alpha C_N - \mu_2 \quad .$$

This result was obtained from the necessary condition of the current valued Hamiltonian. If a present value Hamiltonian has been used, then:

$$\mu_1 e^{-rt} = (P_y Y_N - \alpha C_N - \mu_2)e^{-rt} \qquad \ldots (3.3.7)$$

Furthermore, if the left-hand side is replaced by λ_1, this would yield the following,

$$\lambda_1 = (P_y Y_N - \alpha C_N - \mu_2)e^{-rt} \qquad \ldots (3.3.8)$$

This suggests that λ_1 is nothing but the present value costate variable. The relation between μ_1 and λ_1 is shown in equation (3.3.9).

$$\mu_1 = \lambda_1 e^{rt} \qquad \ldots (3.3.9)$$

This implies that

$$\left.\frac{dx}{dr}\right|_{\frac{dx}{dt}=0} = -\frac{-[g_{\mu_1}(-t)e^{-rt}(P_yY_N - \alpha C_N - \mu_2)]}{f'(x) - g_x}$$

$$= -\frac{tg_{\mu_1}e^{-rt}(P_yY_N - \alpha C_N - \mu_2)}{f'(x) - g_x} \qquad \ldots (3.3.10)$$

To evaluate the sign for this expression, take the limit of equation (3.3.10)

$$\text{(i)} \lim_{r \to \infty} \left.\frac{dx}{dr}\right|_{\frac{dx}{dt}=0} = \lim_{r \to \infty} -\frac{tg_{\mu_1}e^{-rt}(P_yY_N - C_N - \mu_2)}{f'(x) - g_x}$$

$$= -\left[t\frac{g_{\mu_1}}{f'(x) - g_x}(P_yY_N - \alpha C_N - \mu_2)\right]\lim_{r \to \infty} e^{-rt}$$

$$\lim_{r \to \infty} \left.\frac{dx}{dr}\right|_{\frac{dx}{dt}=0} = -[.] \to 0 \qquad \ldots (3.3.11)$$

where [.] is used to signify all the terms in the square braces of equation (3.3.11).

$$\text{(ii)} \lim_{r \to 0} \left.\frac{dx}{dr}\right|_{\frac{dx}{dt}=0} \quad [.] \to -t\frac{g_{\mu_1}}{f'(x) - g_x}(P_yY_N - \alpha C_N - \mu_2)$$

$$= -t\mu_1[g_{\mu_1}/f'(x) - g_x] \qquad \ldots (3.3.12)$$

Thus, it can be argued that if the interest rate used for discounting the present value of the net benefits increases without bound,

then the change in the stock of the extractible natural resource approaches zero. Alternatively, if the interest rate is zero, i.e., no discounting is done, then the entire stock can be used at a later date. In other words, there will be no intertemporal use of the stock or use in every period is equally valued since there is no discount rate used to evaluate the worth at the current time.

The effects on the shadow price of a change in the interest rate is shown in equation (3.3.13),

$$\left.\frac{d\mu_1}{dr}\right|_{\frac{d\mu_1}{dt} = 0} = -\frac{(P_y Y_N - \alpha C_N - \mu_2)e^{-rt} - rte^{-rt}(P_y Y_N - \alpha C_N - \mu_2)}{r - f'(x)} \qquad \ldots (3.3.13)$$

To evaluate the sign of equation (3.3.13) one can take the limit of this equation,

$$= \lim_{r \to \infty} -\frac{(P_y Y_N - \alpha C_N - \mu_2)e^{-rt} - rte^{-rt}(P_y Y_N - \alpha C_N - \mu_2)}{r - f'(x)} \qquad \ldots (3.3.14)$$

If r is allowed to approach infinity, the sign of equation (3.3.14) is undefined because both the numerator and the denominator contain ∞. Furthermore ∞/∞ is meaningless. However, L' Hopitals rule can be used to evaluate the sign. If we allow the numerator in equation (3.3.14) to be designated as m(r) and the denominator as n(r), then by differentiating the numerator and the denominator of equation (3.3.14) with respect to r yields:

$$m'(r) = -[-te^{-rt}(.) - te^{-rt}(.) + rt^2e^{-rt}(.)] \qquad \ldots (3.3.15)$$

and

$$n'(r) = 1 \qquad \ldots (3.3.16)$$

where (.) implies $(P_y Y_N - \alpha C_N - \mu_2)$.

L' Hopitals rule tells that

$$\lim_{r \to \infty} \frac{m(r)}{n(r)} = \lim_{r \to \infty} \frac{m'(r)}{n'(r)} \qquad \ldots (3.3.17)$$

By using equations (3.3.15) and (3.3.16) in (3.3.13), the following equation is obtained.

$$\therefore \lim_{r \to \infty} \left. \frac{d\mu_1}{dr} \right|_{\frac{d\mu_1}{dt} = 0} = \lim_{r \to \infty} \frac{[te^{-rt}(.) - rt^2 e^{-rt}(.) + te^{-rt}(.)]}{1} = [\cdot] \longrightarrow 0 \qquad \ldots (3.3.18)$$

and

$$\lim_{r \to 0} \left. \frac{d\mu_1}{dr} \right|_{\frac{d\mu_1}{dt} = 0} [.] \to 2t\mu_1 \qquad \ldots (3.3.19)$$

where $[.] = [2te^{-rt}(.) - rt^2 e^{-rt}(.)]$.

Assuming that the interest rate increases without bound, the change in the stock size approaches zero. Since there is no change in the stock in the ground, the change in the shadow price or rent also approaches zero. Similarly, if there is no interest rate used for discounting, then the use of stock is pushed forward. As a result, its shadow price also increases.

Similarly, for the other set of equations, i.e., (3.1.18a) and (3.1.19), we get

$$\left.\frac{de}{dr}\right|_{\frac{de}{dt}=0} = -\frac{-[h_{\mu_2}-t)e^{-rt}(P_yY_N - \alpha C_N - \mu_1)]}{\gamma - h_e}$$

$$= -\frac{th_{\mu_2}e^{-rt}(P_yY_N - \alpha C_N - \mu_1)}{\gamma - h_e} \qquad \ldots (3.3.20)$$

Again, to evaluate the sign we take the limit,

$$\text{(i)} \quad \lim_{r \to \infty} \left.\frac{de}{dr}\right|_{\frac{de}{dt}=0} = \lim_{r \to \infty} -\frac{th_{\mu_2}e^{-rt}(P_yY_N - \alpha C_N - \mu_1)}{\gamma - h_e}$$

$$= -t\left(\frac{h_{\mu_2}}{\gamma - h_e}\right)(P_yY_N - \alpha C_N - \mu_1) \lim_{r \to \infty} e^{-rt}$$

$$\therefore \quad \lim_{r \to \infty} \left.\frac{de}{dr}\right|_{\frac{de}{dt}=0} = -[.] \longrightarrow 0 \qquad \ldots (3.3.21)$$

$$\text{(iii)} \quad \lim_{r \to 0} \left.\frac{de}{dr}\right|_{\frac{de}{dt}=0} = [.] \longrightarrow -t\mu_2\left(\frac{h_{\mu_2}}{\gamma - h_e}\right) \qquad \ldots (3.3.22)$$

Here, again, if the interest rate increases without bound, then the change in the stock of the environmental resource goes to zero. It means that the entire capacity of the environmental sink remains at a constant level through time, which suggests that the whole capacity may be used up in the present time. However, for a zero interest rate, the stock could be used at a future date, in other words, all future uses are equally valued as that of present.

In order to evaluate the effects on the shadow price of the environmental resource, the same technique that was used above is employed once again. This is shown in equation 3.3.23,

$$\left.\frac{d\mu_2}{dr}\right|_{\frac{d\mu_2}{dt}=0} = -\frac{(P_yY_N - \alpha C_N - \mu_1)e^{-rt} - rte^{-rt}(P_yY_N - \alpha C_N - \mu_1)}{r - \gamma} \qquad \ldots (3.3.23)$$

If the limit of equation (3.3.23) is taken with respect to r, then the results shown in equation (3.3.24) are obtained.

(i)

$$\lim_{r \to \infty} \left.\frac{d\mu_2}{dr}\right|_{\frac{d\mu_2}{dt}=0} = \lim_{r \to \infty} -\frac{[(P_yY_N - \alpha C_N - \mu_1)e^{-rt} - rte^{-rt}(.)]}{r - \gamma} \qquad \ldots (3.3.24)$$

The sign of equation (3.3.24) is undefined, but the use of L' Hopitals rule yields the result shown in equation (3.3.25).

$$\lim_{r \to \infty} \left.\frac{d\mu_2}{dr}\right|_{\frac{d\mu_2}{dt}=0} = \lim_{r \to \infty} -\frac{[-te^{-rt}(.) + rt^2e^{-rt}(.) - te^{-rt}(.)]}{1}$$

$$\therefore \lim_{r \to \infty} \left.\frac{d\mu_2}{dr}\right|_{\frac{d\mu_2}{dt}=0} = [.] \longrightarrow 0 \qquad \ldots (3.3.25)$$

and,

$$\lim_{r \to 0} \left. \frac{d\mu_2}{dr} \right|_{\frac{d\mu_2}{dt} = 0} = [.] \longrightarrow 2t\mu_2 \qquad \ldots (3.3.26)$$

As the interest rate increases without bound, the change in the stock approaches zero and so the change in the shadow price also approaches zero. If a zero interest rate is charged, the shadow price increases.

Case III: An Exogenous Change in the Level of Technology

In order to evaluate the effect of a change in the level of technology on the stocks of the extractible and the environmental resources and their shadow prices, some modification of equation (3.1.12) is necessary. If equation (3.1.12) is rewritten as it is shown in equation (3.1.12b), then,

$$\mu_1 = P_y Y_N - \alpha C_N - \mu_2 \qquad \ldots (3.1.12b)$$

the change in technology can be evaluated. For this, Y_N and C_N have to be modified, and the following technique can be applied. From equation (3.1.1), we have,

$$Y = Y[N(x, e, K^E, T), e, K^P, T] \qquad \ldots (3.3.27)$$

If K^P and K^E are held constant, then differentiating equation (3.3.27) totally yields

$$dy = Y_N d_N + Y_e d_e + Y_T d_T \qquad \ldots (3.3.28)$$

or

$$dy = Y_N(N_x d_x + N_e d_e + N_T d_T) + Y_e de + Y_T dT \qquad \ldots (3.3.29)$$

If the change in technology affects the final product Y and not the subproduction function, then the expression $N_T dT$ can be treated as zero.

As a result,

$$\therefore \quad Y_N = \frac{dY - Y_T d_T - Y_e de}{N_x dx + N_e de} \qquad \ldots (3.3.30)$$

Similarly,

$$C_N = \frac{dC - C_T d_T - C_e de - C_x d_x}{N_x d_x + N_e de} \qquad \ldots (3.3.31)$$

By substituting equations (3.3.30) and (3.3.31) into equation (3.1.12b), equation (3.3.32) is obtained,

$$\mu_1 = P_Y\left(\frac{dY - Y_e de - Y_T dT}{N_x d_x + N_e de}\right) - \alpha\left(\frac{dC - C_e de - C_x d_x - C_T dT}{N_x d_x + N_e de}\right) - \mu_2$$

$$\ldots (3.3.32)$$

However, μ_2 in equation (3.3.30) can further be replaced by h(e, N), which yields equation (3.3.33),

$$\mu_1 = P_Y\left(\frac{d_y - Y_e de - Y_T dT}{N_x d_x + N_e de}\right) - \alpha\left(\frac{de - C_e de - C_x dx - C_T dT}{N_x d_x + N_e de}\right) - h(e, N)$$

$$\ldots (3.3.33)$$

Now using the implicit function rule

$$\left.\frac{dx}{dT}\right|_{\frac{dx}{dt} = 0} = - \frac{g_{\mu_1}[.]}{f'(x) - [g_x + g_{\mu_1}(.)]} \qquad \ldots (3.3.34)$$

where the items in brackets of equation (3.3.34) [.] are

$$[.] = P_Y \frac{(-Y_{TT} d_T - Y_T d^2T)(N_x d_x + N_e de)}{(N_x d_x + N_e de)^2} - \frac{\alpha(-C_{TT} d_T - C_T d^2T)(N_x d_x + N_e de)}{(N_x d_x + N_e de)^2}$$

$$- h_N N_T$$

and

$$(.) = P_y \frac{(-N_{xx}d_x - N_x d^2x)(d_Y - Y_e de - Y_T dT)}{(N_x d_x + N_e de)^2}$$

$$- \frac{\alpha(-C_{xx}d_x - C_x d^2x)(N_x dx + N_e de) - (N_{xx}de + N_x d^2x)(de - C_e de - C_x dx - C_T d_T) - h_N N_X}{(N_x d_x + N_e de)^2}$$

The sign for equation (3.3.34) is ambiguous. This ambiguous sign suggests that the effect of technology on the stock of the extractible resource cannot be evaluated theoretically. Its effect is an empirical problem. However, the following discussion suggests some of the implications that may be obtained from equation (3.3.34).

As technology improves, it becomes less costly to use the extractible resource in the process of production of the final output. The profit-maximizing firm can use more extractible resources in order to produce a greater level of output given that the marginal cost has now gone down per unit due to improvement in technology. Thus, the stock (both for renewable and nonrenewable) might go down and in that case a negative sign of equation (3.3.34) may be justified. However, this result depends on the assumption that the firm faces an elastic demand curve for the final output. On the other hand, if it is assumed that the demand curve is of a constant elasticity (rectangular hyperbola) type, then for this improvement in technology there might be less extraction of the extractible resource and, in that case, the stock might be augmented. Thus, a positive sign of this partial derivative is a possibility.

However, the polar cases (0 and ∞) are two extreme results of the above two cases. In order to evaluate the effect of an increase in the

level of technology on the shadow price, the same technique is followed and the results are shown in equation (3.3.35)

$$\frac{d\mu 1}{dT} = - \frac{(r - f'(x)[.] + \alpha C_{xT}}{(r - f'(x))} \qquad \ldots (3.3.35)$$

$$\frac{d\mu_1}{dt} = 0$$

where

$$[.] = \frac{P_Y(-Y_{TT}d_T - Y_T d^2T)(N_x d_x + N_e de)}{(N_x d_x + N_e de)^2} - \frac{\alpha(-C_{TT}d_T - C_T d^2T)(N_x d_x + N_e de)}{(N_x d_x + N_e de)^2} - h_N N_T$$

Again, the sign of equation (3.3.35) cannot be determined unambiguously. The same basic result will follow for the environmental resource and its shadow price due to a change in the level of technology. This suggests that the effect of a change in technology on resource use rates and stocks must be evaluated empirically.

Section III.4: Scarcity and Uncertain Stock of Environmental Resource

Uncertainty in the Basic Model

The works of Devarajan and Fisher (1982) and Pindyck (1980, 1984) have addressed the issue of resource scarcity from the point of uncertainty in the stock of the extractible resource. Be it due to uncertainty in the exploration results (D-F), or due to certain random disturbances in the law of motion resulting in a stochastic differential equation of Ito type[3] to describe stock dynamics (Pindyck, 1984) of the extractible resource, the issues relating to extraction rate, and

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[3]For a definition of such type of equation, see Appendix G.

resource scarcity have so far been well-handled. However, in none of the models mentioned above have the environmental issues played any role. It has not been recognized that while extracting and using an extractible resource (coal) the competitive firms also use environment (air) as a joint resource and as a matter of fact the jointness between the extractible and the environmental resource is oftentimes beyond uncoupling. Thus, those models do not address the issue of resource scarcity from an overall society's welfare point of view. It may be mentioned in passing that an improvement in the social welfare (if at all constructed) implicitly means a movement from a lower to a higher indifference curve in the environmental and the extractible resource spaces.

In the lines to follow an attempt has been made to extend the scope of uncertainty in the behavior of the resource scarcity by incorporating it through the stock of the environmental resource.

It is assumed while incorporating uncertainty in the basic theoretical model of Chapter III that the current stock of the environmental resource, e, is known but the future reserve is unknown and depends on the variability in the stock of the extractible resource, x. Since it is assumed in the deterministic model that the capacity of the environmental sink to absorb waste products out of processing of the extractible resource is known (γ) and a constant fraction of the total reserve, it is instructive to modify that in order to incorporate uncertainty. In order to simplify the mathematical construct it is assumed that such capacity of the environmental sink (environmental reserve) is no longer a known parameter but becomes a stochastic variable due to randomness in

the stock of extractible resource. The underlying simplifying assumption is a functional relationship between the environmental and the extractible resource. The current stock of both the resources are known with certainty, it is only the future stock of the environmental resource that is uncertain and depends on the variability of the future stock of extractible resource.

Thus, the stock dynamics of the environmental resource can be written as:

$$de = [\gamma e - N(x, e, K^E, T)]dt + \sigma(x)dZ \qquad \ldots \quad (3.4.1)$$

Note that the difference between equations (3.4.1) and (3.1.3) is only due to the second term in the right-hand side of (3.4.1). Here it is assumed that $\sigma'(x) > 0$, $\sigma(0) = 0$, and dZ are the increments of a stochastic process Z that obeys what is called a Brownian motion or a white noise or is a Weiner process.[4] Thus, equation (3.4.1) becomes a stochastic differential equation. It is still assumed that the competitive firms maximize discounted present value of net surplus with the only exception that here they maximize the expected discounted present value. The problem can be approached using a stochastic optimal control technique.

From a representative firm's point of view (assuming that the competitive firms are homogeneous in nature), the problem can be formulated as follows:

$$\text{Max} \quad E_t \int_0^\infty e^{-rt} \pi(t, x, e, K^E, K^P)dt \qquad \ldots \quad (3.4.2)$$

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[4]For a discussion of a Weiner process, see Appendix G.

$$\text{Subject to } dx = g(x, K^E)dt \qquad \ldots \quad (3.4.3)$$

$$de = h(e, K^E)dt + \sigma(x)dZ \qquad \ldots \quad (3.4.4)$$

Note that $\pi(.)$ is exactly the same as in Section (III.1), i.e.,

$$\pi(.) = P_y\, Y(N(x, e, K^E, T), e, K^P, T) - W\Sigma K_i^P - \alpha C(N, x, e, W, T) \qquad \ldots \quad (3.1.1)$$

without the integral sign before.

Similarly,

$$g(x, K^E) = f(x) - N(x, K^E, e, T)$$

in equation (3.1.2). Note that it is assumed that in $g(x, K^E)$ the other arguments have been treated as constant and that is why they are omitted.

Similarly, the $h(e, K^E)$ in equation (3.4.4) is nothing but $[\gamma e - N(x, K^E, e, T)]$ in equation (3.1.3). Here in equation (3.4.2) E_t takes the expected value at time t.

In order to find the necessary conditions for solution, the method of stochastic dynamic programming (see Kamien and Schwartz, 1981 and/or Malliaris and Brock, 1982) is followed.

Define $J(t_0, x_0, e_0)$ to be the maximum expected value obtainable in a problem of the form of equations (3.4.2) through (3.4.4), starting at time t_0 in state $x(t_0) = x_0$ and $e(t_0) = e_0$. Then the fundamental equation of optimality, which is known as Hamiltonian-Jacobi-Bellman equations[5] of stochastic control theory, can be written as:

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[5]For a derivation of the Hamiltonian-Jacobi-Bellman equation, see Appendix G.

$$-J_t = \text{Max}\ [e^{-rt}\pi(.) + J_x g(x(t), K^E(t)) + J_e h(e(t), K^E(t))$$

$$+ \frac{1}{2}\sigma^2(x(t)J_{ee}] \qquad \ldots \quad (3.4.5)$$

Multiplying both sides of equation (3.4.5) by e^{rt} yields:

$$-J_t e^{-rt} = \text{Max}\ [\pi(.) + e^{rt}J_x g(x, K^E) + e^{rt}J_e h(e, K^E)$$

$$+ \frac{1}{2}\, e^{rt}\sigma^2(x)J_{ee}] \qquad \ldots \quad (3.4.6)$$

The function is maximized by taking the partial derivatives with respect to the control variables (K^E, K^P as in Section (III.1)) and setting them equal to zero.

$$\frac{\partial\pi}{\partial K^E} = PyY_N N_K{}^E - \alpha C_N N_K{}^E + e^{rt}J_x(-N_K{}^E) + e^{rt}J_e(-N_K{}^E) = 0$$

or

$$PyY_N - \alpha C_N - e^{rt}J_x - e^{rt}J_e = 0 \qquad \ldots \quad (3.4.7)$$

Now define

$$e^{rt}J_x = \mu_1 \qquad \ldots \quad (3.4.8)$$

and

$$e^{rt}J_e = \mu_2 \qquad \ldots \quad (3.4.9)$$

Substituting equations (3.4.8) and (3.4.9) into (3.4.7) and by rearranging yields:

$$\mu_1 = PyY_N - \alpha C_N - \mu_2 \qquad \ldots \quad (3.4.10)$$

Notice that equations (3.4.10) and (3.1.12) are exactly the same. In other words, the shadow price of the extractible resource is equated to the value of the marginal product (price) minus the marginal cost of extraction plus the shadow price of the environmental resource (defined

the same way as in Section (III.1)). Note also that J_x is nothing but the partial derivative of the discounted present value of net surplus with respect to the change in the stock of the extractible resource and, thus, is the shadow price of the extractible resource. Multiplying J_x by e^{rt} a current value shadow price (μ_1) is obtained. The same has been done for J_e, it is nothing but the shadow price of the environmental resource.

Now, the other necessary condition of optimality is:

$$\frac{\partial \pi}{\partial K^P} = PyY_K{}^P - W = 0$$

or

$$PyY_K{}^P = W \qquad \ldots (3.4.11)$$

This is exactly the same as equation (3.1.13).

Now, in order to obtain a time path of the shadow price of the stock of extractible resource ($\frac{d\mu_1}{dt}$), such that some analytical conclusions can be made regarding the extraction rate and resource scarcity, the fundamental equation of optimality in equation (3.4.6) has been utilized.

Differentiating equation (3.4.6) with respect to x yields:

$$-e^{rt}J_{tx} = \pi_x + e^{rt}[J_{xx}g(x, K^E) + J_x g_x(x, K^E)] + e^{rt}[J_{ex}h(e, K^E)$$

$$+ J_e h_x(e, K^E)] + e^{rt}[\ (x)J_{ee} + \frac{1}{2}\sigma^2(x)J_{eex}] \qquad \ldots (3.4.12)$$

The rational for taking the partial derivative of equation (3.4.6) is that, for any x, equation (3.4.6) must hold and so it must hold for a slight modification in x. Thus, the partial derivative of equation

(3.4.6) with respect to x must be zero (with K^E, K^P chosen optimally in terms of t, x, e, and J_x) (see Kamein and Schwartz, 1981, p. 240).

Now, in order to eliminate $e^{rt}J_{xx}\, g(x, K^E)$ and $e^{rt}J_{ex}\, h(e, K^E)$ from equation (3.4.12) so that a comparable expression for $\frac{d\mu_1}{dt}$ is obtained, a total derivative of $e^{rt}J_x(t, x, e)$ is taken as follows:

$$\frac{d[e^{rt}Jx(t, x, e)]}{dt} = re^{rt}J_x + e^{rt}[J_{xt} + J_{xx}\frac{dx}{dt} + J_{xe}\frac{de}{dt}] \qquad \ldots (3.4.13)$$

Replace $\frac{dx}{dt}$ and $\frac{de}{dt}$ from equation (3.4.13) by equations (3.4.2) and (3.4.4) which yields:

$$\frac{d[e^{rt}Jx(t, x, e)]}{dt} = re^{rt}J_x + e^{rt}[J_{xt} + J_{xx}g(x, K^E)$$

$$+ J_{xe}[h(e, K^E) + \sigma(x)dZ] \qquad \ldots (3.4.14)$$

$$\frac{d[e^{rt}Jx(t, x, e)]}{dt} = r\mu_1 + e^{rt}[J_{xt} + J_{xx}g(x, K^E)$$

$$+ J_{xe}[h(e, K^E) + \sigma(x)dZ] \qquad \ldots (3.4.15)$$

since $e^{rt}J_x = \mu_1$ from equation (3.4.8), or,

$$e^{rt}[J_{xx}g(x, K^E) + J_{xe}h(e, K^E)] = \frac{de^{rt}Jx(t, x, e)}{dt} - r\mu_1 - e^{rt}Jxt$$

$$- e^{rt}J_{xe}\sigma(x)dZ \qquad \ldots (3.4.16)$$

Now, substituting equation (3.4.16) in (3.4.12) yields:

$$-e^{rt}J_{tx} = \pi_x + \frac{de^{rt}Jx(t, x, e)}{dt} - r\mu_1 - e^{rt}J_{xt} - e^{rt}J_{xe}\,(x)dZ$$

$$+ e^{rt}J_x g_x(x, K^E) + e^{rt}J_e h_x(e, K^E) + e^{rt}\sigma(x)J_{ee} + \frac{1}{2}e^{rt}\sigma^2(x)J_{eex}$$

$$\ldots (3.4.17)$$

By rearranging, equation (3.4.17) yields:

$$- \frac{de^{rt}Jx(t, x, e)}{dt} = \pi_x - r\mu_1 + e^{rt}[J_x g_x(x, K^E) + J_e h_x(e, K^E)$$

$$+ \frac{1}{2}\sigma^2(x)J_{eex} - J_{xe}\sigma(x)dZ + \sigma(x)J_{ee}] \qquad \ldots (3.4.18)$$

Substituting equations (3.4.8) and (3.4.9) in (3.4.18) yields:

$$- \frac{d\mu_1}{dt} = \pi_x - r\mu_1 + \mu_1 g_x(x, K^E) + \mu_2 h_x(e, K^E) + e^{rt}[\sigma(x)J_{ee}$$

$$+ \frac{1}{2}\sigma^2(x)J_{eex} - J_{xe}\sigma(x)dZ] \qquad \ldots (3.4.19)$$

The partial derivatives can now be evaluated,

$$\pi_x = P_y Y_N N_x - \alpha C_N N_x - \alpha C_x \qquad \ldots (3.4.20)$$

since

$$\pi = P_y Y[N(x, K^E, e, T), e, K^P] - W\Sigma_i K_i^P - \alpha C[N(x, K^E, e, T), x, e, W, T]$$

Also,

$$g_x(x, K^E) = f'(x) - N_x \qquad \ldots (3.4.21)$$

since

$$g(x, K^E) = f(x) - N(x, K^E, e, T)$$

and

$$h_x(e, K^E) = -N_x \qquad \ldots (3.4.22)$$

since

$$h(e, K^E) = \gamma e - N(x, K^E, e, T) .$$

Now, substituting equations (3.4.20), (3.4.21) and (3.4.22) in (3.4.19) gives

$$-\frac{d\mu_1}{dt} = (P_y Y_N N_x - \alpha C_N N_x - \alpha C_x) - r\mu_1 + \mu_1(f'(x) - N_x)$$

$$- \mu_2 N_x + e^{rt}[\sigma(x)J_{ee} + \frac{1}{2}\sigma^2(x)J_{eex} - J_{xe}\sigma(x)dZ] \quad . . . (3.4.23)$$

Again, the partial derivatives of the maximum value function J can be obtained as follows. Note that $J(t, x, e) = \pi(t, x, e, K^E, K^P)$

$$J_x = \frac{\partial J(t, x, e)}{\partial x} = \frac{\partial \pi(.)}{\partial x} = P_y Y_N N_x - \alpha C_N N_x - \alpha C_x \quad . . . (3.4.20a)$$

or,

$$J_{xe} = \frac{\partial^2 J(t, x, e)}{\partial x \partial e} = \frac{\partial \pi(.)}{\partial x \partial e} = P_y Y_N N_{xe} - \alpha C_N N_{xe} - \alpha C_{xe}$$

$$. . . (3.4.24)$$

Under the assumption that the level of stock of the extractible natural resource is independent of the stock of the environmental resource (but not vice versa) at least in the current time period, N_{xe} and C_{xe} become zero. It is also assumed that N and e are separable in the production and the cost functions, Y(.) and C(.), respectively. Thus, J_{xe} in equation (3.4.24) becomes zero.

Similarly,

$$J_e = \frac{\partial J(.)}{\partial e} = \frac{\partial \pi(.)}{\partial e} = P_y Y_N N_e - \alpha C_N N_e - \alpha C_e \quad . . . (3.4.25)$$

$$J_{ee} = \frac{\partial^2 J(.)}{\partial e^2} = \frac{\partial^2 \pi(.)}{\partial e^2} = P_y Y_N N_{ee} - \alpha C_N N_{ee} - \alpha C_{ee} \quad . . . (3.4.26)$$

and

$$J_{eex} = \frac{\partial^3 J(.)}{\partial^2 e \partial x} = \frac{\partial^3 \pi(.)}{\partial^2 e \partial x} = P_y Y_N N_{eex} - \alpha C_N N_{eex} - \alpha Ceex \quad . . (3.4.27)$$

Now, under the assumption that the rate of change of environmental stock with respect to the stock of extractible resource is zero, N_{eex} and C_{eex} become zero. Therefore, J_{eex} also becomes zero.

Now substituting equations (3.4.24), (3.4.25), (3.4.26), and (3.4.27) in (3.4.23) yields,

$$-\frac{d\mu_1}{dt} = -r\mu_1 + N_x(P_yY_N - \mu_1 - \mu_2) - \{\alpha C_N N_x + \alpha C_x) + e^{rt}\sigma(x)$$

$$[P_yY_N N_{ee} - \alpha(C_N N_{ee} + C_{ee})] + \mu_1 f'(x) \qquad \ldots (3.4.28)$$

Again, substituting $\mu_2 = P_yY_n - \alpha C_N - \mu_1$ from equation (3.4.10) into (3.4.28) and rearranging yields,

$$\frac{d\mu_1}{dt} = r\mu_1 + \alpha C_x - \mu_1 f'(x) - e^{rt}\sigma(x)[P_yY_N N_{ee} - \alpha(C_N N_{ee} + C_{ee})]$$

or,

$$\frac{d\mu_1}{dt} = \mu_1(r - f'(x)) + \alpha C_x - e^{rt}\sigma(x)A(x, e, t) \qquad \ldots (3.4.29)$$

where A(x, e, t) are the items in the square braces.

Thus, equation (3.4.29) describes the time path of the shadow price of the extractible resource. Here the shadow price or the dynamics of shadow price depends on the net biological growth rate, $(r - f'(x))$ plus the marginal cost of extraction due to a change in the stock, C_x, minus the current value of the net benefit due to a change in the stock of environmental resource adjusted for the standard deviation of the current stock of extractible resource. Notice that equation (3.4.29) captures equation (3.1.14), i.e., the time path of shadow price of the extractible resource as a special case. In case $\sigma(x) = 0$ then equation (3.4.29) reduces to equation (3.1.14). The variance effect,

$\sigma(x)$, becomes the determining factor in future extraction rates. If it is positive and large, also assuming $A(x, e, t)$ is positive, the $\frac{d\mu_1}{dt}$ decreases, which implies that the marginal loss in profit from future extraction decreases. Alternatively, it can be said that the marginal gain in profit increases due to not extracting today. As a result, current extraction would be lower. For an exhaustible resource, this might imply a slower depletion of the stock of resource in absence of positive exploration activities. However, emergence of a steady state equilibrium both in case of an exhaustible and for a renewable resource can be conceived in terms of probability distributions and moments, since the resource stock, at least for one in this model, grows stochastically. Whether there exists such a probabilistic steady state equilibrium has not been explored here and may be thought of as scope for further study.

CHAPTER IV

GENERAL EMPIRICAL MODEL AND ESTIMATION PROCEDURE

Section IV.1: The Empirical Model

The economic model developed in Sections III.1 through III.2 of Chapter III addresses the scarcity issue from a theoretical perspective. It has been shown that under the assumptions of a resource market composed of competitive profit-maximizing firms and the existence of a jointness between the extractible and the environmental resource, traditional measures of resource scarcity need to be modified in order to include the full opportunities foregone by the society (firm). Analytical results suggest that if the end users of the extractible resource have to account for the cost of maintaining or rehabilitating the environment, then a modified measure of resource scarcity, such as developed above, may show a different trend compared to that determined following traditional scarcity measures.

An empirical estimate of scarcity of an exhaustible resource, coal from its use point of view, on the basis of this modified definition, is included in this chapter. As a counterpart of the dynamic, theoretical model of Chapter III, an empirical model following the "duality approach" is developed in this chapter. However, a suitable econometric specification is needed before the empirical model can be put to test. In Section III.1, the theoretical model has been cast as an intertemporal control problem. There the profit-maximizing firm

was hypothesized as an intertemporal maximizer of net surplus. However, in the empirical analysis the said firm is assumed to be a cost minimizer and the model is recast in a static cost-minimizing framework. It may be pointed out that this way of reformulation is quite consistent with the intertemporal control problem of Chapter III since the Hamiltonian essentially summarizes an infinite series of static optimization problem (see Halvorsen and Smith 1984). It also should be noted that at steady state both the static and the dynamic problem would yield the same result. Therefore, there is no loss of generality in using the static optimization technique for empirical model building.

The Model

Assume that the representative firm's problem is to minimize cost and there exists n firms. The objective, then, is to:

$$\underset{K^j}{\text{minimize}} \sum_i W_i K_i^j \qquad \ldots (4.1.1)$$

$$\text{subject to } Y = Y(N, e, K^P, T) \qquad \ldots (4.1.2)$$

$$N = N(X, e, K^E, T) \qquad \ldots (4.1.3)$$

where $J = P, E$ and $i = 1, 2, \ldots n$

Here equation (4.1.1) represents the total cost of producing the final output where W_i is the hiring price of the composite capital and labor input (K^P and K^E) that is used in production of the final output and resource extraction, respectively. The constraints faced by the firm include the production function of the final output (equation 4.1.2) and the extraction function (equation 4.1.3) for the natural resource.

Equations (4.1.1) through (4.1.3) can be expressed as a Lagrangian function, such as that shown in equation (4.1.4).

$$L = \sum_i W_i K_i^j + \theta[Y - Y(N, e, K^P, T)]$$

$$+ \mu[N - N(X, e, K^E, T)] \qquad \ldots \quad (4.1.4)$$

Note that and μ are the two Lagrangian multipliers of this problem that can be interpreted as the shadow prices associated with the optimal level the final output (Y) and the extracted natural resource (N).

The first-order conditions (f.o.c) of this minimization problem are shown in equations (4.1.5) and (4.1.6).

i) $$\frac{\partial L}{\partial K_i^P} = W_i - \theta \frac{\partial Y}{\partial K^P} = 0$$

$$\text{or, } W_i = \theta \frac{\partial Y}{\partial K^P} \qquad \ldots \quad (4.1.5)$$

ii) $$\frac{\partial L}{\partial K_i^E} = W_i - \mu \frac{\partial N}{\partial K^E} = 0$$

$$\text{or, } W_i = \mu \frac{\partial N}{\partial K^E} \qquad \ldots \quad (4.1.6)$$

In these equations, Y and N are at their wealth-maximizing (solution) levels. Since Y is the optimal level of output, therefore, $\frac{\partial Y}{\partial N} \cdot \frac{\partial N}{\partial K^E} = 0$.

The solution to this cost-minimization problem yields the reproducible cost function:

$$CR = CR(Y, W, N, X, e, T) \qquad \ldots \quad (4.1.7)$$

Now, by applying the Envelope[1] theorem one can obtain the following:

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[1]See Silberberg (1978) for a detailed discussion of the Envelope Theorem.

$$\frac{\partial CR}{\partial N} = \frac{\partial L}{\partial N} = - \theta \frac{\partial Y}{\partial N} + \mu \qquad \ldots \quad (4.1.8)$$

The right-hand side of equation (4.1.8) can be evaluated by considering the solution of the cost minimization problem with N unrestricted. For this, the Lagrangian of the unconstrained problem is:

$$\bar{L} = \sum_i W_i K_i^j + \bar{\theta}[Y - Y(N, e, K^P, T)] + \bar{\mu} N(X, e, K^E, T) \qquad \ldots \quad (4.1.9)$$

Here, as noted above, $\bar{\theta}$ and $\bar{\mu}$ are the Lagrangian multipliers. The first-order necessary conditions are shown in equations (4.1.10) and (4.1.11).

i) $$\frac{\partial \bar{L}}{\partial K_i^P} = W_i - \bar{\theta} \frac{\partial Y}{\partial K^P}$$

$$\text{or, } W_i = \bar{\theta} \frac{\partial Y}{\partial K^P} \qquad \ldots (4.1.10)$$

ii) $$\frac{\partial \bar{L}}{\partial K_i^E} = W_i - \theta \frac{\partial Y}{\partial N} \cdot \frac{\partial N}{\partial K^E} + \bar{\mu} \frac{\partial N}{\partial K^E}$$

$$\text{or, } W_i = \frac{\partial N}{\partial K^E} \left(- \bar{\mu} + \bar{\theta} \frac{\partial Y}{\partial N}\right) \qquad \ldots (4.1.11)$$

Notice that in equation (4.1.11) $\frac{\partial Y}{\partial N}$ did not vanish since, in this unconstrained problem, Y and N are not at their solution levels.

Now, in order to derive the shadow price, $\bar{\mu}$, some further algebraic manipulations are required. From equations (4.1.5) and (4.1.10), equation (4.1.12) is obtained:

$$\theta = \bar{\theta} \qquad \ldots \quad (4.1.12)$$

and also from equations (4.1.6) and (4.1.11), equation (4.1.13) is obtained

$$\mu \frac{\partial N}{\partial K^E} = \frac{\partial N}{\partial K^E} (- \bar{\mu} + \bar{\theta} \frac{\partial Y}{\partial N}) \qquad \ldots (4.1.13)$$

$$\text{or, } \mu = - \bar{\mu} + \bar{\theta} \frac{\partial Y}{\partial N} \qquad \ldots (4.1.14)$$

Now, substituting equation (4.1.14) in (4.1.8) yields:

$$\frac{\partial CR}{\partial N} = - \theta \frac{\partial Y}{\partial N} - \bar{\mu} + \bar{\theta} \frac{\partial Y}{\partial N} \qquad \ldots (4.1.15)$$

Again, substituting equation (4.1.12) in (4.1.15) yields:

$$- \frac{\partial CR}{\partial N} = \bar{\mu} \qquad \ldots (4.1.16)$$

The negative of the partial derivative of the reproducible cost function with respect to the output of the extraction subproduction function yields a shadow price. Note that $\bar{\mu}$ and not μ would give the shadow price of the natural resource in use since $\bar{\mu}$ is attached to the unconstrained Lagrangian function.

For this derivation, a suitable functional form needs to be specified in order to make the cost function estimable.

Section IV.2: Functional Form: Translog Production and Cost Function

The reproducible cost function CR(Y, W, N, X, e, T) can be represented by using numerous functional forms. However, a Translog functional form is used in this study.

The Transcendental Logarithmic functional form, better known as Translog, is due to Christensen, Jorgenson and Lau (1973). They claim

that it provides a valid second-order approximation to an arbitrary functional form and the crucial "separability" hypothesis in production analysis does not have to be imbedded in the functional form as maintained hypothesis. Rather, statistical tests regarding separability could be performed on this. Another advantage of this form is that the CES and the Cobb-Douglas functions are special cases of the Translog. Christensen, Jorgenson and Lau (1973) have shown in depth the merits and applicability of this functional form. However, certain salient features of this special function need to be clarified. One first such clarification is related to the Transcendental function itself. Simon (1982, pp. 44) defines a Transcendental function as any function that is not algebraic, and the term algebraic means that the function can be expressed as a finite number of sums, differences, products, quotients, or root of polynomials. Examples of Transcendental functions include exponential, logarithmic, and trigonometric functions.

In a note on the Transcendental production function, Halter et al. (1957) introduced the general form of such functions in production economics. They claimed that the function is consistent with classical production functions (e.g., it can exhibit increasing, decreasing, and negative marginal returns, singularly, in pairs, or simultaneously). The general form of these functions is shown in equation (4.2.1).

$$Y = CX_1^{a_1} e^{b_1X_1} X_2^{a_2} e^{b_2X_2} \ldots X^{an} e^{b_nX_n} \qquad \ldots \quad (4.2.1)$$

where Y = total output, X_i = inputs, and C, a_i, b_i, (i = 1,...,n) are estimation parameters.

Although Halter et al. claimed that this function can exhibit different rates of return, it is restricted in terms of the partial

elasticity of substitution. In fact, it has a partial elasticity of substitution = $(1 - a_2 + b_1X_1)\ (a_2 + b_2X_2)/[(1 - a_2)\ (a_2 + b_2X_2)^2 + a_2\ (1 - a_2 + b_1X_1)^2]$, which reduces to unity when $b_1 = b_2 = 0$ (Fuss et al., 1978, pp. 242). Hence, the Transcendental production function of Halter et al. is a restricted one and does not possess much more general applicability when compared to an earlier Cobb-Douglas type.

In order to circumvent these types of problems, Christensen, Jorgenson and Lau came up with their Transcendental Logarithmic function which represents the underlying production process (Brendt and Christensen 1973).

Let the true production function be represented by

$$Y = g(X_1, \ldots, X_n) \qquad \ldots \quad (4.2.4)$$

where Y is output and X are inputs. Now taking logarithms of both sides, one can write:

$$\ln Y = \ln g(e^{\ln X_1}, \ldots, e^{\ln X_n}) \qquad \ldots \quad (4.2.5)$$

which can be written without loss of generality as

$$\ln Y = f(\ln X_1, \ldots, \ln X_n) \qquad \ldots \quad (4.2.6)$$

So, the production function $Y = g(X_1, \ldots, X_n)$ or its transcendental logarithmic transformation $[\ln Y = \ln g(e^{\ln X_1}, \ldots, e^{\ln X_n})]$ are equivalent (Denney and Fuss 1977). The quadratic approximation of this Translog function (or of any arbitrary production function of the form $\ln Y = f(\ln X_1, \ldots, \ln X_n)$ would be

$$\ln Y = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln X_1 + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma ij \ln X_i \ln X_j \quad . . . \quad (4.2.7)$$

Christensen, Jorgenson and Lau, followed by Brendt and Christensen, have shown that this transcendental logarithmic production function (or the transcendental logarithmic cost function) captures the Cobb-Douglas, CES types of production or cost functions as special cases and no a priori restriction regarding homotheticity of the production structure, separability with respect to inputs and outputs, and the constancy of the partial elasticity of substitution have to be maintained.

Separability is crucial in the sense that it allows decentralization in decision making or optimization by stages--which allows consistent multistage estimation. This may be the only feasible procedure when large numbers of inputs and outputs are involved specifically, when the relative simple concept of a production function is applied to complex organizations. One important application of separability is in the derivation of value-added functions. If the gross output production function is weakly separable in primary inputs, then a net output or value-added function can be defined and used for analysis (Denney and May 1978). In defining separability, first denote the set of n inputs by

$$N = (1, \ldots, n) \quad . . . \quad (4.2.8)$$

A partition S of N is given by

$$(N_1, \ldots, N_s) \quad . . . \quad (4.2.9)$$

where $N = N_1 \cup N_2 \cup \ldots \cup N_s$ and $N_r \cap N_t$ is empty for $r \neq t$. Separability is characterized by the independence of the marginal rate of

substitution between a pair of inputs from changes in the level of another input. This is shown in equation (4.2.10):

$$\frac{\partial(f_i/f_j)}{\partial V_K} = 0 \qquad \ldots (4.2.10)$$

where f_i/f_j is the marginal rate of substitution, because f_i and f_j are the first partial derivatives of the production function with respect to ith and jth input, and V_K is the level of another input.

An equivalent condition of separability is

$$f_i f_{iK} - f_i f_{jK} = 0 \qquad \ldots (4.2.11)$$

If either of these conditions hold, then one can say that "f" (the production function) is strongly separable with respect to the partition S if equation (4.2.10) holds for all $i \in N_r$ and $j \in N_t$ and $K \notin N_r \cup N_t$. The function "f" is weakly separable with respect to the partition S if equation (4.2.10) holds for all $i, j \in N_r$ and $K \notin N_r$. These properties may hold at a point or globally.

Historically, separability[2] has played an important role in the specification of functional forms. The Cobb-Douglas and CES functions are explicitly strongly separable, Sato's nested CES is also strongly separable with respect to the highest level partition and then strongly separable within each subaggregative. Since the separability conditions (equation 4.2.11) depend on the second-order partial derivatives, functional forms linear in parameters must be at least of the second-order in the variables to contain separability as a testable implication. The

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[2]The paragraph on separability has been adapted from Fuss et al. 1978.

class of Cobb-Douglas functions, which is of the first-order in logarithms maintain separability a priori. Thus, the need for having a flexible functional form at least up to the second-order was great until Christensen, Jorgenson and Lau developed the Translog function.

In a recent article on the choice of functional forms, Eli Applebunn (1979) carried out parametric tests to discriminate among the Translog, generalized Leontief and square rooted quadratic functional forms for production and indirect production functions. He concluded that the generalized Leontief and the square rooted quadratic are preferred choices for the primal and dual representation of technology, respectively. However, due to computational ease and wide popularity, the Translog functional form has been chosen for the empirical analysis that follows.

Section IV.3: Econometric Specification and Estimation Procedure

The reproducible cost function of section (IV.1) is CR = CR(Y, W, N, X, e, T) where W is the vector of input prices, e.g., prices of capital (K), labor (L), natural resource (coal) (n), and the prices of equipment and material used for protecting air pollution (e). However, data limitation on the stock of extractible and the environmental resource make these two variables, X and e, drop out of the estimating equation.

Thus, one can write the Translog, functional form of the reproducible cost function, CR(Y, W, N, T), as follows:

$$\ln CR = a_0 + a_y \ln Y + \sum_i a_i \ln W_i + a_N \ln N + a_T T$$

$$+ \frac{1}{2}[b_{yy}(\ln Y)^2 + \sum_i\sum_j b_{ij} \ln W_i \ln W_j + b_{NN}(\ln N)^2 + b_{TT}T^2]$$

$$+ \sum_i C_{iy} \ln W_i \ln Y + \sum_i C_{iN} \ln W_i \ln N + \sum_i C_{iT} \ln W_i T$$

$$+ C_{yN} \ln Y \ln N + C_{yT}(\ln Y)T + C_{NT}(\ln N)T \quad . . . \quad (4.3.1)$$

where i = K, L, n, e.

In order to correspond to a well-behaved production function, a cost function must be homogeneous of degree one in prices, i.e., for a fixed level of output, total cost must increase proportionately when all prices increase proportionately (see Christensen and Greene 1976). This, together with the symmetry condition, imply the set of restriction on the parameters shown in equations (4.3.2) and (4.3.3).

$$\sum_i a_i = 1 \quad . . . \quad (4.3.2)$$

$$\sum_i bij = \sum_j bji = 0 \quad . . . \quad (4.3.3)$$

where i, j = K, L, n and e.

Also, if it is assumed that there exist Hicks neutral technical change and homotheticity of the production function in reproducible inputs, then some additional restrictions follow, which are shown in equations (4.3.4) and (4.3.5),

$$C_{iT} = 0 \quad . . . \quad (4.3.4)$$

and

$$C_{yi} = 0;\ C_{yT} = 0 \quad . . . \quad (4.3.5)$$

where i = K, L, N, e.

Thus, the translog cost function in equation (4.3.1) with the homotheticity and Hicks neutrality assumptions imposed reduces to the function shown in equation (4.3.6).

$$\begin{aligned}\ln CR = {} & a_o + a_y \ln Y + a_K \ln W_K + a_L \ln W_L + a_n \ln W_n + a_e \ln W_e \\ & + a_N \ln N + a_T T + \tfrac{1}{2}\,[b_{yy}(\ln Y)^2 + b_{KK}(\ln W_K)^2 + b_{LL}(\ln W_L)^2 \\ & + b_{nn}(\ln W_n)^2 + b_{ee}(\ln W_e)^2 + 2b_{KL} \ln W_K \ln W_L \\ & + 2b_{Kn} \ln W_K \ln W_n + 2b_{Ke} \ln W_K \ln W_e + 2b_{Ln} \ln W_L \ln W_n \\ & + 2b_{Le} \ln W_L \ln W_e + 2b_{ne} \ln W_n \ln W_e + b_{TT}T^2] \\ & + C_{KN} \ln W_K \ln N + C_{LN} \ln W_L \ln N + C_{nN} \ln W_n \ln N \\ & + C_{eN} \ln W_e \ln N \qquad \ldots \quad (4.3.6)\end{aligned}$$

In order to estimate equation (4.3.6) econometrically, a disturbance term is <u>added</u> to the above equation with the following assumptions regarding the error term. Let the random disturbance term be denoted as U_t. U_t is a random variable which can assume any value $(0, \infty]$. Stochasticity in the cost function can be incorporated this way. It is reasonable to assume standard behavior of such a disturbance term such that statistical tests can be performed. Thus, it is assumed that this random disturbance term is distributed normally, i.e.,

$$E(U_t) = 0 \qquad \ldots \quad (4.3.7)$$

$$E(U_t^2) = \sigma^2 \qquad \ldots \quad (4.3.8)$$

and

$$U_t \sim N(0, \sigma^2) \qquad \ldots \quad (4.3.9)$$

Here, equations (4.3.7) and (4.3.8) imply that the error term has a mean zero and constant variance (σ^2) and equation (4.3.9) imply that it is distributed normally with mean zero and variance σ^2 (constant). However, in the real estimation the homoskedasticity assumption (i.e., constant variance) has been relaxed since the data used for estimation are pooled.

Thus, equation (4.3.6) with an additive error term U_t becomes the equation to be estimated. Note that in the parent production function (equation 4.2.2), the error term might have entered as a multiplicative term and when a logarithmic transformation has been made, it has been expressed as an additive term. The multiplicative assumption is reasonable if it is admitted that the errors are proportional to the scale of operation (see Dhrymes 1970).

Thus, equation (4.3.6) can be rewritten as:

$$\begin{aligned}
\ln CR = {} & a_o + a_y \ln Y + a_K \ln W_K + a_L \ln W_L + a_n \ln W_n + a_e \ln W_e \\
& + a_N \ln N + a_T T + \tfrac{1}{2} [b_{yy}(\ln Y)^2 + b_{KK}(\ln W_K)^2 + b_{LL}(\ln W_L)^2 \\
& + b_{nn}(\ln W_n)^2 + b_{ee}(\ln W_e)^2 + 2b_{KL} \ln W_K \ln W_L \\
& + 2b_{Kn} \ln W_K \ln W_n + 2b_{Ke} \ln W_K \ln W_e + 2b_{Ln} \ln W_L \ln W_n \\
& + 2b_{Le} \ln W_L \ln W_e + 2b_{ne} \ln W_n \ln W_e + b_{TT}T^2] \\
& + C_{KN} \ln W_K \ln N + C_{LN} \ln W_L \ln N + C_{nN} \ln W_n \ln N \\
& + C_{eN} \ln W_e \ln N + U_t \qquad \ldots \quad (4.3.10)
\end{aligned}$$

Given this large number of parameters to be estimated, a large data set is required in order to obtain enough degrees of freedom to statistically test the significance of the estimated parameters. One way of obtaining a large number of observations is to use cross sectional and time series data sets that have been pooled. However, pooling of data has certain advantages and disadvantages. One of the primary advantages is that it increases the number of observations (e.g., if there are T time periods and N cross sectional units which yield N x T observations). A major disadvantage is that of correctly specifying a model that will adequately allow for differences in behavior over cross sectional units as well as any differences in behavior over time for a given cross sectional unit. Furthermore, once a model is specified, there is an additional problem of determining the most efficient procedure to use for testing hypotheses about the parameters (see Judge et al. 1982). In order to avoid controversy regarding which specification and what estimation technique is best, a simple but rigorous model is specified for the estimation of the Translog cost function in equation (4.3.10).

A cross sectionally heteroskedastic and time-wise autoregressive model has been selected for estimation. The reason this specification has been selected is due to an assumption that the cross sectional observations, i.e., the observations on different firms (e.g., cost of capital, labor, etc.) will vary substantially in magnitude. As a result, the assumption of homoskedasticity (constant variance) is not plausible on a priori grounds since it is reasonable to expect that there will be less variation in the cost of some inputs (e.g., capital,

labor) among small firms than there may be for large firms. The time series data would also suggest that the disturbances are autoregressive though not necessarily heteroskedastic. Thus, combining these two assumptions, a cross sectionally heteroskedastic and time-wise autoregressive model has been selected (see Kmenta 1986).

The assumed characteristics of the disturbance term U_t are shown in equations (4.3.11) through (4.3.16).

$$E(U_{it}^2) = \sigma_i^2 \qquad \ldots (4.3.11)$$

$$E(U_{it}\, U_{jt}) = 0, \text{ for } i \neq j \qquad \ldots (4.3.12)$$

and

$$U_{it} = \rho_i U_{i,\, t-1} + \varepsilon_{it} \qquad \ldots (4.3.13)$$

where

$$\varepsilon_{it} \sim N(0, \sigma_{\varepsilon i}^2) \qquad \ldots (4.3.14)$$

and

$$U_{io} \sim N(0, \frac{\sigma_{\varepsilon i}^2}{1 - \rho_i^2}) \qquad \ldots (4.3.15)$$

and

$$E(U_{i,\, t-1},\, \varepsilon_{jt}) = 0 \text{ for all } i, j \qquad \ldots (4.3.16)$$

Here equation (4.3.11) imply heteroskedasticity; cross sectional independence is implied by equation (4.3.12). Since it has been mentioned that the model presupposes time series wise autoregressive, the disturbance term, U_{it}, is assumed to follow an autoregression of order one (AR(1)), which is indicated in equation (4.3.13). Note that ρ is the autoregression coefficient. Equations (4.3.14) and (4.3.15) indicate

that the disturbances in the model are normally distributed and are independent of each other as reflected in equation (4.3.16).

Thus, the estimating model becomes a "generalized linear regression" model (see Kmenta 1986). Consistent and best linear unbiased estimates of the parameters of equation (4.3.10) can be obtained by using the formula shown in equation (4.3.17).

$$\beta = (X'\Omega^{-1}X)^{-1}(X'\Omega^{-1}Y) \qquad \ldots (4.3.17)$$

where β stands for the estimated parameter (in the model under estimation, i.e., equation (4.3.10), this is equivalent to any of the b_{ij}s), X stands for the matrix of the explanatory variables, Y is the vector of the dependent variables, and Ω stands for the estimated variance-covariance matrix of the random disturbance term of the model. In actual estimation, the following steps have been followed.

First, ordinary least squares (OLS) method has been applied to all N x T observations. Here N represents the number of cross sectional units and T represents time. The resulting estimates of the regression coefficients are unbiased and consistent and can be used to calculate the regression residuals, e_{it}. From these residuals, estimates of the autoregression coefficients, ρ_i, can be obtained by using this formula:

$$\rho_i = \frac{\Sigma e_{it} e_{i,t-1}}{\Sigma e^2_{i,t-1}} \qquad \ldots (4.3.18)$$

where t = 2, 3, ..., T, and i = 1, 2, ..., N.

The purpose of estimating these autoregression coefficients is to remove autocorrelation from the model. This is achieved in the next

step by transforming the variables as follows. For notational convenience let equation (4.3.10) be represented by the following standard multiple linear regression equation.

$$Y_{it} = \beta_1 X_{it,1} + \beta_2 X_{it,2} + \ldots + \beta_K X_{it,K} + \varepsilon_{it} \qquad \ldots (4.3.19)$$

where i = 1, 2, ..., N, and t = 1, 2, ..., T.

Corresponding to equation (4.3.10), the variable in the left-hand side, the dependent variable Y_{it} in the above equation, stands for Ln CR and β_j's (j = 1,2, ..., K) stand for the coefficients of the variables in the right-hand side of equation (4.3.10).

In the first step mentioned above, equation (4.3.19) is estimated (in actuality 4.3.10 is estimated), and then the ρ_i's are utilized to transform the variables as follows:

$$Y^*_{it} = \beta_i X^*_{it,1} + \beta_2 X^*_{it,2} + \ldots + \beta_K X^*_{it,K} + \varepsilon^*_{it} \qquad \ldots (4.3.20)$$

where

$$Y^*_{it} = Y_{it} - \rho_i Y_{i,t-1} \qquad \ldots (4.3.21)$$

$$X^*_{it,K} = X_{it,K} - \rho_i X_{i,t-1,K} \qquad \ldots (4.3.22)$$

and

$$\varepsilon^*_{it} = U_{it} - \rho_i U_{i,t-1} \qquad \ldots (4.3.23)$$

The purpose of transforming the variables in the above way is to estimate the variances, $\sigma^2_{\varepsilon i}$ from observations that are at least asymptotically nonautoregressive. For this, OLS has been applied to equation (4.3.20) on N(T-1) number of observations. The resulting regression residuals can be used to estimate the variances, $\sigma^2_{\varepsilon i}$ by using

$$S^2_i = \frac{1}{T-K-1} \sum_{t=2}^{T} \varepsilon^{*2}_{it} \qquad \ldots (4.3.24)$$

Note that since ρ_i is a consistent estimator of ρ_i, S^2_i is a consistent estimator of $\sigma^2_{\varepsilon i}$.

So far, the procedure has illustrated how to obtain consistent estimators of ρ_i and $\sigma^2_{\varepsilon i}$, but nothing has been mentioned about the parameter estimates. In order to complete that task, equation (4.3.17) must be rewritten as:

$$\beta = (X'\Omega^{-1}X)^{-1}(X'\Omega^{-1}Y) \qquad \ldots (4.3.17a)$$

where Ω, mentioned above, is the variance covariance matrix of the error term Ω. The variance-covariance matrix can be written as:

$$\Omega = \begin{bmatrix} \sigma_1^2 \rho_1 & 0 \ldots & 0 \\ 0 & \sigma_2^2 \rho_2 \ldots & 0 \\ 0 & 0 \ldots \sigma_N^2 & \rho_N \end{bmatrix} \qquad \ldots (4.3.25)$$

$$\text{where} \quad \rho_i = \begin{bmatrix} 1 & \rho_i & \rho_i^2 & \ldots & \rho_i^{T-1} \\ \rho_i & 1 & \rho_i & \ldots & \rho_i^{T-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \rho_i^{T-1} & \rho_i^{T-2} & \rho_i^{T-3} & & 1 \end{bmatrix} \qquad \ldots (4.3.26)$$

where the ρ_i's are the autoregression coefficients of the first-order autoregressive scheme.

Thus, the task of deriving the estimates of the elements of the variance-covariance matrix has been completed. In order to obtain the

consistent estimators of the parameters the estimated variance-covariance matrix Ω can be placed in equation (4.3.17). However, the evaluation of equation (4.3.17) is computationally burdensome. For this, a slightly different procedure has been applied. Since the generalized least squares estimator is equivalent to applying ordinary least squares to a set of transformed data (see Johnston 1972), the following transformation has been made before applying ordinary least squares. Thus, the results obtained from this procedure would yield almost identical results to the computationally burdensome exercise described above (see Kmenta 1986). For this, the observations have to be subjected to a double transformation. The first transformation, which has been done in step one and two, was to remove autoregression. The third step is to remove heteroskedasticity. After estimating the variances from the autoregression-free observations, there still remains another transformation of the observations, which is achieved by dividing both sides of equation (4.3.20) by $S_{\varepsilon i}$ obtained from equation (4.3.24). Thus, the final estimating equation becomes:

$$Y^{**}_{it} = \beta_1 X^{**}_{it} + \beta_2 X^{**}_{it,2} + \ldots + \beta_K X^{**}_{it,K} + \varepsilon^{**}_{it} \qquad \ldots (4.3.27)$$

where $$Y^{**}_{it} = \frac{Y^{*}_{it}}{S_{\varepsilon i}} \qquad \ldots (4.3.28)$$

$$X^{**}_{it,K} = \frac{X^{*}_{it,K}}{S_{\varepsilon i}} \quad (K = 1, 2, \ldots, K) \qquad \ldots (4.3.29)$$

and $$\varepsilon^{**}_{it} = \frac{\varepsilon^{*}_{it}}{S_{\varepsilon i}} \qquad \ldots (4.3.30)$$

for t = 2, 3,..., T, and i = 1, 2,..., N.

The disturbances term, ε_{it}^{**}, is asymptotically nonautoregressive and homoskedastic.[3] Finally, equation (4.3.27) can be estimated using ordinary least squares and all the N(T-1) pooled observations. The results are reported in the following chapter.

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[3]The section on the estimation procedure has been adopted from Kmenta (1986).

CHAPTER V

ESTIMATING EQUATION AND RESULTS OF ANALYSIS

Section V.1: Introduction

Coal is one of the most important sources of energy in present America. Since the escalation of oil prices in mid-1970s, a rapid shift towards coal has been observed in the electric power generation industry. Over a period of 35 years (1949-84), net generation of electricity from the use of coal has been increased from 135 billion kilowatt hours to 1,341 billion kilowatt hours, an increase of almost 900 percent or an average yearly growth of 26 percent (see Annual Energy Review 1984). Consumption of coal by the electric industry has also increased from 84 million short tons to 664 million short tons, about 20 percent per annum. This massive increase in the consumption of coal by the electricity industry utilizes most of the coal mined. Today, almost 75 percent of the entire annual coal extracted goes to the electricity industry. Future projections by the Energy Information Administration (1984) indicate that it is going to increase even more.

Traditional measures of scarcity have suggested this natural resource as relatively abundant. Renewed interest in the environmental impact of coal use in terms of sulphur oxide, nitrus oxide, and other emissions by the electric industries has prompted the selection of coal as the representative resource for this study.

In a study on U.S. coal and the electric power industry, Richard L. Gordon (1975) pointed out that developments in coal and its increased use has significant environmental conflicts which have resulted in numerous public policies and regulations intended to control its use. This, in turn, has driven up the cost of coal use. If these costs of regulations are adequately accounted for, the resource may become increasingly scarce.

For actual estimation, data for cost of generating electricity where coal is the major source of fuel has been utilized. Historical steam electric plant construction costs and production expenditures which were reported annually by the Federal Power Commission up until 1975 and then reported by the Energy Information Administration of the Department of Energy (DOE) of the U.S. government have been utilized as the major sources of data. See Appendix H for a detailed discussion of the data set.

Resource scarcity is an intertemporal concept. A resource may be scarce today but might not be so in future due to new exploration, substitution, etc. Impacts considered over time may be a reflection of true scarcity. For this study, a total time period of 43 years (1940-82) has been selected as a reference period. Over this time period, certain regulatory changes have been imposed on the coal-fired electric utility industry. During 1969 and then again in 1976, the Environmental Protection Agency (EPA) imposed certain standards on coal use with particular emphasis on sulphur oxide emissions and particulate controls. In order to meet these regulations, additional costs were incurred by electric power generating industries. The "free" environment

(air) where waste products formerly could be discharged ceased to be "free." Thus, it became more costly to use coal. This additional cost from 1970 through 1982 has been incorporated in the model.

Section V.2: Sampling Technique

Over the period of study (1940-82) there have been a number of additions and deletions in the total number of electric power plants using coal as the major source of energy. Therefore, this information has been included in the estimation process. Moreover, in order to account for variations (heteroskedasticity) due to size differences in terms of capacity and net generation of electricity, a stratified random sampling without replacement (SRSWOR) method has been adopted for selecting the sample power plants.

The selection procedure consisted of the following steps. First, four strata were selected depending on the installed generating capacity. Stratum one has been designated to those plants that have a generating capacity between 1 and 250 megawatt. Stratum two represents those plants that have a generating capacity from 251 up to 500 megawatt, and stratum three and four represent capacity from 501 up to 1000 and 1001 and above, respectively. The total number of plants in each category were earmarked serially. A total sample of size 10 was picked from these stratum population such that the proportion in the stratum population was reflected in the sample size. That is, if the proportion of plants in the stratum population sizes was 40:20:20:20, then the samples from the first strata become 4 and 2 each from the rest of the 3 strata. Each of the 10 samples were picked by using a four-digit random number table. Once a number was selected, the same number has not been used in

the sample drawing process. This process has been repeated for each year. For the entire period (i.e., 1940-82), 430 observations have been selected.

Section V.3: Empirical Results

The study period (1940-82) was split up into two periods for estimation purposes because of changes in the manner in which environmental protection costs were incorporated into the plants' operating costs. It has been assumed that prior to the regulations by the EPA in 1969, few electric power plants incurred any sizeable expenditure for environmental protection. Even if they had, no consistent estimates are obtainable from the individual plant statistics. However, following the imposition of federal regulations, utility industries have been compelled to abide by the law. Hence, environmental protection has become a serious (albeit, forced) concern in the coal-fired electric utility companies. Thus, in the empirical estimation, the period (1940-69) represents Scenario I where little explicit concern towards environmental protection was taken. Expenditures regarding air quality protection were not incurred and are not reflected in cost function estimates. The other period (1970-82) represents Scenario II where the individual firms had to incur additional expenditures to protect air quality. Thus, two models following equation (4.3.10) have been estimated.

Rewriting the estimating equation of (4.3.10) under these two different scenarios may help illustrate the exact specification of the two models. The model for Scenario I is as follows:

$$L_nCR = a_o + a_y \text{ Ln } Y + a_K \text{ Ln } W_K + a_L \ln W_L + a_n \ln W_n + a_N \ln N$$
$$+ a_T T + \tfrac{1}{2}[b_{yy}(\ln Y)^2 + b_{KK}(\ln W_K)^2 + b_{LL}(\ln W_L)^2$$
$$+ b_{nn}(\ln W_n)^2 + 2b_{KL} \ln W_K \ln W_L + 2b_{Kn} \ln W_K \ln W_n$$
$$+ 2b_{Ln} \ln W_L \ln W_n + b_{TT}T^2] + C_{Kn} \ln W_K \ln N$$
$$+ C_{LN} \ln W_L \ln N + C_{nN} \ln W_N \ln N + U_t \qquad \ldots \quad (5.3.1)$$

The model for Scenario II is:

$$\ln CR = a_o + a_y \ln Y + a_K \ln W_K + a_L \ln W_L + a_n \ln W_n + a_e \ln W_e$$
$$+ a_N \ln N + a_T T + \tfrac{1}{2}[b_{yy}(\ln Y)^2 + b_{KK}(\ln W_K)^2 + b_{LL}(\ln W_L)^2$$
$$+ b_{nn}(\ln W_n)^2 + b_{ee}(\ln W_e)^2 + 2b_{KL} \ln W_K \ln W_L$$
$$+ 2b_{Kn} \ln W_K \ln W_n + 2b_{Ke} \ln W_K \ln W_e + 2b_{Ln} \ln W_L \ln W_n$$
$$+ 2b_{Le} \ln W_L \ln W_e + 2b_{ne} \ln W_n \ln W_e + b_{TT}T^2]$$
$$+ C_{KN} \ln W_K \ln N + C_{LN} \ln W_L \ln N + C_{nN} \ln W_n \ln N$$
$$+ C_{eN} \ln W_e \ln N + U_t \qquad \ldots \quad (5.3.2)$$

<u>Scenario I (1940-1969)</u>

Notice that there are six explanatory variables absent in the estimating equation under Scenario I. The estimated coefficients with their appropriate "t" statistics are reported in Table 5.3.1.

In order to calculate the shadow prices from the estimated reproducible cost function equation (4.1.16) is referred again. In that

TABLE 5.3.1

Parameter Estimates of the Reproducible Cost Function Under Scenario I (Equation 5.3.1)

Coefficients	Estimated Values	"t" Values	$R^2 = 0.5358$ $F = 19.078$
Constant	-5.2399	-0.62081	
a_y	-0.0795	-0.8204	
a_K	-0.2552	-0.9170	
a_L	0.7927	1.0571	
a_n	1.5901	1.2743*	
a_N	0.1231	0.1830	
a_T	0.0199	1.2494*	
b_{yy}	-0.0137	-0.8967	
b_{KK}	-0.0198	-0.6066	
b_{LL}	0.2968	1.2221*	
b_{NN}	-0.2481	-0.7314	
b_{KL}	-0.6458	-0.9415	
b_{KN}	0.0499	0.7329	
b_{LN}	-0.1969	-1.0375	
b_{TT}	-0.0003	-0.3456	
C_{KN}	-0.0829	-0.9831	
C_{LN}	-0.1105	-0.7381	
C_{nN}	0.0669	0.3869	

*Indicates the values are significant at 85 percent level of confidence.

equation it has been shown that the negative of the partial derivative of the estimated cost function with respect to N yields shadow price. This is obtained as follows:

Differentiating ln CR with respect to N yields:

$$\frac{d(\ln CR)}{d(\ln N)} = \frac{dCR}{dN} \cdot \frac{N}{CR} \qquad \ldots \quad (5.3.3)$$

The left-hand side can be obtained from equation (5.3.1) and is shown in equation (5.3.4).

$$\frac{d \ln CR}{d \ln N} = a_N + C_{KN} \ln W_K + C_{LN} \ln W_L + C_{nN} \ln W_n \qquad \ldots \quad (5.3.4)$$

Now, equations (5.3.4) and (5.3.3) can be used to derive equations (5.3.5) and (5.3.6).

$$\frac{dCR}{dN} \cdot \frac{N}{CR} = a_N + C_{KN} \ln W_K + C_{LN} \ln W_L + C_{nN} \ln W_n \qquad \ldots \quad (5.3.5)$$

$$\frac{dCR}{dN} = (a_N + C_{KN} \ln W_K + C_{LN} \ln W_L + C_{nN} \ln W_n)\frac{CR}{N}^{1} \qquad \ldots \quad (5.3.6)$$

The required shadow prices are thus obtained just by taking the negative of the right-hand side of equation (5.3.6). This procedure has been followed in order to calculate the shadow prices for different years under Scenario I. Note that the parameters under the parentheses of equation (5.3.6) are obtained directly from the estimated regression equation, i.e., from Table 5.3.1.

---

[1]CR is the estimated cost function. Since exponentiation results in bias, Goldberger's (1968) suggested technique has been followed.

However, sampling weights are used to obtain the weighted average of the variables in equation (5.3.6). The shadow prices are reported in Table 5.3.2.

In Table 5.3.2 the estimated shadow prices (μ_1) only (because μ_2 is zero in this model due to absence of the expenditure on environmental protection in the cost function) are compared against alternative measures of scarcity. One such alternative indicator is the market price of the resource, here the market price of coal. As shown in Figure (5.3.1), the real market price has not changed much over this entire period (1940-69). Compared to the market price, the estimated shadow price has fallen significantly. The third alternative measure of scarcity, unit extraction cost shown in Table 5.3.2 and Figure (5.3.1), also indicates a downward trend. So, it is interesting to note that these three <u>different</u> indicators show a downward trend. The argument against unit extraction cost is that it does not incorporate the substitution effect. In addition, it can be considered a very rough estimate in computational sense. The unit extraction cost as reflected here has been obtained by dividing the average wage rate in the electric industry by the output per man hour (average productivity of labor), then, converted to cents per million btu. Thus, it is recognized that such a measure is only a proxy and is not a true measure of unit extraction cost since the contribution of capital is missing from this statistic. As a measure of resource scarcity, though, it has a similar trend to that of the estimated shadow price, it cannot be accepted. Whether or not scarcity is reflected in the market price has already been addressed in Chapter II. However, an additional point should be made. Market

TABLE 5.3.2

Scarcity Indexes for Coal During 1940-1969

Year	Estimated Real Shadow Price of Unextracted Coal (¢/mln btu) Actual	Index (1970=100)	Real Market[1] Price of Coal (¢/mln btu) Actual	Index (1970=100)	Real Unit[2] Extraction Cost of coal (¢/mln btu) Actual	Index (1970=100)
1940	0.47	235.0	0.22	67.0	0.16	194.0
1941	0.43	215.0	0.23	71.0	0.15	182.0
1942	0.38	190.0	0.22	67.0	0.16	194.0
1943	0.36	180.0	0.23	71.0	0.15	182.0
1944	0.36	180.0	0.24	74.0	0.15	182.0
1945	0.35	175.0	0.26	80.0	0.15	182.0
1946	0.25	125.0	0.25	77.0	0.15	182.0
1947	0.22	110.0	0.25	77.0	0.14	170.0
1948	0.23	115.0	0.28	86.0	0.14	170.0
1949	0.23	115.0	0.28	86.0	0.15	182.0
1950	0.22	110.0	0.27	83.0	0.14	170.0
1951	0.18	90.0	0.25	77.0	0.13	158.0
1952	0.22	110.0	0.25	77.0	0.13	158.0
1953	0.23	115.0	0.26	80.0	0.12	146.0
1954	0.23	115.0	0.24	74.0	0.11	133.0
1955	0.24	120.0	0.23	71.0	0.11	133.0
1956	0.21	105.0	0.24	74.0	0.12	146.0
1957	0.20	100.0	0.25	77.0	0.12	146.0
1958	0.20	100.0	0.24	74.0	0.11	133.0
1959	0.20	100.0	0.23	71.0	0.10	121.0
1960	0.20	100.0	0.23	71.0	0.10	121.0
1961	0.20	100.0	0.22	67.0	0.09	109.0
1962	0.20	100.0	0.22	67.0	0.08	97.0
1963	0.20	100.0	0.21	64.0	0.08	97.0
1964	0.20	100.0	0.21	64.0	0.08	97.0
1965	0.20	100.0	0.21	64.0	0.08	97.0
1966	0.19	95.0	0.21	64.0	0.08	97.0
1967	0.19	95.0	0.21	64.0	0.07	85.0
1968	0.18	90.0	0.21	64.0	0.07	85.0
1969	0.18	90.0	0.22	67.0	0.07	85.0

[1]Real market price of coal has been obtained by dividing the market price by the wholesale price index.

[2]Real unit cost has been calculated by dividing the wage by output per manhour (labor productivity) and then deflated by the wholesale price index.

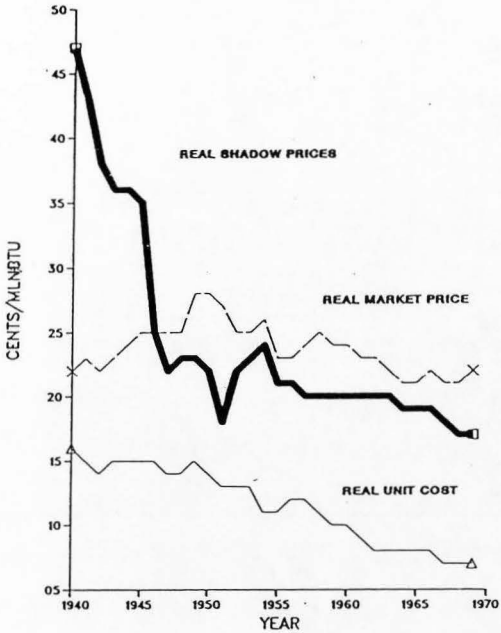

Figure 5.3.1. A plot of estimated real shadow price, real market price and real unit cost during 1940-1969.

price is an average price, an average of spot price, contract price, and administered price. While spot price might reflect competition and probably reflects perceived scarcity to some degree, the others do not. Personal inquiries reflect that in some cases contractual prices are higher than market price and so on (Ghosh and Southern California Edison Company, 1985). Thus, for a resource in use, market price has certain deficiencies in reflecting scarcity. That leaves only the estimated shadow price. In this empirical study a downward trend is observed implying that coal in use was becoming less scarce over the period 1940-69. The supply of coal for electricity generation has been consistently increasing in relation to total use of demand. As noted in various Bureau of Mines Reports (1950, 1971, 1976, 1978, 1980, 1981, 1982, and 1983), there has been a tremendous increase in the recoverable reserve base, only 1.2 billion short tons were reported in 1950, whereas in 1982 the recoverable reserve base has been increased to 26.3 billion. Since N, the extraction of coal, depends on the stock or recoverable reserve (see equation (3.1)), it is likely to see a negative trend in the shadow price of coal. Thus, following the traditional approach where environmental protection is not explicitly treated in a cost function and given these results, it may be concluded coal in use was not becoming scarce over the period 1940-69.

A hypothesis of a positive trend of the shadow price was tested by regressing the shadow prices against time. The null hypothesis could not be accepted on the basis of "t" statistics. The following regression equation was estimated.

$$\mu_t = A + BT \qquad \ldots \quad (5.3.7)$$

where μ_t, represents the estimated shadow prices from equation (5.3.6), A is a constant, T represents time (e.g., 1940 = 1,..., 1969 = 30), and B is the coefficient of regression.

The estimated equation is:

$$\mu_t = \underset{(19.8541)}{0.3553} - \underset{(-7.0104)}{0.0071}T \qquad \ldots (5.3.8)$$

Here the "t" statistics are in parentheses. Thus, one can say that coal in use was not becoming scarce, at least during the period 1940-69.

In order to determine if the shadow price of coal in use changed as environmental constraints were imposed, the estimation procedure was repeated. The only difference was the specification of the model used in the estimation procedure.

Scenario II (1970-1982)

The estimating equation is:

$$\begin{aligned} L_n CR = {} & a_o + a_y \, Ln\, Y + a_K \, Ln\, W_K + a_L \ln W_L + a_n \ln W_n + a_e \ln W_e \\ & + a_N \ln N + a_T T + \tfrac{1}{2}[b_{yy}(\ln Y)^2 + b_{KK}(\ln W_K)^2 + b_{LL}(\ln W_L)^2 \\ & + b_{nn}(\ln W_n)^2 + b_{ee}(\ln W_e)^2 + 2b_{KL} \ln W_K \ln W_L \\ & + 2b_{Kn} \ln W_K \ln W_n + 2b_{Ke} \ln W_K \ln W_e + 2b_{Ln} \ln W_L \ln W_n \\ & + 2b_{Le} \ln W_L \ln W_e + 2b_{ne} \ln W_n \ln W_e + b_{TT}T^2] \\ & + C_{Kn} \ln W_K \ln N + C_{LN} \ln W_L \ln N + C_{nN} \ln W_n \ln N \\ & + C_{eN} \ln W_e \ln N + U_t \qquad \ldots (5.3.9) \end{aligned}$$

The estimated parameters with their "t" statistics is reported in Table 5.3.3.

Again, in order to calculate the shadow prices from the estimated regression equation, the same partial derivative of the reproducible cost function with respect to N is taken, as it has been done in equation (5.3.4) above.

Since there is a difference in the estimating reproducible cost function, so there also will be a difference in the estimating equation for the shadow prices. This is obtained by taking the partial derivative of equation (4.3.10) with respect to N. Finally,

$$\frac{dCR}{dN} = (a_N + C_{KN} \ln W_K + C_{LN} \ln W_L + C_{nN} \ln W_n + C_{eN} \ln W_e)\frac{CR}{N} \quad \ldots (5.3.10)$$

The shadow prices are thus obtained by utilizing the equation (5.3.9) and taking a negative sign of the results from equation (5.3.10). This procedure has been followed in obtaining the shadow prices as recorded in Table 5.3.4.

Once again, the estimated shadow prices are compared against alternative measures of scarcity, e.g., real market price and real unit cost (based on labor productivity). As shown in Figure (5.3.2), the real market price was relatively steady until the oil price shock of 1973. Since then, it has increased sharply. However, during the later years in this period it has been decreasing slightly. The real unit cost has experienced a declining trend. On the other hand, the shadow price ($\mu_1 + \mu_2$) has exhibited a slightly increasing trend in the later part of

TABLE 5.3.3

Parameter Estimates of the Reproducible Cost Function Under Scenario II (Equation 5.3.8)

Estimated Coefficients		"t" Values	$R^2 = 0.5358$ $f = 19.078$
Constant	39068.0	0.2273	
a_y	-0.3944	-3.1706*	
a_K	2.6214	0.8149	
a_L	0.2590	0.2932	
a_n	0.0893	0.1458	
a_e	-2.4958	-0.7874	
a_N	-8.8503	-0.6608	
a_T	-0.1175	-2.0482*	
b_{yy}	0.1421	1.5455*	
b_{KK}	-3325.7	-0.2273	
b_{LL}	0.0348	0.2253	
b_{nn}	0.0101	0.1092	
b_{ee}	-3325.7	-0.2273	
b_{KL}	-0.2383	-0.2906	
b_{Kn}	-0.1381	-2.2573*	
b_{ke}	6651.4	0.2273	
b_{Ln}	0.0868	0.4378	
b_{Le}	0.1356	0.9674	
b_{ne}	-0.0814	-0.6736	
b_{TT}	6.1062	0.7998	
C_{KN}	0.3307	0.2498	
C_{LN}	-1.3172	-1.3791*	
C_{nN}	-5.4032	-0.6996	
C_{eN}	0.0086	1.6563*	

*Values significant at 90 percent confidence level.

TABLE 5.3.4

Scarcity Indexes for Coal During 1970-82

Year	Estimated Real* Shadow Price of Unextracted Coal		Real Unit* Cost of Extraction of Coal		Real Market* Price of Coal	
	Actual	Index	Actual	Index	Actual	Index
1970	19.71	100.00	8.24	100.00	32.60	100.00
1971	19.46	98.73	9.00	109.22	35.57	109.11
1972	18.56	94.16	9.64	116.99	37.39	114.69
1973	15.39	78.08	9.23	112.01	39.33	120.64
1974	15.70	79.65	8.02	97.33	67.60	207.36
1975	18.10	91.83	10.97	133.13	76.04	233.25
1976	19.67	99.79	11.68	141.75	73.19	224.51
1977	18.18	92.24	11.75	142.60	71.36	218.89
1978	17.71	89.85	11.29	137.01	74.21	227.64
1979	19.92	101.06	11.83	143.56	73.42	225.21
1980	19.20	97.41	10.31	125.12	69.88	214.35
1981	22.92	116.29	9.55	115.90	68.74	210.86
1982	21.93	111.26	9.95	120.75	67.15	205.98

*The actual real shadow price, the real marginal extraction cost, and the real market price are in cents per million b.t.u.

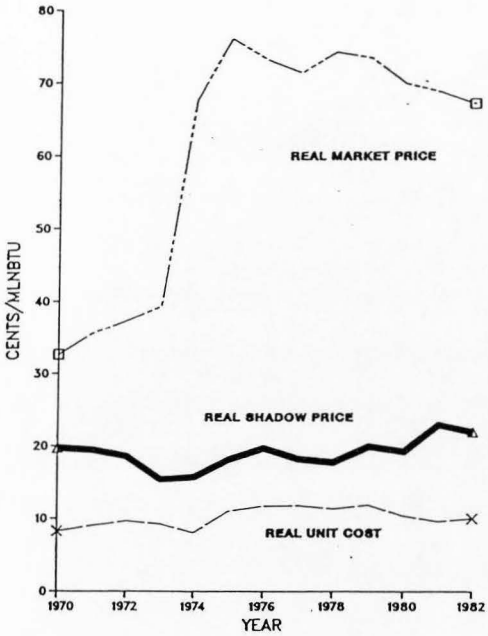

Figure 5.3.2. A plot of real market price, estimated real shadow price and real unit cost during 1970-1982.

this period. Following the same line of reasoning expressed above as in favor of shadow price as a proper measure of resource scarcity, these results suggest that coal in use was becoming relatively more scarce during this period.

A null hypothesis of decreasing trend in the shadow price has been tested by using equation (5.3.7) under Scenario II. The estimated equation is reported below.

$$\mu_t = \underset{(15.3420)}{17.0012} + \underset{(2.0020)^*}{0.2795}T$$

Here, also, the null hypothesis of falling trend has been rejected on the basis of the "t" statistics.

These results might suggest that since the environmental regulations have been imposed, the apparent abundant natural resource coal becomes more scarce at least as it is used in the electricity industry. Note that the electricity industry consumes about 75 percent of entire coal consumption in the present United States. On the other hand, one may suggest that this environmental regulation induced a sudden change in the trend of the shadow prices. This increasing trend in the scarcity indicator may come from the supply side only. In other words, if one observes a decreasing trend in the recoverable reserve base of coal, which is treated as an argument in the extraction subproduction function, and also experiences an increasing demand for coal by the electricity industry, then this positive trend in the scarcity indicator

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*"t" statistics are reported in the parentheses.

might follow. In that case, only the supply demand relationship is sufficient to explain the phenomenon.

However, that is not the case. No doubt that there is a significant increase in the demand for coal by the electricity industry (e.g., consumption has increased from 320.2 million short tons in 1970 to 593.7 million short tons in 1982, or an average of 6.5 percent per year). But at the same time, the recoverable reserve base (defined as the amount of coal that can be recovered (mined) from the coal reserves at existing mines as of the end of the year, see Bureau of Mines Reports) has significantly increased, i.e., from 0.8 billion in 1970 to 26 billion in 1982, or about 24 percent annually on an average over this period. This suggests that the positive trend in the shadow price is not due to a deficit in supply. That again points out to the fact that the environmental regulation for protecting the atmosphere might have caused the scarcity indicator to exhibit a positive trend.

Thus, the empirical study may suggest two significant conclusions. First, during 1940-69, when environmental protection was not of much concern, coal in use, as exhibited by the estimated shadow prices, was not relatively scarce. If this situation were continued, i.e., EPA would not compel the coal users to protect the environment, it is likely to foresee a continuation of the similar trend, as obtained in Figure 5.3.1. However, this is a guess, which is beyond the scope of verification. The second empirical result (for the period 1970-82) may suggest that due to the binding constraint of environmental protection by the EPA, the electric industries had to incur additional expenditures and this is only due to the use of coal. Thus, coal in use becomes

increasingly costlier, which might suggest that coal as it is viewed from its use point has become more scarce.

CHAPTER VI

SUMMARY, CONCLUSIONS, POLICY IMPLICATIONS, AND SCOPE FOR FURTHER RESEARCH

Section VI.1: Summary and Conclusions

Whether natural resources are becoming scarce or not has been addressed in the literature on the basis of various measures of resource scarcity developed over time. The shadow price (marginal user cost), unit cost, and relative market price of a resource are the most commonly used indicators for natural resource scarcity. All of these indicators address the question of a particular resource being scarce from a partial equilibrium standpoint. In other words, these indicators do not view an extractible natural resource and the environment in which it belongs as joint and sometimes inseparable. As such, they underscore the importance of a changing environment due to a change in the stock of an extractible resource. By assuming that the environment remains unchanged or implicitly recognizing it as a common property, these economic indicators of scarcity address the issue of scarcity only in a limited sense. In reality, the extracting and use of an extractible resource (e.g., coal) requires that additional costs be borne by society (firms) in order to maintain the environment (air, land, water) at some level exogeneously determined to be socially optimal. Jointness between an extractible resource and an environmental resource has not yet been examined with its bearing on measures of resource scarcity. The present

study purports to bridge that gap. This is considered to be a more general approach to resource scarcity than modelled previously.

The analysis as shown in Chapter III incorporated the cost of cleaning up or maintaining the environment (air) into the model where profit-maximizing firms have been hypothesized to maximize the discounted net benefits under dynamic conditions in a world of certainty. The results obtained here are much more general and captures earlier results as special cases. It has been shown that the measure developed in this study would analytically indicate a different measure of scarcity if the jointness between the two resources mentioned above is accounted for. It is also shown that the theoretical model built here has at least local stable solutions and under certain conditions it also has a globally stable solution.

The theoretical model in Sections I through III in Chapter III and its empirical counterparts in Chapters IV and V are confined to a deterministic environment. The real world confronted by a society (individual firms) is subjected to uncertainties of various kinds. One such uncertainty may be from the stock of the environmental resource. Since the environmental and the extractible resources have been viewed as joint inputs, the variability in one may induce a certain degree of randomness in the stock of the other. The model in Chapter VI thus attempts to address the question of an uncertain stock size of one of these two joint resources and evaluate its impact on the measure of resource scarcity. Since the basic theoretical model in Sections II and III in Chapter III has utilized optimal control (maximum principle) as the tool of analysis, it is naturally instructive to bring in

uncertainty in the same basic framework. Stochastic optimal control has been specially developed in the literature to handle this type of problem. In this section a stochastic optimal control model has been developed in order to handle uncertainty of the kind mentioned above. It is shown analytically that the scarcity indicator (shadow price) would indicate a slower current extraction of a resource in the event of an uncertain environmental resource stock. In case the resource under consideration is an exhaustible one and assuming there is no positive exploration activities, the scarcity indicator in this chapter would indicate a slower depletion. That means it will be scarcer relatively slowly compared to that of a deterministic world.

In Chapters IV and V an empirical model, consistent with theoretical constructs of Chapter III, has been presented. The shadow price of unextracted coal, which is viewed as an indicator of scarcity, has been estimated and reported. The results in Chapter V show that given the presently mandated level of environmental control, the shadow price appears to be increasing which implies that the resource (coal) as it is in use is becoming scarce. These empirical findings thus strengthen the theoretical underpinning of viewing the extractible and the environmental resource as joint inputs (resources). The inclusion of the environmental costs will result in the resource becoming more scarce relative to a situation where no control or no cost of environmental protection is accounted for.

Section VI.2: Policy Implications

One of the major policy implications of this study may be to point to the necessity of identifying the socially optimal amount of

pollution. In a developing society, more so for an already highly industrialized society, there has to be tradeoffs between the amounts of cleaner environment and material objects. Unless a socially desirable amount of both these goods are determined simultaneously, a partial approach to the problem will be attained. In a quasi-free market economy like the United States, the environment or its optimal amount of pollution is controlled by the government (nonmarket force) whereas the material object or its optimal supply is determined by the marketplace. Thus, there is a controversy or a breakdown in the simultaneous attainment of optimal amounts of both these goods (in a two-good world). This phenomenon may disrupt the true scarcity position of a natural resource. The Department of Energy, instead of issuing a license to generate electricity and a permit to emit pollutants up to a specified level, may issue just one license. In that, the negative externality of discharging pollutants might be internalized by the electricity-generating firms in their cost calculations and thus an optimal amount of both electricity and pollutants might result. In other words, the idea is to establish more property rights by not controlling the amount of pollution that is permissible but by increasing the cost of attaining that property right and then let the market forces take care. Alternatively, a socially desirable amount of both the goods may be dictated upon.

Since uncertainty in the stock of the environmental resource may also cause a resource to be scarce, it might be fruitful in identifying the root cause of such uncertainty and then reduce it to the possible extent. The model built in Section IV of Chapter III presupposes that the randomness in the stock of environmental resource is partly due to

the variability in the stock of the extractible resource. It is also shown there that if there is no variability, then there is no difference between the results obtained here and in a deterministic model. So the immediate policy implication that one might visualize is to attempt to reduce the variability in the stock size of the extractible resource. This might be attained by continuous monitoring of the resource stock and investing more in dissemination of information regarding the stock. Furthermore, research efforts toward identification of the change in the capacity of the environmental sink due to change in the use of extractible resource (coal) might reduce uncertainty and may help in more accurately predicting scarcity.

Section VI.3: Scope for Research

The present study has attempted to generalize the concept of a jointness between an extractible and an environmental resource. As an empirical test, coal has been selected and the shadow prices of coal have been estimated. Before further generalization, more empirical work needs to be done. One of such might be to evaluate the scarcity trend in alternative natural resources. For example, if coal in use is found becoming scarce, then one may be interested to find out the situation for natural gas, petroleum, water resources, etc., from which electricity can be generated. Then rank these resources according to their relative positions. The Department of Energy (DOE) may then use this ranking order in issuing licenses to upcoming electric utility companies in the future. In other words, a proper evaluation in terms of the availability of alternative natural resources might serve the purpose of

a better intertemporal and intergenerational planning than just looking at one.

The model in Section IV of Chapter III has incorporated uncertainty in a very simplistic manner. More rigorous works to that direction can be made. Uncertainty can be brought in through exploration activities as well as through the environmental stock and thus give the result in Chapter VI a much more general approach. One other interesting possibility in incorporating uncertainty may be by approaching the problem from a Baysian point of view. In other words, a prior distribution function of the stock of the extractible resource may be used in obtaining a conditional posterior density function of the stock of the environmental resource and then evaluate the results of scarcity. Empirical works in an uncertain world would be equally important.

One of the major assumptions that has been made in estimating scarcity is that the environmental cost is proportional to other capital cost and fixed. This might not be so. Further research may treat this also as a variable and then evalute the scarcity indicator.

Another important point regarding further empirical research could be in utilizing a different type of functional form for the cost function. A generalized Leontief type cost function can be utilized for this purpose.

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APPENDICES

Appendix A

The firm's objective function in Chapter III is to maximize the discounted present value of net benefits from selling its final output over an infinite time horizon. While producing its output in order to maximize profit, certain assumptions regarding the production functions are deemed necessary. These are as follows:

The production function for the final output is $Y = Y(N(X, K^E, e, T), e, K^P, T)$. Here the final output Y depends on the amount of extractible resource N, which itself is also a production function and the composite capital labor input (K^P), the surrounding environment (e), and the technology (T).

It is assumed here that as N increases, the final output increases or the production function Y is concave with respect to N, i.e.,

$$\frac{\partial Y}{\partial N} = Y_N > 0 \quad \text{and} \quad \frac{\partial^2 Y}{\partial N^2} = Y_{NN} < 0 .$$

Similarly, as the composite capital labor input is employed more and more, the output increases at a diminishing rate, which implies,

$$\frac{\partial Y}{\partial K^P} = Y_{K^P} > 0 \quad \text{and} \quad \frac{\partial^2 Y}{\partial_{K^P}^2} = Y_{K^P K^P} < 0 .$$

However, the change in the stock of the environmental resource has been assumed to be directly related to the change in the output Y.

or, $$\frac{\partial Y}{\partial e} = Y_e > 0 \quad \text{and} \quad \frac{\partial^2 Y}{\partial e^2} = Y_{ee} > 0 .$$

As technology improves, the production of the final output increases but at a diminishing rate. This has been expressed as $\frac{\partial Y}{\partial T} = Y_T > 0$ and $\frac{\partial^2 Y}{\partial T^2} > 0$. It also has been assumed that as the level of the stock of the environmental resource increases along with an increase of the stock of the extractible resource, the output, Y, may increase or may remain the same.

$$\frac{\partial^2 Y}{\partial N \partial e} = Y_{Ne} \geq 0 .$$

Similarly, the extraction subproduction function $N = N(X, e, K^E, T)$ has been assumed to be concave with respect to the composite capital labor input. In other words, if more of this input is employed, then production (extraction) increases but at a diminishing rate, i.e.,

$$\frac{\partial N}{\partial K^E} = N_{K^E} > 0 \quad \text{and} \quad \frac{\partial^2 N}{\partial_{K^2 E}} = N_{K^E K^E} < 0 .$$

It also is assumed that as the stock of the extractible resource increases, the extraction (production) also increases, which is expressed as

$$\frac{\partial N}{\partial x} = N_x > 0 \quad \text{and} \quad \frac{\partial^2 N}{\partial x^2} = N_{xx} > 0 .$$

Similarly, for the stock of the environmental resource, i.e.,

$$\frac{\partial N}{\partial e} = N_e > 0 \quad \text{and} \quad \frac{\partial^2 N}{\partial e^2} = N_{ee} > 0 .$$

N is also assumed to be concave with respect to the level of technology. As technology improves, N increases but at a diminishing rate, i.e,

$$\frac{\partial N}{\partial T} = N_T > 0 \quad \text{and} \quad \frac{\partial^2 N}{\partial T^2} = N_{TT} < 0 .$$

However, it has been assumed that there is no cross effect among the inputs that can result in a change in the production of the extractible resource. This has been expressed as

$$\frac{\partial^2 N}{\partial K^E \partial e} = N_K{}^E e = 0; \quad \frac{\partial^2 N}{\partial K^E \partial x} = N_K{}^E X = 0;$$

$$\frac{\partial^2 N}{\partial K^E \partial T} = N_K{}^E T = 0; \quad \frac{\partial^2 N}{\partial x \partial e} = N_x e = 0; \quad \frac{\partial^2 N}{\partial x \partial T} = N_x T = 0$$

$$\frac{\partial^2 N}{\partial T \partial e} = N_T e = 0 .$$

The cost function has been assumed to be inversely related with respect to the level of stock, i.e.,

$$\frac{\partial C}{\partial x} = C_x < 0 \quad \text{and} \quad \frac{\partial^2 C}{\partial x^2} = C_{xx} < 0 .$$

A similar relationship with the other resource (environment) holds good, i.e.,

$$\frac{\partial C}{\partial e} = Ce < 0 \quad \text{and} \quad \frac{\partial^2 C}{\partial e^2} = Cee < 0 .$$

On the other hand, it has been assumed that the cost function is

directly related with the hiring prices, i.e, $\frac{\partial C}{\partial w} = C_w > 0$ and $\frac{\partial^2 C}{\partial w^2} = C_{ww} > 0$. As the extraction rate, N, increases, the cost also increases at a diminishing rate. This has been technically expressed as:

$$\frac{\partial C}{\partial N} = C_N > 0 \quad \text{and} \quad \frac{\partial^2 C}{\partial N^2} = C_{NN} < 0 .$$

The cost is assumed to decrease as the level of technology improves, i.e.,

$$\frac{\partial C}{\partial T} = C_T < 0 \quad \text{and} \quad C_{TT} > 0 .$$

Thus, in general, it can be said that the production function for the final output (Y) is concave with respect to the composite capital labor input (K^P) used for processing, the level of extracted resource (N), and the level of technology (T). While it has been assumed that this production function is convex with respect to the level of the stock of environmental resource, the extraction subproduction function has been assumed to be concave with respect to the composite capital labor input (K^E) and the level of technology (T). However, the same extraction subproduction function is assumed to be convex with respect to the stocks of the two resources (x and e). The cost function, which is dual to the extraction subproduction function, has been assumed to be concave with respect to the level of output (N) and the stocks of resources, whereas it is convex with respect to the hiring price and the level of technology.

Appendix B

From the necessary first-order condition in equation (3.1.9),

$$H_K^E = \frac{\partial H}{\partial K^E} = P_y Y_N N_K^E - C_N N_K^E - {}_2N_K^E$$

From equation (3.1.4), the second partial derivative of the Hamiltonian is:

$$H_{KK}^{EE} = \frac{\partial^2 H}{\partial_K E^2} = P_y Y_N N_{KK}^{EE} - C_N N_{KK}^{EE} - \mu_2 N_{KK}^{EE} \qquad \ldots \qquad (B.1)$$

Since N_{KK}^{EE} is less than zero by assumption, the sign of this equation could be less than, equal to, or greater than zero. However, it is reasonable to assume that the absolute value of the marginal product (i.e., $P_y Y_N$) is greater than (or equal to) the marginal cost (αC_N). Otherwise, it would not be profitable to continue production. Hence, the above expression has a negative sign.

Each of the components of the quadratic form shown in equation (3.1.8) must be evaluated. The sign of first component,

$$H_{KK}^{PP} = \frac{\partial^2 H}{\partial_K P^2} = P_y Y_K P_K P \qquad \ldots \qquad (B.2)$$

is negative since $Y_K P_K P$ is less than zero by assumption.

The sign of

$$H_{xx} = \frac{\partial^2 H}{\partial x^2} = \frac{\partial}{\partial x}[P_y Y_N N_x - \alpha C_x + \mu_1(f'(x) - N_x) - \mu_2 N_x]$$

$$= P_y Y_N N_{xx} - \alpha C_{xx} + \mu_1 f''(x) - \mu_1 N_{xx} - \mu_2 N_{xx} \qquad \ldots \qquad (B.3)$$

can be positive, negative, or equal to zero.

Next, the sign of

$$H_{ee} = \frac{\partial^2 H}{\partial e^2} = \frac{\partial}{\partial e}[P_y Y_N N_e + P_y Y_e - \alpha C_N N_e - \alpha C_e - \mu_1 N_e + \mu_2 - \mu_2 N_e]$$

$$= P_y Y_N N_e + P_y Y_{ee} - \alpha C_{ee} - \alpha C_{Ne} N_e \qquad \ldots \qquad (B.4)$$

could be positive, negative, or equal to zero. However, the sign could be said to be positive under the assumption that the rate of change of extraction cost with respect to the level of the stock of the environmental resource is zero.

In order to evaluate the sign for

$$H_{xe} = \frac{\partial}{\partial e}[P_y Y_N N_x - \alpha C_N N_x - \alpha C_x + \mu_1(f'(x) - N_x) - \mu_2 N_x]$$

$$= P_y Y_{Ne} N_x - N_{xe}(\alpha C_{Ne} + \alpha C_{xe}) \qquad \ldots \qquad (B.5)$$

It must be assumed that the level of stock of the extractible natural resource is independent of the stock of the environmental asset. N_{xe} and C_{xe} become zero and, since $Y_{Ne} \geq 0$ by assumption, the expression is either equal to zero or has a positive sign.

Thus,

$$\underset{-}{H_{KK}^{EE}}\underset{-}{H_{KK}^{PP}}\underset{+}{H_{xx}}\underset{+}{H_{ee}} - \underset{+}{H_{xe}^2}\underset{-}{H_{KK}^{EE}}\underset{-}{H_{KK}^{PP}} \gtreqless 0$$

However, the possibility of the quadratic form being negative is ruled out by the concavity assumption.

Appendix C

Figure C.1 under the following condition shows that there is an optimal steady state time path, $S^R S^R$ which is convergent at Q.

(i-b) $d(\frac{d\mu_1}{dt}) > 0$ and $d(\frac{dx}{dt}) < 0$

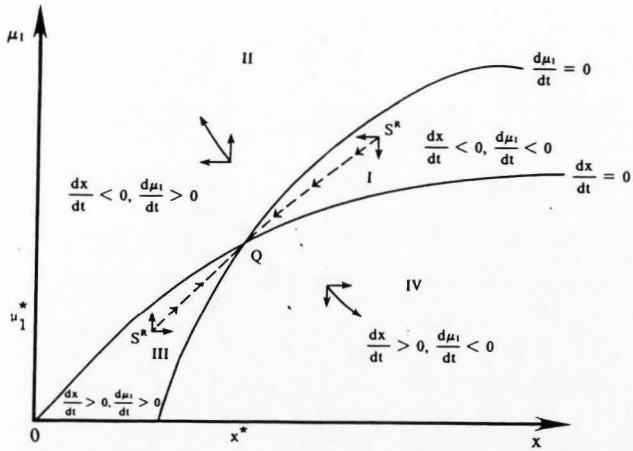

Figure C.1. A phase diagram of stock of a renewable resource and its shadow price when $d(\frac{dx}{dt}) < 0$ and $d(\frac{d\mu_1}{dt}) > 0$.

Under the set of conditions in (i-c), no vector movement is found in Figure C.2 that could lead the system to an optimal steady state. This is shown in Figure C.2.

(i-c) $d(\frac{d\mu_1}{dt}) > 0$ and $d(\frac{dx}{dt}) = 0$

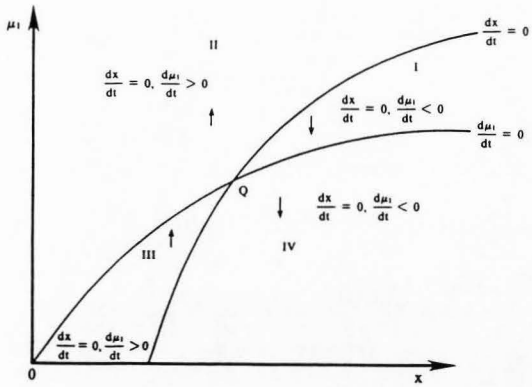

Figure C.2. A phase diagram of stock of renewable resource and its shadow price when $d(\frac{dx}{dt}) = 0$ and $d(\frac{d\mu_1}{dt}) > 0$.

Figure C.3 under the set of conditions in (i-d) shows that there is no optimal steady state time path which is convergent at Q.

(i-d) $d(\frac{d\mu_1}{dt}) < 0$ and $d(\frac{dx}{dt}) < 0$

Figure C.4, under the set of conditions (i-e) shows that there is an optimal steady state time path, $S^R S^R$. In all other quadrants, the vector movements are such that the system is divergent, but the vector movements in quadrants II and IV lead the system to converge at Q'.

(i-e) $d(\frac{d\mu_1}{dt}) < 0$ and $d(\frac{dx}{dt}) > 0$

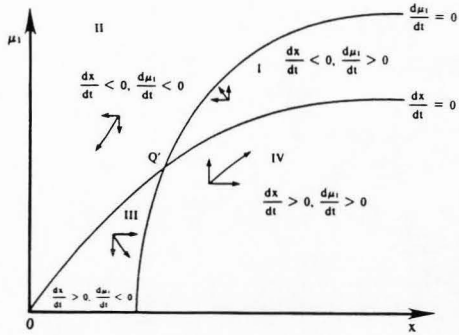

Figure C.3. A phase diagram of stock of renewable resource and its shadow price when $d(\frac{dx}{dt}) < 0$ and $d(\frac{d\mu_1}{dt}) < 0$.

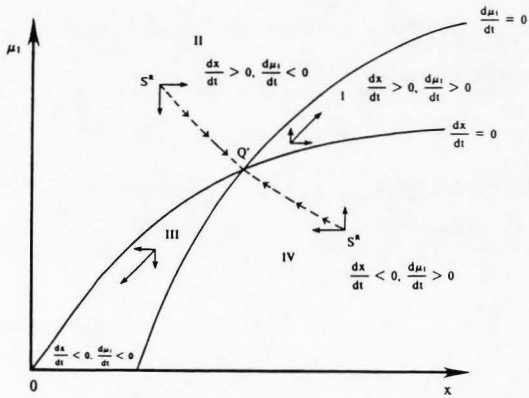

Figure C.4. A phase diagram of stock of renewable resource and its shadow price when $d(\frac{dx}{dt}) > 0$ and $d(\frac{d\mu_1}{dt}) < 0$.

(i-f) $d(\frac{d\mu_1}{dt}) < 0$ and $d(\frac{dx}{dt}) = 0$

Here, again, under the condition in (i-f), there is no optimal steady state solution (see Figure C.5). In fact, there is no vector movement that can lead the system to an optimal solution. Similarly, it can be stated that if any of the signs in the half spaces (i.e., $d(\frac{d\mu_1}{dt})$ or $d(\frac{dx}{dt})$) is zero, then there is no solution.

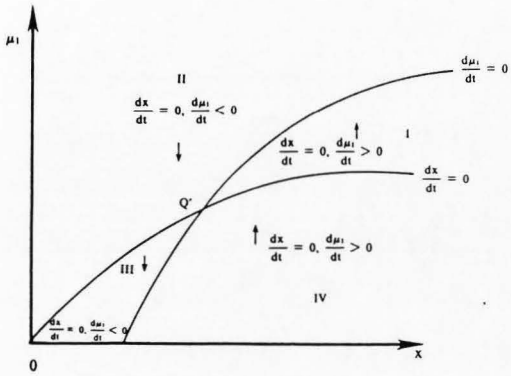

Figure C.5. A phase diagram of stock of renewable resource and its shadow price when $d(\frac{dx}{dt}) = 0$ and $d(\frac{d\mu_1}{dt}) < 0$.

Appendix D

(ii) Given

$$\left.\frac{d\mu_1}{dx}\right|_{\frac{dx}{dt}=0} > 0 \quad \text{and} \quad \left.\frac{d\mu_1}{dx}\right|_{\frac{d\mu_1}{dt}=0} < 0$$

Figures D.1 and D.2 show phase diagrams for a positively sloped time path of stock of resource and a negatively sloped time path of its shadow price under two different conditions.

(ii-b) $d(\frac{d\mu_1}{dt}) > 0$ and $d(\frac{dx}{dt}) < 0$

In other words, given the shapes of the two time paths, the conditions in (b), i.e., the signs in the half spaces, would determine the optimal steady state time path. Figure D.1 shows that there is no optimal steady state time path which is convergent at Q under this condition.

(ii-e) $d(\frac{d\mu_1}{dt}) < 0$ and $d(\frac{dx}{dt}) > 0$

In Figure D.2, v'v' or v"v" is the optimal steady state time path, depending on where the system starts.

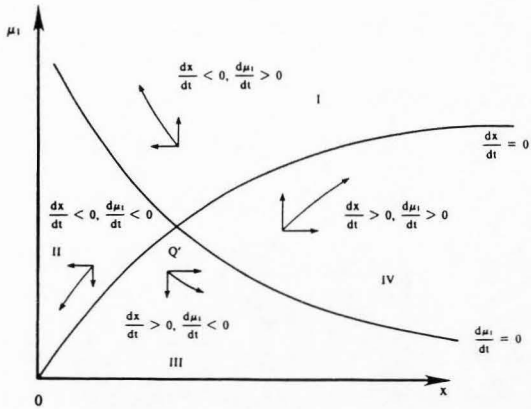

Figure D.1. A phase diagram of stock of renewable resource and its shadow price when $d(\frac{dx}{dt}) < 0$ and $d(\frac{d\mu_1}{dt}) > 0$.

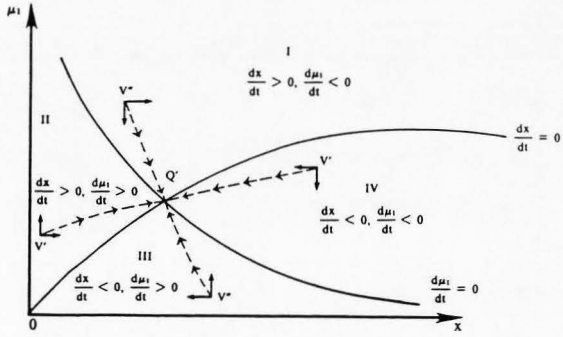

Figure D.2. A phase diagram of stock of renewable resource and its shadow price when $d(\frac{dx}{dt}) > 0$ and $d(\frac{d\mu_1}{dt}) < 0$.

(iii) Given

$$\left.\frac{d\mu_1}{dx}\right|_{\frac{dx}{dt}=0} > 0 \quad \text{and} \quad \left.\frac{d\mu_1}{dx}\right|_{\frac{d\mu_1}{dt}=0} = 0$$

Figures D.3 and D.4 describe the phase diagrams under the following two conditions:

(iii-b) $\quad d(\frac{d\mu_1}{dt}) > 0 \quad$ and $\quad d(\frac{dx}{dt}) < 0$

Here, also, there is no optimal steady state time path which is convergent at Q' (see Figure D.3).

Similarly, for

(iii-e) $\quad d(\frac{d\mu_1}{dt}) < 0 \quad$ and $\quad d(\frac{dx}{dt}) > 0$

It is found out that there exists an optimal steady state time path, v'v' or v"v". Here in all the four quadrants, the vector movements are such that the system will converge to Q' (see Figure D.4). So it can be said that 'Q' is globally stable.

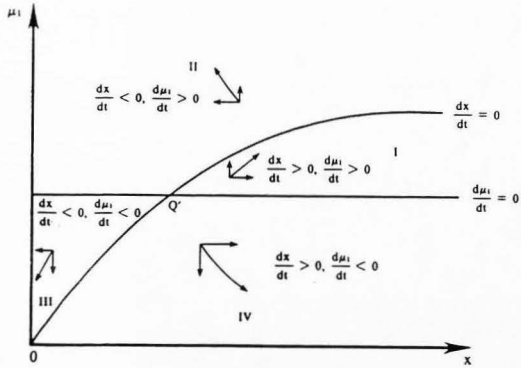

Figure D.3. A phase diagram of stock of renewable resource and its shadow price when $d(\frac{dx}{dt}) < 0$ and $d(\frac{d\mu_1}{dt}) > 0$.

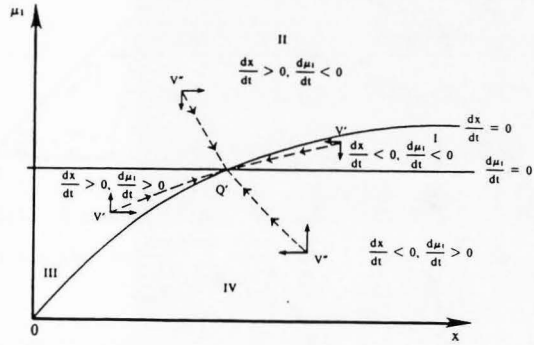

Figure D.4. A phase diagram of stock and the shadow price of a renewable resource when $d(\frac{dx}{dt}) > 0$ and $d(\frac{d\mu_1}{dt}) < 0$.

Appendix E

Under the set of conditions in (i-b),

(i-b) $d(\frac{de}{dt}) > 0$ and $d(\frac{d\mu_2}{dt}) < 0$,

Figure E.1 shows that in two of the four quadrants, the vector movements are convergent and so there does exist an optimal solution for this case.

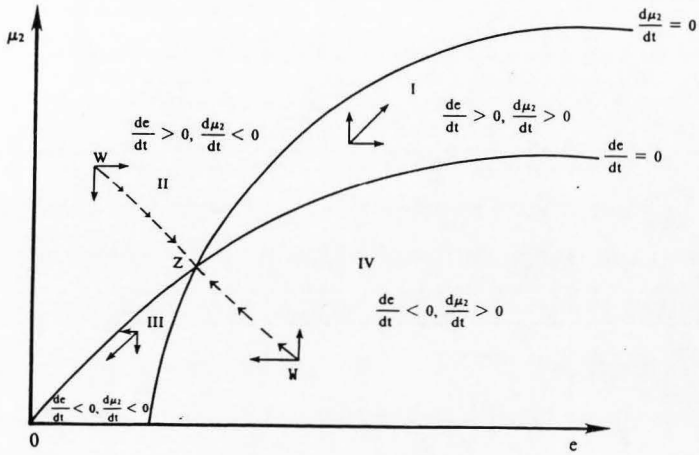

Figure E.1. A phase diagram of stock of environmental resource and its shadow price when $d(\frac{de}{dt}) > 0$ and $d(\frac{d\mu_2}{dt}) < 0$.

Given the conditions in (i-c),

(i-c) $d(\frac{de}{dt}) > 0$ and $d(\frac{d\mu_2}{dt}) = 0$

There does not exist any optimal steady state time path. This is shown in Figure E.2.

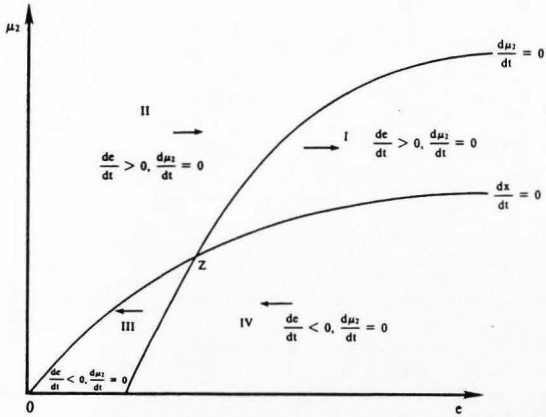

Figure E.2. A phase diagram of the stock and the shadow price of an environmental resource when $d(\frac{de}{dt}) > 0$ and $d(\frac{d\mu_2}{dt}) = 0$.

Given the conditions in (i-d),

(i-d) $\quad d(\frac{de}{dt}) < 0 \quad$ and $\quad d(\frac{d\mu_2}{dt}) < 0$

Figure E.3 shows that there does not exist an optimal steady state time path which is convergent at Z.

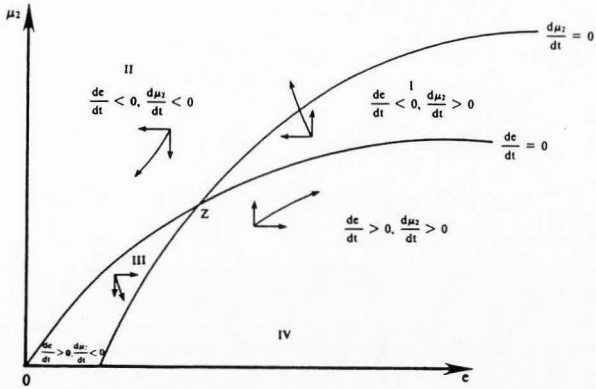

Figure E.3. A phase diagram of stock and the shadow price of an environmental resource when $d(\frac{de}{dt}) < 0$ and $d(\frac{d\mu_2}{dt}) < 0$.

Under the conditions in (i-e),

(i-e) $\quad d(\frac{de}{dt}) < 0 \quad$ and $\quad d(\frac{d\mu_2}{dt}) > 0$

There is an optimal time path for this condition, which is demonstrated in Figure E.4.

Here, again, under the conditions in (i-f),

(i-f) $\quad d(\frac{de}{dt}) < 0 \quad$ and $\quad d(\frac{d\mu_2}{dt}) = 0$

Figure E.5 indicates that there is no vector movement in any quadrant that can lead the system in having an optimal time path in steady state.

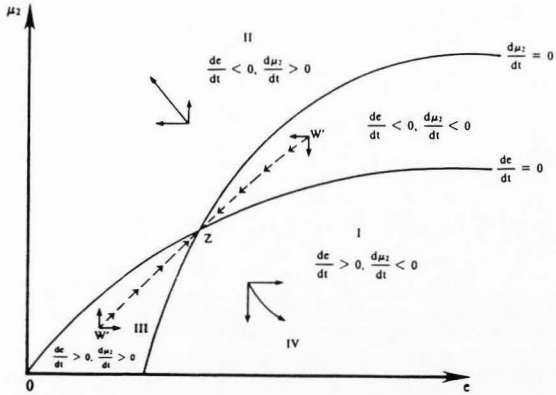

Figure E.4. A phase diagram of stock and shadow price of an environmental resource when $d(\frac{de}{dt}) < 0$ and $d(\frac{d\mu_2}{dt}) > 0$.

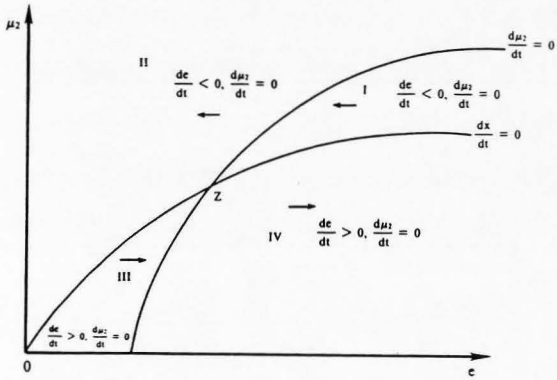

Figure E.5. A phase diagram of stock and shadow price of an environmental resource when $d(\frac{de}{dt}) < 0$ and $d(\frac{d\mu_2}{dt}) = 0$.

Appendix F

(ii) Given

$$\left.\frac{d\mu_2}{de}\right|_{\frac{de}{dt}=0} > 0 \quad \text{and} \quad \left.\frac{d\mu_2}{de}\right|_{\frac{d\mu_2}{dt}=0} < 0 \quad \text{and}$$

Figures F.1 and F.2 show phase diagrams of a positively sloped time path of the stock of environmental resource and a negatively sloped time path of its shadow price under the following two conditions.

(ii-b) $d(\frac{de}{dt}) > 0$ and $d(\frac{d\mu_2}{dt}) < 0$

Then the phase diagram in Figure F.1 shows that there exists optimal steady state time paths w'w' and/or w"w" which are convergent at Z. The

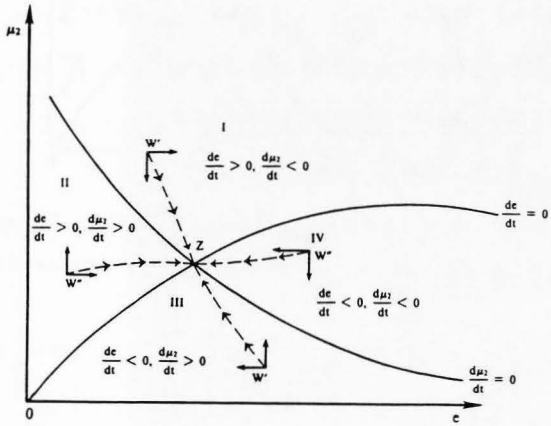

Figure F.1. A phase diagram of stock and shadow price of an environmental resource when $d(\frac{de}{dt}) > 0$ and $d(\frac{d\mu_2}{dt}) < 0$.

optimal steady state stock of the environmental resource and its shadow price is marked at levels corresponding to the point Z.

Again, under the conditions in (ii-e), it is shown in Figure F.2 that there is no optimal steady state time path which is convergent at Z.

(ii-e) $d(\frac{de}{dt}) < 0$ and $d(\frac{d\mu_2}{dt}) > 0$

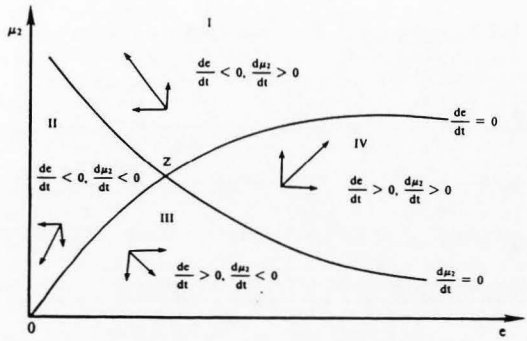

Figure F.2. A phase diagram of stock and shadow price of an environmental resource when $d(\frac{de}{dt}) < 0$ and $d(\frac{d\mu_2}{dt}) > 0$.

(iii) Given

$$\left.\frac{d\mu_2}{de}\right|_{\frac{de}{dt} = 0} > 0 \quad \text{and} \quad \left.\frac{d\mu_2}{de}\right|_{\frac{d\mu_2}{dt} = 0}$$

Figures F.3 and F.4 show phase diagrams of positively and infinitely sloped time paths of stock of environmental resource and its shadow price, respectively. Under the following conditions that

(iii-b) $d(\frac{de}{dt}) > 0$ also $d(\frac{d\mu_2}{dt}) < 0$;

the phase diagram in Figure F.3 shows that there does not exist an optimal steady state time path which is convergent at Z.

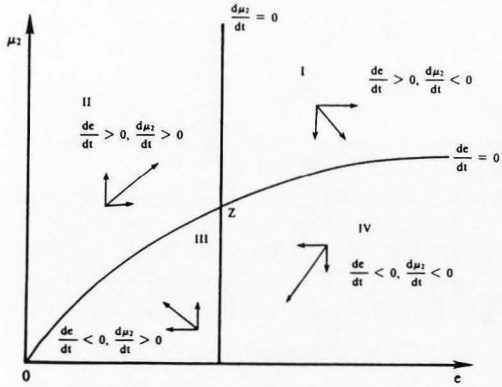

Figure F.3. A phase diagram of stock and shadow price of an environmental resource when $d(\frac{de}{dt}) > 0$ and $d(\frac{d\mu_2}{dt}) < 0$.

Also, under the condition that

(iii-e) $d(\frac{de}{dt}) < 0$ and $d(\frac{d\mu_2}{dt}) > 0$;

here again in Figure F.4, it is shown that there is no optimal steady state time path that is convergent at Z.

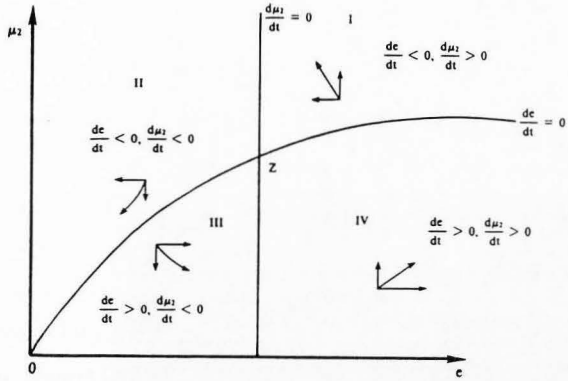

Figure F.4. A phase diagram of stock and shadow price of an environmental resource when $d(\frac{de}{dt}) < 0$ and $d(\frac{d\mu_2}{dt}) > 0$.

Appendix G

The Ito type stochastic differential equation has been used in Chapter VI in order to incorporate uncertainty.

An Ito type stochastic differential equation is composed of a Weiner process, e.g.,

$$dx = f(t, x)dt + \sigma(t, x)dZ \qquad \ldots \qquad (G.1)$$

here Z is a Weiner process. A Weiner process or a Brownian motion process $[Z_t, t\ [o, \infty)]$ is a stochastic process on a probability space $(\Omega, \mathfrak{F}, P)$ with certain properties. However, first define the probability space $(\Omega, \mathfrak{F}, P)$ as a triple, where Ω is a nonempty space of trials, $\mathfrak{F}$ is a σ-field of subsets of Ω representing various events, and P is a probability measure defined on $\mathfrak{F}$.

The properties of the Weiner process are:

i) $Z_0(\omega) = 0$ with probability 1; i.e., the process starts at 0

ii) If $0 \leq t_0 \leq t_1 \leq \ldots \leq t_n$ are time points, then for $H \in R^1$,

$$P[Z_{ti} - Z_{ti-1} \in H_i \text{ for } i \leq n] = \prod_{i \leq n} P[Z_{ti} - Z_{ti-1} \in H_i] \qquad \ldots \qquad (G.2)$$

This implies that the increments of the process $Z_{ti} - Z_{ti-1}$, $i \leq K$ are independent random variables.

iii) For $0 < S < t$, the increment $Z_t - Z_s$ has distribution

$$p[Z_t - Z_s \in H] = \frac{1}{\sqrt{2\pi(t - s)}} \int_H \exp\left(- \frac{x^2}{2(t - s)}\right)dx \qquad \ldots \qquad (G.3)$$

This implies that every increment $Z_t - Z_s$ is normally distributed with mean 0 and variance $\sigma^2(t - s)$.

iv) For each $w \varepsilon \Omega$, $Z_t(\omega)$ is continuous in t, for $t \geq 0$.

Note that condition (ii) reflects a kind of lack of memory. That is, the past history of the process does not influence its future position. The future position of the process depends on its present position but does not depend on how the process got there. Formally, if $0 \leq t_0 < t_1 < t_2 \ldots < t_n < t$, then for real $x, x_0, \ldots, x_n$.

$$P[z_t < x \mid Z_{to} = x_0, \ldots, Z_{tn} = x_n] = P[Z_t \leq x \mid Z_{tn} = x_n] \quad \ldots \quad (G.4)$$

(G.4) is called a Markov property.

Condition (iii) implies that the increments of a Weiner process are stationary in the sense that the distribution of $Z_t - Z_s$ depends only on the difference t - s. From property (i) $Z_0 = 0$ that enables to describe the behavior of increments by claiming that Z_t is normally distributed with $E(Z_t) = 0$ and $E(Z_t^2) = t$. The covariance for $0 \leq s < t$, i.e.,

$$Cov(Z_s, Z_t) = \text{minimum } (t, s) .$$

There is another important thereom about the Weiner process--its nondifferentiability.

For a detailed discussion, see Malliaris and Brock (1982). The important characteristics of the Ito type stochastic differential equation is in Ito's Lemma, which has been applied in deriving the results in Section IV of Chapter III.

Ito's Lemma can be described as:

Let $u(t, x)$: $[0, T] x R^d \rightarrow R^K$ denote continuous nonrandom function such that its partial derivatives U_t, U_{xi}, U_{xixj} are continuous where T is an indexed set and

$$U_t = \frac{\partial}{\partial t} U(t, x) \qquad \ldots \qquad (G.5)$$

$$U_{xi} = \frac{\partial}{\partial xi} U(t, x);\ i = 1, 2, \ldots, d \qquad \ldots \qquad (G.6)$$

$$U_{xixj} = \frac{\partial^2}{\partial xi \partial xj} U(t, x),\ i, j < d \qquad \ldots \qquad (G.7)$$

Suppose that $x(t) = x(t, \omega)$: $[0, T] x \Omega R^d$ is a process with stochastic differential.

$$dx(t) = f(t)dt + \sigma(t)dZ(t) \qquad \ldots \qquad (G.8)$$

where $f(t) = f(t, \omega)$: $[0, T] x \Omega R^d$ is measurable in (t, w), i.e., measurable in both arguments and

$$\sigma(t) = \sigma(t, w): [0, T] x \Omega R^d x R^m .$$

Here σ is a (dxm) matrix and $Z(t) = Z(t, w) x \Omega R^m$ is an m dimensional Weiner process or,

$$\underset{(R^d x R^1)}{dx(t)} = \underset{(R^d x R^1)}{f(t)dt} + \underset{(R^d x R^m)}{\sigma(t)} \ \underset{(R^m x R^1)}{dZ(t)} \qquad \ldots \qquad (G.9)$$

Now let $Y(t) = U(t, x(t))$. Then the process $Y(t)$ has a differential on $[0, T]$ given by

$$dy(t) = U_t dt + U_x dx(t) + \frac{1}{2} tr(\sigma\sigma' U_{xx})dt$$

$$= (U_t + U_x f + \frac{1}{2} U_{xx}\sigma^2)dt + U_x \sigma dZ \qquad \ldots \qquad (G.10)$$

(G.10) is the result of Ito's Lemma which is generally used for stochastic differential equation solutions. Note that in order to obtain the results in equation (G.10), Taylor's Theorem and Ito's multiplication rule, $[(dt)^2 = 0,\ (dZ)^2 = dt;\ dt.dZ = 0]$ has been applied.

The Hamiltonian-Jacobi-Bellman equation (3.4.5) has been obtained as follows:

$$\text{Let } J(t, x, e) \cong \max_{K^E, K^P} E[e^{-rt}\pi(.)\Delta t + J(t + \Delta t, x + \Delta x, e + \Delta e)] \qquad \ldots \quad (G.11)$$

Assuming that J is twice continuously differentiable, expand the function on the right around (t, x, e) by Taylors Series:

$$J(t + \Delta t, x + \Delta x, e + \Delta e) = J(t, x, e) + J_t(t, x, e)\Delta t$$

$$+ J_x(t, x, e)\Delta x + J_e(t, x, e)\Delta e + \frac{1}{2} J_{xx}(t, x, e)(\Delta x)^2$$

$$+ \frac{1}{2} J_{ee}(t, x, e)(\Delta e)^2 + \text{h.o.t.} \qquad \ldots \quad (G.12)$$

Here, h.o.t. stands for higher order terms.

Now, making use of equations (3.4.3) and (3.4.4), i.e.,

$$(\Delta x)^2 = g^2(.)(\Delta t)^2 \qquad \ldots \quad (G.13)$$

and

$$(\Delta e)^2 = h^2(.)(\Delta t)^2 + \sigma^2(\Delta Z)^2 + 2h\sigma\Delta t\Delta Z \qquad \ldots \quad (G.14)$$

Now, using Ito's multiplication rule, equations (G.13) and (G.14) can be expressed as:

$$(\Delta x)^2 = 0 \qquad \ldots \quad (G.15)$$

$$(\Delta e)^2 = \sigma^2 dt \qquad \ldots \quad (G.16)$$

Now, substituting equations (G.15) and (G.16) in equation (G.12) yields:

$$J(t + \Delta t,\ x + \Delta x,\ e + \Delta e) = J(t,\ x,\ e) + J_t(.)\Delta t + J_x(.)\Delta x$$

$$+ J_e(.)\Delta e + \frac{1}{2} J_{ee}(.)\sigma^2 dt \qquad \ldots \quad (G.17)$$

Now, substituting equation (G.17) in (G.11) yields:

$$J(t,\ x,\ e) = \max_{K^E, K^P} E[e^{-rt}\pi(.)\Delta t + J + J_t\Delta t + J_x\Delta x + J_e\Delta e$$

$$+ \frac{1}{2} J_{ee}\sigma^2 dt] \qquad \ldots \quad (G.18)$$

$$= \max E[e^{-rt}\pi(.)\Delta t + J + J_t\Delta t + J_x g\Delta t + J_e h\Delta t$$

$$+ J_e\sigma(x)\Delta Z + \frac{1}{2} J_{ee}\sigma^2\Delta t] \qquad \ldots \quad (G.19)$$

Now, take expectation in equation (G.19), the only stochastic term is ΔZ, and its expectation is zero by assumption. Also, subtract $J(t,\ x,\ e)$ from each side and then divide through Δt and finally let $t \longrightarrow 0$ to get:

$$0 = \max[e^{-rt}\pi(.) + J_t + J_x g + J_e h + \frac{1}{2}\sigma^2 J_{ee}] \qquad \ldots \quad (G.20)$$

By rearranging and multiplying both sides by e^{rt} yields:

$$-J_t e^{rt} = \max[\pi(.) + e^{rt}J_x g(.) + e^{rt}J_e h(.) + \frac{1}{2} e^{rt}\sigma^2(.)J_{ee}]$$

$$\ldots \quad (G.21)$$

which is equivalent to equation (3.4.6).

Appendix H

The translog cost function that has been utilized under two scenarios have made use of the following data set directly from the individual coal-fired plant statistics reported by the Federal Power Commission (FPC) annually.

Output Variable

Y, the final output, is the net generation of electricity in million kilowatt hours for the sampled plants as reported by the FPC annual reports.

Input Prices: Price of Capital

W_K, the price of capital for each of the sampled plants, have been calculated by using the concept of service price of capital of Christensen, Jorgenson, and Lau (1969). Price of capital has been developed by using the formula:

$$P_K = q_K(r + \quad)$$

where q_K is the acquisition cost of capital which is obtained directly from the FPC reports, and r and are the real rate of interest (nominal minus the inflation rate) and the depreciation, respectively.

Price of Labor

W_L, the price of labor, has been calculated for each of the sampled plants in each year by utilizing the average number of employees in each plant and the production expenses, exclusive of fuel, directly obtained from the FPC reports. Finally, in order to come up with hourly wage, Bureau of Labor Statistics reports an average hourly income,

number of hours worked in the utility industry (coal-fired) has been utilized. This has been done by using the following formula:

$$W_L = \frac{a_L \cdot P_{Ex}}{T_h}$$

Here a_L represents the average number of labor in each plant, P_{Ex} is the production expenses for labor, and T_h represents the total number of hours worked by the employees in each plant.

Price of Fuel

W_n, the price of fuel (coal), has been directly taken from the Federal Power Commission reports (1948-1976, 1973). This is just dollars per million b.t.u.

Price of Inputs for Environmental Protection

A fixed proportion (15 percent of the price of capital) has been utilized for the price of inputs for environmental protection. This proportion has been accepted on the basis of reports by Electric Power Research Institute (1973, 1984, 1985a, 1985b), Environmental Protection Agency reports on air quality control, and personal enquiry at the Southern California Edison Company.

Fixed Output

N, the ratio of total production of coal and the consumption by the Electric Utility Industry, has been collected from the Annual Energy Review of the Energy Information Administration (1977-78, 1979-81, 1982).

Cost of Production

C_R, the production cost, has been directly taken from the FPC reports. This is nothing but the total production cost per million b.t.u.

The other variables used in estimating the models under Scenarios I and II are combinations of the above variables. Note that all the variables are used in their natural logarithmic forms.

VITA

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